Simulation of Nonlinear RF Circuits Driven By Multi-Carrier Modulated Signals

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Abstract — **The simulation of nonlinear RF circuits driven by digitally-modulated signals with multiple carriers is addressed by combining the Envelope Transient Harmonic Balance Method with the Artificial Frequency Mapping Technique. Specific application examples are considered, in particular the modeling of OFDM-based communications circuits. A mixer subject to a modulated RF signal and an amplifier excited by a multi-carrier modulated stimulus are studied. Here an envelope time-marching scheme is coupled to an artificial frequency mapping based harmonic balance engine.**

Index Terms — **Nonlinear differential equations, Nonlinear distortion, Circuit simulation.**

I. INTRODUCTION

The need for increased wireless data transmission rates, and thus, greater spectral efficiency and larger bandwidths, is driving the application of new types of modulation formats. Many of these use multi-carrier modulation schemes as is the case of Orthogonal Frequency Division Multiplexing, OFDM. More efficient utilization of RF hardware is leading to multi-carrier systems and in particular multicarrier CDMA [1]. Beyond that, the use of the same RF physical channels by different modulated signals, such as GSM and UMTS, or the desired coexistence of mobile telephony and position/localization services in the same handset, are imposing new nonlinear RF/microwave simulation challenges. New concepts such as MIMO-enhanced GSM (which can be treated as multi-carrier communications) also impose modeling challenges which are beyond the capabilities of current simulators. Addressing the demands resulting from the above systems is the focus of this paper.

One of the more important advances in modeling digitally modulated signals is the development of the Envelope Transient Harmonic Balance (ETHB) techniques [2-8] which was initially developed to handled an slowly modulated single RF carrier. It has since been extended in commercial simulators [9] to handle two or more carriers although the technique has not been described. This is achieved by combining multi-tone Harmonic Balance (HB) with transient solvers. Unfortunately, the multi-rate nature of the problem is kept, and the complexity is still very high. In the various modeling problems outlined the stimulus is composed of several un-modulated and modulated RF carriers.. Therefore, even using a multi-rate simulator, we no longer would have a single time-domain envelope and frequency-domain carrier, as in ETHB, but a possibly very large series of modulations and carriers.

The main aim of this paper is to combine the multi-tone Artificial Frequency Mapping Techniques, AFMT, with ETHB and so permit rapid and memory-efficient modeling of multi-carrier digital communication systems. In this paper the theoretical generalization of the ETHB and AFMT algorithm will be presented first, and then it will be applied for two special RF/microwave cases of practical significance: a mixer whose RF input is driven by a modulated signal, and a simplified nonlinear amplifier excited by the newly multi-carrier modulated signals (OFDM).

Section II addresses the mathematical formulation underlying this generalization, while Sections III discusses the implementation details. Finally, Section IV is devoted to some simulation examples.

II. THEORETICAL FORMULATION

In general, an RF circuit can be modeled by a system of ordinary differential equations in time:

$$\mathbf{i}[\mathbf{y}(t)] + \frac{d\,\mathbf{q}[\mathbf{y}(t)]}{d\,t} = \mathbf{x}(t) \tag{1}$$

where $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are the excitation and the state-variable vectors, respectively, $\mathbf{i}[\mathbf{y}(t)]$ represents memoryless linear or nonlinear elements, and $\mathbf{q}[\mathbf{y}(t)]$ models memoryless linear or nonlinear charges (capacitors) or fluxes (inductors). In the present case, $\mathbf{x}(t)$ is assumed to be a composite signal where (up to) N different envelope signals modulate N carriers:

$$\mathbf{x}(t) = \mathbf{X}_0 + \sum_{n=1}^{N} \left[\mathbf{x}_{m_n}(t) \left(e^{-j\omega_n t} + e^{j\omega_n t} \right) \right] \tag{2}$$

In its highest level of generality these carriers of frequency ω_n can either be correlated or uncorrelated with each other, or with any of the modulating envelopes, $x_{m_n}(t)$. If the envelope time and frequency variables, $t_{en} \leftrightarrow \Omega_n$ and $t_{cn} \leftrightarrow \omega_{cn}$ can be considered independent from each other, the stimulus and state vectors become dependent on these $2N$ different time-scales and the nonlinear ODE of (1) is turned into a multi-rate partial differential equation, MPDE, [10]:

$$\mathbf{i}[\mathbf{y}(t_{e1}, t_{e2}, \cdots, t_{eN}, t_{c1}, t_{c2}, \cdots, t_{cN})]$$
$$+ \frac{\partial \mathbf{q}[\mathbf{y}(t_{e1}, \cdots, t_{eN}, t_{c1}, \cdots, t_{cN})]}{\partial t_{e1}} + \cdots$$
$$+ \frac{\partial \mathbf{q}[\mathbf{y}(t_{e1}, \cdots, t_{eN}, t_{c1}, \cdots, t_{cN})]}{\partial t_{cN}} + \cdots \qquad (3)$$
$$= \mathbf{x}(t_{e1}, \cdots, t_{eN}, t_{c1}, \cdots, t_{cN})$$

This MPDE can now either be solved in the time-domain, using a $2N$-dimensional time-marching scheme, in frequency-domain using an appropriate HB algorithm, or in any combination of time and frequency using a mixed-mode technique. The mixed-mode approach has proven to be particularly attractive for modeling communications circuits with digitally-modulated carriers.

In many multi-carrier communication systems there are only a few master clocks, and so in a large number of situations of practical interest the number of orthogonal time scales can be significantly reduced from the original $2N$. Furthermore, since the information envelopes are necessarily aperiodic while the carriers are periodic, and the period of the envelopes is several orders of magnitude longer than that of the carriers, for all practical situations (3) must be solved using a mixed-mode method. In this method the envelopes are represented by a succession of time samples (eventually separated by constant or dynamic time-steps) and the carriers are described by a vector of complex Fourier coefficients. This naturally leads us to a multi-dimensional Fourier transform-based harmonic balance algorithm, MDFT-HB [11], for the simulation of the carriers, and a multi-dimensional time-step integration for the envelopes. Nevertheless the use of MDFT-HB is known to be very costly in both simulation time and memory utilization and thus its use is generally prohibitive for situations with more than two modulated carriers.

Following the procedure with the conventional single modulated carrier ETHB, the modulated multi-carrier signals will thus be represented as a series of envelope time-dependent Fourier coefficients [5].

When such an time-varying envelope frequency-domain representation is substituted into (3) the MPDE becomes:

$$\mathbf{I}(t_{e1}, \cdots, t_{eN}, k\omega_{c1}, \cdots, k\omega_{cN})$$
$$+ \frac{\partial \mathbf{Q}[\mathbf{Y}(t_{e1}, \cdots, t_{eN}, k\omega_{c1}, \cdots, k\omega_{cN})]}{\partial t_{e1}} + \cdots \qquad (4)$$
$$+ jk\omega_{c1}\mathbf{Q}(t_{e1}, \cdots, t_{eK}, k\omega_{c1}, \cdots, k\omega_{cN}) + \cdots$$
$$= \mathbf{X}(t_{e1}, \cdots, t_{eN}, k\omega_{c1}, \cdots, k\omega_{cN})$$

in which $\mathbf{I}(t_{en}, k\omega_{cn})$, $\mathbf{Q}(t_{en}, k\omega_{cn})$, $\mathbf{Y}(t_{en}, k\omega_{cn})$ and $\mathbf{X}(t_{en}, k\omega_{cn})$ are the t_{en} time varying Fourier components of the memoryless nonlinearities, the state variables, and the excitation, respectively. The discretization of (4) using the backward Euler rule leads to the following system of difference equations in the above Fourier coefficients:

$$h_{1_i} \cdots h_{N_i} \mathbf{I}(t_{e1_i}, \cdots, t_{eN_i}, k\omega_{c1}, \cdots, k\omega_{cN})$$
$$+ h_{2_i} \cdots h_{N_i} \mathbf{Q}[\mathbf{Y}(t_{e1_i}, \cdots, t_{eN_i}, k\omega_{c1}, \cdots, k\omega_{cN})] + \cdots$$
$$+ h_{1_i} \cdots h_{N_i} j\mathbf{\Omega}_{c1} \mathbf{Q}(t_{e1_i}, \cdots, t_{eN_i}, k\omega_{c1}, \cdots, k\omega_{cN}) + \cdots$$
$$= h_{1_i} \cdots h_{N_i} \mathbf{X}(t_{e1_i}, \cdots, t_{eN_i}, k\omega_{c1}, \cdots, k\omega_{cN})$$
$$+ h_{2_i} \cdots h_{N_i} \mathbf{Q}[\mathbf{Y}(t_{e1_{i-1}}, \cdots, t_{eN_i}, k\omega_{c1}, \cdots, k\omega_{cN})] + \cdots$$
$$\qquad (5)$$

The proposed mixed-mode method operates by integrating (5) in a t_{e_i} time-step by time-step basis (h_{n_i}), starting from the initial conditions $\mathbf{X}(t_{e1_0}, \ldots, t_{eN_0}, k\omega_{c1}, \ldots, k\omega_{cN})$ and $\mathbf{Y}(t_{e1_0}, \ldots, t_{eN_0}, k\omega_{c1}, \ldots, k\omega_{cN})$ and solving for each of the successive time-samples t_{en_i} using a frequency-domain (HB) algorithm. Here the frequency-domain HB solution for the carriers is efficiently solved using AFMT.

In the above most general formulation, orthogonal time scales were considered for each envelope and carrier signals. However, in many real situations, as in multi-carrier modulation formats such as OFDM, the carriers can share the same reference. In most telecommunication systems, even the different envelope bit streams share a common clock reference and bandwidth. This can provide a significant reduction in the problem dimensionality, allowing for particularly efficient use in many practical situations.

In the following, the presented method will be illustrated by two special implementations. The first one is the simulation of an RF circuit with a multi-carrier signal. The second is a mixer subject to a modulated RF signal. It will be treated as a two-input nonlinear system where one of the high frequency carriers is modulated (the RF input) while the other is a CW signal (the local oscillator, LO).

III. MULTI-CARRIER IMPLEMENTATION

One particular scenario where the number of independent time scales can be significantly reduced is the simulation of circuits excited by an OFDM signal. As this modulation format uses equally separated carriers, the HB analysis can be carried out in a much more efficient

manner using an appropriate AFMT technique, AFM-HB [12]. Furthermore, since all independent carrier envelopes share the same clock time base, the envelope time-step integration can be performed in a synchronous way.

So, a judicious use of AFM allows multi-dimensional HB to be converted into a one dimensional, or sinusoidal, HB implementation [12], while the synchronous time-step integration turns the multi-dimensional time-domain analysis into a one-dimensional initial condition problem (although still in N different envelopes).

Accordingly, the N-envelope time variables are described by a single t_e time-base, discretized in a single time-step, h_i. The N carriers are represented by a central RF frequency, ω_c, and a constant carrier separation, $\Delta\omega$. After the appropriate AFM transformation, these two frequency-domain variables are mapped onto the harmonics of a single artificial frequency, λ, and (5) becomes:

$$h_i.\mathbf{I}(t_{e_i}, k\lambda) + \mathbf{Q}\left[\mathbf{Y}(t_{e_i}, k\lambda)\right] + h_i.j\mathbf{\Lambda}\,\mathbf{Q}(t_{e_i}, k\lambda)$$
$$= h_i.\mathbf{X}(t_{e_i}, k\lambda) + \mathbf{Q}\left[\mathbf{Y}(t_{e_{i-1}}, k\lambda)\right] \quad (6)$$

This demonstrates that any complex OFDM scheme can be simulated using a slightly modified version of the conventional ETHB, combining a conventional sinusoidal HB with a time-step integration engine.

IV. ILLUSTRATIVE APPLICATION EXAMPLES

To illustrate the practical capabilities of the proposed algorithms, the response of the circuit shown in Fig. 1 [5], will now be studied for the two addressed cases, the mixer and the multi-carrier modulated signal. The nonlinear circuit of Fig. 1 is simply a transfer nonlinearity that drives an output parallel resonant circuit. It can be viewed as a behavioral model or a very simplified representation of either an output tuned FET-based amplifier, or gate mixer.
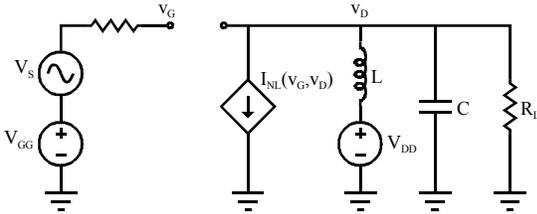
Fig. 1: Nonlinear circuit example.

a) Mixer Test

The mixer configuration considered is a gate mixer in which both the RF and LO signals are applied to the gate. It will behave as a down converter. The circuit excitation is the sum of a DC bias voltage, V_{GG}, plus a (Binary Phase Shift Keying (BPSK)-modulated RF carrier, and a CW LO, i.e:

$$v_s(t_e, t_{c1}, t_{c2}) = V_s + v_m(t_e)\left(e^{-j\omega_{c1}t_{c1}} + e^{j\omega_{c1}t_{c1}}\right) +$$
$$+ V_{LO}\left(e^{-j\omega_{c2}t_{c2}} + e^{j\omega_{c2}t_{c2}}\right) \quad (8)$$

The RF carrier frequency is at f_{c1} = 2.1GHz, and the LO at f_{c2} = 2GHz while the output band-pass IF filter is centered at 100MHz. The modulating signal waveform, $v_m(t_e)$, is the pseudo-random sequence shown in Fig. 2(a), which corresponds to the eye-diagram plotted in Fig. 2(b).

a)   b)
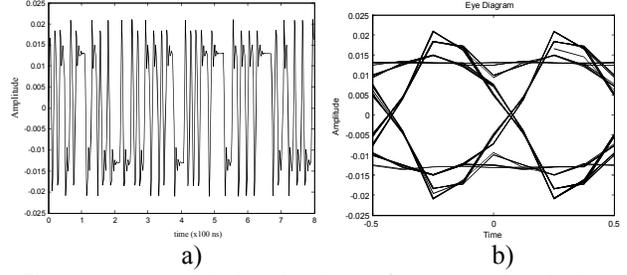
Fig. 2 – Input modulation signal waveform, a), and excitation eye-diagram, b).

Fig. 3. depicts the output eye diagram of the IF signal, while Fig. 4 presents the resulting simulated spectrum around the IF carrier.
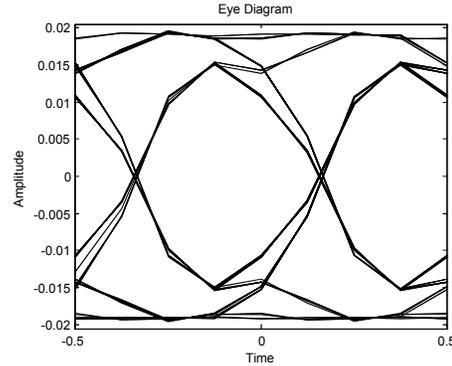
Fig. 3 – Output eye-diagram at the IF frequency, 100MHz.
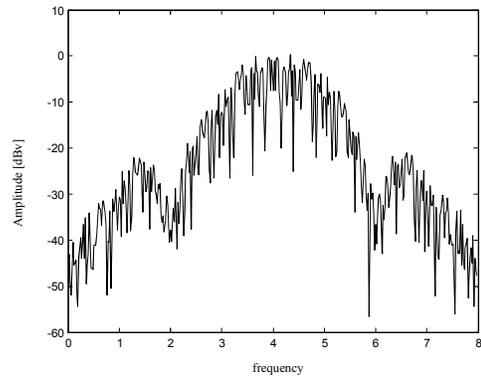
Fig. 4 – Simulated output spectrum centered at the IF carrier.

b) Multi-Carrier Modulated Amplifier Test

In the second case the circuit of Fig.1 was biased, and the output filter tuned, so that it behaves as an amplifier. The input excitation is a multi-carrier (5 tones) modulated signal centered at 2GHz, and with a frequency separation of 100kHz. Each of the five carriers is modulated by one different envelope signal. So, the excitation is:

$$v_s(t_{e1},\ldots,t_{e5},t_c) = V_s + v_{m1}(t_{e1})\left(e^{-j\omega_{c1}t_{c1}} + e^{j\omega_{c1}t_{c1}}\right)$$
$$+ \cdots + v_{m5}(t_{e5})\left(e^{-j\omega_{c5}t_{c5}} + e^{j\omega_{c5}t_{c5}}\right)$$

(9)

where $\omega_{c_n} = \omega_{c1} + n\Delta\omega$. Fig. 5 presents the output envelope of each fundamental carrier.
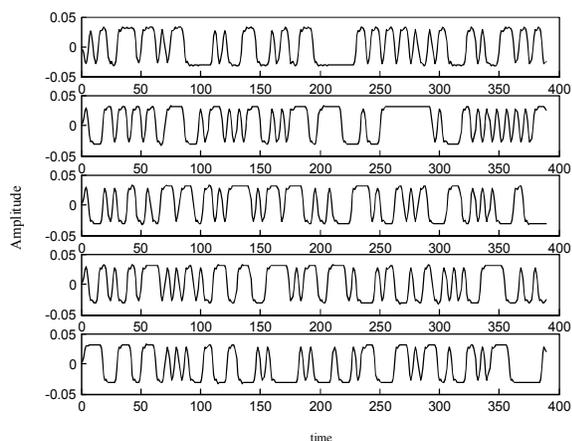
Fig. 5 – Output envelope signals at each carrier.

V. Conclusion

In this paper a combination of the multi-envelope transient harmonic balance algorithm with AFMT is proposed. However, to render this generalization practical, the number of independent time/frequency variables must be reduced. The algorithm takes advantage of particular properties of the stimulus to couple a conventional one-dimensional time-step integration scheme with the artificial frequency mapping techniques based HB. To illustrate the methods capabilities, two examples of practical relevance in the microwave and wireless fields were studied: a mixer driven by a CW LO and modulated RF signal; and an amplifier excited by an OFDM digital modulated multi-carrier signal.

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