Origin of the Half-Wavelength Errors in Microwave Measurements Using Through–Line Calibrations

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Abstract—The through–line calibration family uses measurements of a through connection and an inserted line to determine the propagation constant of the line. This is typically used with measurements of a reflect standard to characterize fixturing errors. Subsequently, the S-parameters of a device under test are de-embedded. Measurement uncertainties occur at frequencies where the length of the line standard is an odd multiple of a half-wavelength. The origin of these errors is identified as fixture inconsistencies between the through and the line measurements. Error formulations are developed, and it is shown that a small fixturing error can be accounted for as a multiplicative transmission error. An error sensitivity function is developed and highlights the importance of fixture repeatability.

Index Terms—De-embedding, genetic algorithm, propagation constant, through–line (TL) calibration, through–reflect–line (TRL).

I. INTRODUCTION

The THROUGH–reflect–line (TRL) technique [1] is a calibration procedure used for microwave measurements when classical standards such as open, short, and matched terminations cannot be realized. There are now a family of related techniques, here denoted as through–x–line (TxL) methods, with the commonality being the two-port measurement of a through connection and of the same structure with an inserted transmission line. These two measurements yield the complex propagation constant of the line but have errors that become evident at frequencies where the length of the line is an odd–integer multiple of a half-wavelength, i.e., the critical lengths [2], [3]. These errors affect the entire calibration and subsequent de-embedded measurements. A common frustration is that the size of these errors is seemingly unpredictable. The common strategy to mitigate these errors is to avoid using measurements where the line standard has an electrical length within 20° of a critical length.

It is well-known that the TxL half-wavelength peculiarities exist and are modeled using various approaches. Rolain et al. uses a mixed lumped-distributed perturbation model to ease the detection of errors seen in transmission line quantities [4]. Stumper uses a technique similar to that proposed here where sensitivity coefficients that describe “disturbed” scattering matrices for the calibration standards are derived [5], [6]. Additionally, efforts have been made to provide methods of correction for such errors using iterative and statistical techniques, such as those found in [7] and [8], respectively.

This paper discusses the origin of the half-wavelength errors in the TxL calibration family. It is shown that the errors are a result of limitations on the reproducibility of fixtures between the through and line measurements. An error analysis is presented that highlights the role of fixturing errors. It is shown that the errors can be captured by a transmission error in the fixtures, and the error is reduced if the line is lossy. The effects of fixture port coupling are not addressed in these formulations.

II. THROUGH–LINE CALIBRATION

The through and line measurement structures include fixtures as well as the direct connection (for the through) and the inserted line (for the line) (see Fig. 1). The two-port measurements of these two structures yield the propagation constant $\gamma_L = \alpha + j\beta$ of the line [1]. The structures include error networks cascaded with their associated fixtures. For the through structure, the error networks between the ideal internal port of a...
network analyzer and the desired measurement reference plane
are designated as Fixture A at Port 1 and Fixture B at Port 2.
For the line measurement, fixturing is reestablished (following
the through measurement) with the line inserted. In general, the
fixturing cannot be faithfully reproduced and therefore becomes
Fixtures A’ and B’, respectively (see Fig. 1).
Following the original derivation [1], \( \gamma_L \) is developed using
cascading matrices \( R \), each of which is related to the two-port
scattering parameters \( S \) by
\[
R = \begin{bmatrix}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{S_{12}S_{21} - S_{11}S_{22}}{S_{22}} & \frac{S_{11}}{S_{22}} \\
\frac{S_{21}}{S_{22}} & \frac{1}{S_{22}}
\end{bmatrix}.
\]
(1)
Thus, the parameters describing the A and B fixtures are \( S_A \),
\( S_B \) and \( R_A \), \( R_B \), respectively. The cascading matrix of the
through is simply a unity matrix, and the line of length \( \ell_L \) is
described by
\[
R_L = \begin{bmatrix}
e^{-\gamma_L\ell_L} & 0 \\
0 & e^{\gamma_L\ell_L}
\end{bmatrix}.
\]
(2)
The characteristic impedance of the line is taken as the refer-
cence impedance \( Z_0 \) of the measurement system, however, the
actual value of the line impedance is not required in determin-
ing \( \gamma_L \). The cascading matrix of the through structure is
\[
R_t = R_AR_B \quad \text{(Through)}
\]
(3)
and the line structure (without fixturing errors such that \( R_A' = R_A \)
and \( R_B' = R_B \)) has the following cascading matrix:
\[
R_d = R_AR_d R_B \quad \text{(Line)}.
\]
(4)
Solving (3) for \( R_d \) and substituting into (4) yields what will be
referred to as the through–line (TL) equation, i.e.,
\[
TR_A = R_AR_L
\]
(5)
where
\[
T = R_d R_t^{-1}.
\]
(6)
Expanding the previous equations leads to the following system
of equations:
\[
t_{11}R_{11} + t_{12}R_{12} = R_{11}e^{-\gamma_L\ell_L} \quad \text{(7a)}
\]
\[
t_{21}R_{11} + t_{22}R_{12} = R_{21}e^{-\gamma_L\ell_L} \quad \text{(7b)}
\]
\[
t_{11}R_{12} + t_{12}R_{22} = R_{12}e^{\gamma_L\ell_L} \quad \text{(7c)}
\]
\[
t_{21}R_{12} + t_{22}R_{22} = R_{22}e^{\gamma_L\ell_L} \quad \text{(7d)}
\]
which upon solution yields [1]
\[
\gamma_L = \frac{1}{2\ell_L} \ln \left( \frac{t_{11} + t_{22} \pm \zeta}{t_{11} + t_{22} \mp \zeta} \right)
\]
(8)
where
\[
\zeta = \left( t_{11}^2 - 4t_{11}t_{22} + t_{22}^2 + 4t_{12}t_{12} \right)^{1/2}.
\]
III. INSERTION OF FIXTURE ERROR

Errors in the repeatability of the fixtures can be described by
an inserted two-port network, and three of the possible error
configurations are considered here, as shown in Fig. 1. The
errors are represented variously as follows: in Case (a), as an
asymmetrical error located outside the fixture; in Case (b), as an
asymmetrical error located inside the fixture; and in Case (c), as
dual fixture error located inside the fixtures. In the following,
we solve for the propagation constant incorporating the impact
of various error configurations.

Case (a)

In Case (a) shown in Fig. 1, Fixture A is preceded by a two-
port error with cascading parameters, i.e.,
\[
R_e = \begin{bmatrix}
\Lambda_{A11} & \Lambda_{A12} \\
\Lambda_{A21} & \Lambda_{A22}
\end{bmatrix}
\]
(10)
and the TL equation (5) becomes
\[
TR_A = R_e A R_L
\]
(11)
where
\[
R_e = \begin{bmatrix}
e^{-\Delta} & 0 \\
0 & e^{\Delta}
\end{bmatrix}
\]
(13)
where \( \Delta \) is complex. The quantity \( \Delta \) does not necessarily
 correspond to delay and does not need to vary smoothly with
 respect to frequency. (As expected, \( R_e \) is an identity matrix
 when the error is zero, i.e., \( \Delta = 0 \).) As will be shown in
Section V, this fixturing error fully captures the error as far as
\( \gamma \) extraction is concerned. With (13), (12) becomes
\[
R_e = \begin{bmatrix}
R_{A11}e^{-\Delta} & R_{A12}e^{-\Delta} \\
R_{A21}e^{\Delta} & R_{A22}e^{\Delta}
\end{bmatrix}
\]
(14)
and the TL equations are now given as follows:
\[
e^{\Delta}(t_{11}R_{A11} + t_{12}R_{A21}) = R_{A11}e^{-\gamma_L\ell_L} \quad \text{(15a)}
\]
\[
e^{-\Delta}(t_{21}R_{A11} + t_{22}R_{A21}) = R_{A21}e^{-\gamma_L\ell_L} \quad \text{(15b)}
\]
\[
e^\Delta(t_{11}R_{12} + t_{12}R_{22}) = R_{A12}e^{\gamma_L\ell_L} \quad \text{(15c)}
\]
\[
e^{-\Delta}(t_{21}R_{12} + t_{22}R_{22}) = R_{A22}e^{\gamma_L\ell_L} \quad \text{(15d)}
\]
Solving these as before produces
\[
\gamma_L = \frac{1}{2L} \ln \left( \frac{e^{\Delta t_{11}} + e^{-\Delta t_{22}} \pm \Psi}{e^{\Delta t_{11}} + e^{-\Delta t_{22}} \mp \Psi} \right)
\] (16)

where
\[
\Psi = \left( e^{2\Delta t_{11}^2} - 2t_{11}t_{22} + t_{22}^2 e^{-2\Delta} + 4t_{21}t_{12} \right)^{1/2}.
\] (17)

**Case (b)**

With the error network located inside Fixture A (Case (b) in Fig. 1) and using \( R_e \) from (13), the TL equation becomes
\[
TR_A = R_{Ac}R_L
\] (18)

where
\[
R_{Ac} = R_A R_e.
\] (19)

These error variants can become useful when there is a need to solve for either a single- or dual-ended fixture error. For the single-ended error of Case (b), the equations to solve are given as follows:
\[
\begin{align*}
t_{11}R_{A11} + t_{12}R_{A21} &= R_{A11}e^{-\Delta}e^{-\gamma LL} \quad (20a) \\
t_{21}R_{A11} + t_{22}R_{A21} &= R_{A21}e^{-\Delta}e^{-\gamma LL} \quad (20b) \\
t_{11}R_{A12} + t_{12}R_{A22} &= R_{A12}e^{\Delta}e^{\gamma LL} \quad (20c) \\
t_{21}R_{A12} + t_{22}R_{A22} &= R_{A22}e^{\Delta}e^{\gamma LL}. \quad (20d)
\end{align*}
\]

Solving the system of equations in the same manner as before produces
\[
\gamma_L = \frac{1}{2L} \left[ \ln \left( \frac{t_{11} + t_{22} \pm \zeta}{t_{11} + t_{22} + \zeta} \right) - 2\Delta \right]
\] (21)

where \( \zeta \) is the same as that in (9).

**Case (c)**

Case (c) contains two error networks as follows: the first being that described by (13) and the second being denoted by
\[
R_{e'} = \begin{bmatrix} e^{-\Delta'} & 0 \\ 0 & e^{\Delta'} \end{bmatrix}.
\] (22)

The propagation constant becomes
\[
\gamma_L = \frac{1}{2L} \left[ \ln \left( \frac{-2t_{11}t_{21} - t_{12}t_{11} + t_{12}t_{22} \pm t_{12} \zeta}{-2t_{11}t_{21} - t_{12}t_{11} + t_{12}t_{22} \mp t_{12} \zeta} \right) - 2(\Delta + \Delta') \right].
\] (23)

In summary, in the three cases it is observed that the transmission error introduces an error in the extraction of \( \gamma_L \). In effect, there is an uncertainty in the electrical length.

**IV. Fixture Error Investigation**

For verification of the aforementioned error formulation, a series of 50-\( \Omega \) microstrip lines is fabricated on a 62-mil-thick FR-4 laminate substrate with a dielectric constant of 4.6 and a loss tangent of 0.016. Two sets of measurements were performed yielding errored and nonerrored measurements. In one set, i.e., the nonerrored measurements, the through and line structures were measured with the fixtures reproduced as faithfully as possible. Fixtures consisted of a SubMiniature version A connector and a short section of microstrip line that is approximately 1 cm long. The components of the propagation constant that were extracted from these measurements are shown in Fig. 2 and designated as “no introduced fixture error.” In the second set, which is the errored measurements, the error was deliberately introduced in Fixture A during the line measurement by placing a small piece of substrate on top of the microstrip line that composes Fixture A. This altered the two-port describing the fixture making it dissimilar to that of the through measurement. The components of the propagation constant that were extracted from these measurements are also shown in Fig. 2 and designated as “introduced fixture error.” In both cases, \( \beta \) in Fig. 2(b) is reasonably smooth, but errors are seen...
clearly in $\alpha$ [Fig. 2(a)]. In the measurements with no fixture error, the glitches in $\alpha$ at the half-wavelength frequencies are small. They are greatly exaggerated however with the errored measurements where the fixture error was introduced. While the error is nominally equivalent at all frequencies, the error clearly has the most significant impact at half-wavelength frequencies. The “extracted” results shown in Fig. 2 were produced using an optimization technique with the results of (16). A genetic algorithm was employed to fit the errored propagation constant $\gamma_L$ to a smoothed version of the nonerrored $\gamma_L$, producing a fit with an average error of $2 \cdot 10^{-5}$ and a total error of 0.002. The significance of this good fit is that it indicates that the transmission error described by $R_e$ [(13)] fully describes the fixture error.

V. FIXTURE ERROR CAPTURE

In the TxL calibration procedure, fixturing error is reflected in inaccuracies in extracting fixture scattering parameters. The purpose of this section is to identify the contributions of the transmission errors captured by $\Delta$ to the effective fixture two-port parameters. The errored fixture is defined in terms of $S$-parameters in the following form:

$$\begin{bmatrix} S_{11}\alpha (1 + \delta_{11}\alpha) & S_{12}\alpha (1 + \delta_{12}\alpha) \\ S_{21}\alpha (1 + \delta_{21}\alpha) & S_{22}\alpha (1 + \delta_{22}\alpha) \end{bmatrix}$$

(24)

or as a cascading matrix as

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

(25)

where

$$R_{11} = \frac{S_{A12}(1 + \delta_{A12})S_{A21}(1 + \delta_{A21})}{S_{A21}(1 + \delta_{A21})}$$

$$= \frac{S_{A11}(1 + \delta_{A11})S_{A22}(1 + \delta_{A22})}{S_{A21}(1 + \delta_{A21})}$$

$$R_{12} = \frac{S_{A11}(1 + \delta_{A11})}{S_{A21}(1 + \delta_{A21})}$$

$$R_{21} = -\frac{S_{A22}(1 + \delta_{A22})}{S_{A21}(1 + \delta_{A21})}$$

$$R_{22} = \frac{1}{S_{A21}(1 + \delta_{A21})}.$$  

It is in this form that we ultimately want to represent the fixture error described by $R_{\alpha e}$ or $R_{\alpha a}$. For Case (a), the desired error term $R$ in (25) is equated to the cascading fixture error term $R_{\alpha e}$, leading to

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} e^{-\Delta}(S_{A12}S_{A21} - S_{A11}S_{A22}) & e^{-\Delta}S_{A11} \\ S_{A21}e^{\Delta}S_{A22} & S_{A21} \end{bmatrix}.$$  

(26)

This produces the following governing relationship between the errored fixture $S$-parameters and the cascading fixture error term $R_{\alpha e}$ [(12)]:

$$\begin{bmatrix} \delta_{A11} & \delta_{A12} \\ \delta_{A21} & \delta_{A22} \end{bmatrix} = \begin{bmatrix} e^{-2\Delta} - 1 & e^{-\Delta} - 1 \\ e^{-\Delta} - 1 & 0 \end{bmatrix}. $$

(27)

When the fixture error term $R_e$ is placed inside the fixture, as in Cases (b) and (c), then the matrix that relates $R_{\alpha o}R_e$ to the errored fixture $S$-parameters becomes

$$\begin{bmatrix} \delta_{A11} & \delta_{A12} \\ \delta_{A21} & \delta_{A22} \end{bmatrix} = \begin{bmatrix} 0 & e^{-\Delta} - 1 \\ e^{-\Delta} - 1 & e^{-2\Delta} - 1 \end{bmatrix}. $$

(28)

These formulations are required to associate the cascade error matrices of (16) and (21) to the errored $S$-parameters of the fixture. The resultant complex error quantities produced by the genetic algorithm needed to de-embed the nonerrored solution in terms of $\Delta$, $\delta_{11}$, $\delta_{12}$, and $\delta_{21}$ are shown in Fig. 3. This demonstrates that the transmission error represented by (13) fully captures the errors in fixture inconsistencies between the through and line measurements.
VI. SENSITIVITY ANALYSIS

The errors in TxL calibration result in significantly enhanced errors at frequencies corresponding to the length of the line being multiples of a half-wavelength. We explore this further by developing the sensitivity of the extracted propagation constant. This is performed by analyzing how $\gamma_L$ in (16) varies as a function of $\Delta$ by taking the derivative with respect to the fixture error, producing the error sensitivity function (ESF) as follows:

$$\text{ESF} = \frac{\partial \gamma_L}{\partial \Delta} = \frac{4}{l \Psi} \left( t_{11} e^{\Delta} - t_{22} e^{-\Delta} \right) \left( t_{11} t_{22} - t_{21} t_{12} \right) \left( -t_{11} e^{\Delta} - t_{22} e^{-\Delta} + \Psi \right).$$

(29)

ESF is a function of $\Delta$ and can be separated into its real and imaginary components as follows: $\alpha$ ESF and $\beta$ ESF. In Fig. 4, the magnitude of $\delta$ is varied while the angle is kept constant at 0 rad so that the sensitivity of $\gamma_L$ to error can be accessed. Additionally, in Fig. 5 the magnitude of $\delta$ is held constant at 0.1 while the angle is varied from 0 to $\pi$. The ESF is obviously dependent on the measured quantities $T$. Figs. 4 and 5 demonstrate a dramatic increase in sensitivity to transmission error at frequencies corresponding to multiples of a half-wavelength.

VII. CONCLUSION

It has been shown how fixture variation in the transition from the through to the line in TL calibration procedures can lead to the well-known $\lambda/2$ error. It was shown that the impact of errors in fixture reproducibility on propagation constant extraction can be accounted for by a transmission error. An ESF that allows the sensitivity of $\gamma_L$ to fixture variation error to be assessed has been presented. This also indicates a high sensitivity to fixture errors at frequencies corresponding to multiples of a half-wavelength.

REFERENCES

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