Statistical Modeling of Cross Modulation in Multichannel Power Amplifiers Using a New Behavioral Modeling Technique

Khaled Gharaibeh and Michael Steer
Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, NC 27695-7914, USA, email: {kmgharai,mbs}@ncsu.edu

Abstract - **A statistically based analysis technique is developed for multichannel amplifier environments enabling in-band and out-of-band intermodulation and cross modulation distortions to be predicted with very little computation. This is coupled with the introduction of a multiple envelope behavioral modeling technique that captures the black box characteristics of multichannel amplifiers. A CDMA two channel amplifier with a two-channel signal is characterized experimentally and verifies the approach presented here.**

I INTRODUCTION

The nonlinear interaction of multiple signals, or cross modulation, is a fundamental performance limiting phenomena in many wireless environments. The same phenomenon is responsible for co-site interference, intentional and unintentional jamming, distortion in multi-carrier systems, and limits what can be achieved with multifunctional systems. This paper specifically focuses on distortion in multichannel amplifiers which are a preferred approach to generating high power signals in base stations. Two RF architectures yield high power multiple channel signals as shown in Fig. 1 [1]. In the traditional configuration, shown in Fig. 1(a), the individual channel are applied to narrowband power amplifier and then the output of each combined to obtain a multichannel high power signal. Generally zonal filters are used with each narrowband power amplifier prior to power combining to eliminate out-of-band distortion and particularly to eliminate the presentation of undesired signals to the output nonlinearity of the amplifiers. Multichannel amplifiers amplify several channels at once, see Fig. 1(b), and can be expected to be a much lower cost and smaller way of generating the high power composite signal. However it is difficult to achieve the low levels of distortion required in advanced wireless systems such as wideband CDMA. One critical problem is that the Peak to Average Ratio (PAR) is greater when there are multiple channels than when there is a single channel. Thus saturation and limiting effects occur at lower average power levels than when there is a single channel signal. A second problem that arises is that cross modulation and intermodulation result from the interaction of the multiple channels in a nonlinear environment. These manifest themselves as extra in-band and out-of-band distortions.

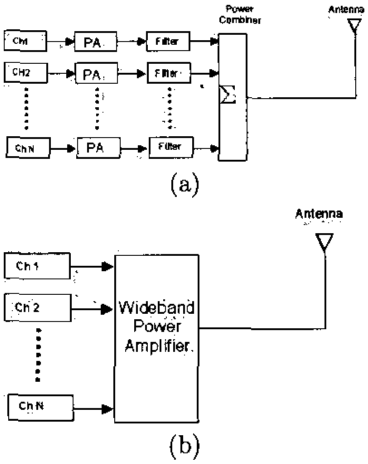


Figure 1: Traditional, (a), multiple single channel amplifier; and, (b), multichannel amplifier systems.

The traditional way for characterizing distortion in multichannel power amplifiers is to test the amplifier with n tones and examine the intermodulation capability of the power amplifier [2]. However, this cannot accurately characterize cross modulation when the amplifier is driven by digitally modulated signals.

This paper extends an analysis for estimating single channel amplifier distortion [3] to a multiple-envelope statistical technique for estimating distortion in multichannel amplifiers. A general autocorrelation function is presented for the output signal when the input is the sum of two or more digitally modulated carriers passed through a memoryless nonlinear model. As well as presenting the new statistical model, a novel multiple-envelope behavioral model is presented which captures the interaction of multiple signals (or envelopes). This cross-interaction effect is not captured in behavioral models developed by single envelope-based characterizations.

II MULTI ENVELOPE BEHAVIORAL MODEL

A memoryless model accurately characterizes a nonlinear system (or device) when it has no significant memory within the signal bandwidth. Such a model can always be represented as a complex power series:

$$y(t) = \sum_{n=1}^{N} a_n w^n(t) \tag{1}$$

where a_n is the n-th instantaneous complex power series coefficient, $y(t)$ is the output waveform, and $w(t)$ is the input signal. The nonlinear gain characteristic above may not always be the most efficient form to represent a bandpass nonlinearity but it is sufficient. The model applies to arbitrary $w(t)$ and so the model (1), is applicable to both sinusoids and digitally modulated signals. The key modeling issue here is developing the coefficients from envelope measurements such as AM-AM but this has been addressed previously for one-dimensional behavioral models [4]. These concepts are extended to establish a model that includes cross modulation effects.

Consider an input consisting of two modulated carriers $w_1(t)$ and $w_2(t)$ applied to an amplifier so that $w(t) = w_1(t) + w_2(t)$. The signals $w_1(t)$ and $w_2(t)$ are modulated RF carriers with center frequencies $\omega_1(t)$ and $\omega_2(t)$ respectively. Let $z(t)$ and $u(t)$ be the complex envelopes of $w_1(t)$ and $w_2(t)$ respectively, then the sum signal can be written in complex envelope form as:

$$\begin{aligned} w(t) &= w_1(t) + w_2(t) \\ &= A_1(t)cos(\omega_1 t + \theta_1) + A_2(t)cos(\omega_2 t + \theta_2) \\ &= \tfrac{1}{2}[z(t)e^{j\omega_1 t} + z^*(t)e^{-j\omega_1 t} \\ &\quad + u(t)e^{j\omega_2 t} + u^*(t)e^{-j\omega_2 t}] \end{aligned} \tag{2}$$

Defining the nonlinear model using the input-output relationship (1), and applying the signal in (2) and using multinomial expansion, yields:

$$w^n(t) = \frac{1}{2^n} \sum_{n_1+n_2+n_3+n_4=n} \frac{n!}{n_1!n_2!n_3!n_4!} \times [z^{n_1} {z^*}^{n_2} u^{n_3} {u^*}^{n_4} e^{j(n_1-n_2)\omega_1 t} e^{j(n_3-n_4)\omega_2 t}]$$

Considering the components of the output signal centered at the first carrier, the output complex envelope can be expressed as:

$$\tilde{y}^{(1)}(t) = zG(|z|, |u|) \tag{3}$$

where G is the complex gain compression function and is a function of the levels of both signal envelopes; it can be written as:

$$G(|z|, |u|) = \sum_{n=1}^{N} \sum_{k=0}^{\frac{n-1}{2}} b_{n,k} |z|^{n-2k-1} |u|^{2k} \tag{4}$$

Where

$$b_{n,k} = \frac{a_n}{2^{n-1}} M(n,k)$$

$$M(n,k) = \binom{n}{\frac{n+1}{2} - k, k, k}$$

and

$$\binom{n}{n_1, n_2, \ldots, n_p} = \binom{n}{n_1}\binom{n-n_1}{n_2} \cdots \cdots \binom{n - n_1 - \ldots - n_{p-1}}{n_p}$$

is the multinomial coefficient. The above expression for the gain function is derived assuming that $\omega_2 - \omega_1 > 2B$ and $\omega_2 < 2\omega_1$ so that intermodulation products do not lie inside the bandwidth of each of the input signals. The new set of coefficients $b_{n,k}$ represents the relationship between the input and the output complex envelopes of the first carrier. The coefficients are obtained using a new two-tone test. The idea is that the input power of the first tone is swept while the power of the second tone is fixed at a certain power level. The process is repeated for each power step of the second tone. In this way multiple curves for the nonlinear characteristics (AM-AM and AM-PM conversions) are obtained. A two-dimensional (2D) curve fitting yields the set of coefficients $b_{n,k}$(the number of coefficients is equal to $[\frac{N+1}{2} \times \frac{N+3}{4}]$.

The two dimensional complex power series coefficients $b_{n,k}$ are obtained using a Vector Network Analyzer (VNA) to extract the AM-AM and AM-PM characteristics at $f_1 = 2.1$ GHz of a wideband C-band amplifier. In addition, an external 2.0 GHz signal source

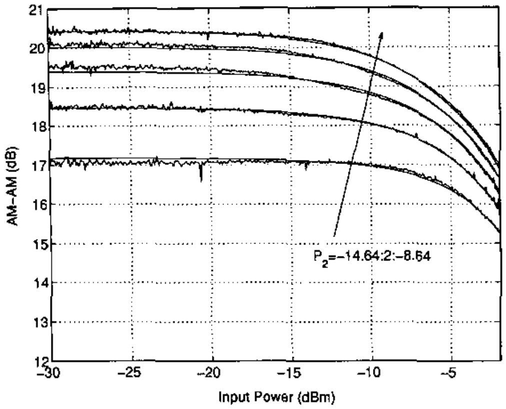


Figure 2: Measured and Fitted AM-AM characteristics.

injects a second tone at frequency f_2 at various power levels. The AM-AM and AM-PM characteristics at f_1 are determined at each power level of the second source. The power of the VNA signal was swept between -30 dBm to 0 dBm while the power of the second tone was swept manually between -20 dBm and 0 dBm in a 0.5 dB steps. The nonlinear gain characteristics were fitted using least square polynomial fitting. Fig. 2 shows the AM-AM characteristics for each power level of f_2 and their corresponding polynomial fit.

III Statistical Analysis

With a digitally modulated signal, distortion introduced by nonlinearities is measured as the amount of spectral regrowth. In [3] Gard *et. al.* developed a generalized autocorrelation formulation for a single CDMA signal. Here we extend the analysis to the case of multiple channels. The output autocorrelation function at the first carrier is defined as:

$$R_{yy}^{(1)}(\tau) = E[\tilde{y}^{(1)}(t)\tilde{y}^{(1)}(t+\tau)]$$

Now, let $z_1 = z(t), z_2 = z(t+\tau), u_1 = u(t)$ and $u_2 = u(t+\tau)$ then, the autocorrelation function of the signal at the first carrier frequency is:

$$R_{yy}^{(1)}(\tau) = E[\tilde{y}^{(1)}(z_1, u_1)\tilde{y}^{*^{(1)}}(z_2, u_2)]$$

Using (3) , the output autocorrelation can be formulated as:

$$R_{yy}^{(1)}(\tau) = \sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=0}^{\frac{n-1}{2}}\sum_{k=0}^{\frac{m-1}{2}} b_{n,l}b_{m,k}^{*}R_{z_n z_m u_l u_k}(\tau)$$

Where $R_{z_n z_m u_l u_k}(\tau)$ is defined as:

$$R_{z_n z_m u_l u_k}(\tau) =$$
$$E[z_1^{\frac{(n+1)}{2}-l} z_1^{*^{\frac{(n-1)}{2}-l}} \quad z_2^{\frac{(n-1)}{2}-k} z_2^{*^{\frac{(n+1)}{2}-k}} u_1^l u_1^{*^l} u_2^k u_2^{*^k}]$$

The above expression reduces to the single channel formulation as in [3] when $u(t) = 0$ and setting k, l to 0. The autocorrelation function of stationary random processes can be obtained from their time averages assuming ergodicity [3]. Therefore: The power spectrum of the output signal can be found from the Fourier transform of the autocorrelation function as:

$$S_{yy}^{(1)}(f) = \sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{l=0}^{\frac{n-1}{2}}\sum_{k=0}^{\frac{m-1}{2}} b_{n,l}b_{m,k}^{*}S_{nmlk}(\tau) \qquad (5)$$

where $S_{nmlk}(\tau)$ is:

$$S_{nmlk}(f) = \int_{-\infty}^{\infty} R_{z_n z_m u_l u_k}(\tau)e^{-j\omega\tau d\tau}$$

The expression (5) is a general output power spectral density and consists of $\left[\left(\frac{N+1}{2}\right)^2 \times \left(\frac{N+3}{4}\right)^2\right]$ terms. These terms can be divided into three major groups: The first group represent the linear output with gain compression ($l = 0, k = 0, n = 1$ or $m = 1$), the second group represent the intermodulation distortions ($l = 0, k = 0$, $n > 1$ and $m > 1$), and the third group represents the cross modulation distortion caused by the presence of the second signal ($l > 0, k > 0$). In this way, the development of the statistical model allows the accurate characterization of cross modulation distortion.

IV Measurements And Simulation Results

The statistical model was used along with the behavioral model coefficients to simulate the distortion introduced by the interaction of multiple signals in a forward link IS-95 CDMA system. The CDMA signals was generated in *SystemView* by *Elanix* and the autocorrelation function and its Fourier transform were computed in MATLAB. Fig. 3 shows the output spectrum of a CDMA signal divided into linear, intermodulation, and cross modulation components assuming that the nonlinear model is excited by the sum of two CDMA channels centered at 2.0 and 2.1 GHz. The output spectrum shows the increase in spectral regrowth and In-Channel distortion over single channel excitation due to cross modulation. Fig. 4 shows gain compression and Adjacent Channel Power Ratio (ACPR) of the first channel as a function of its output power and the second channel input power.

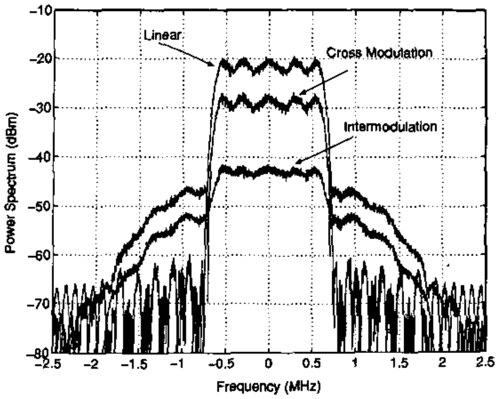
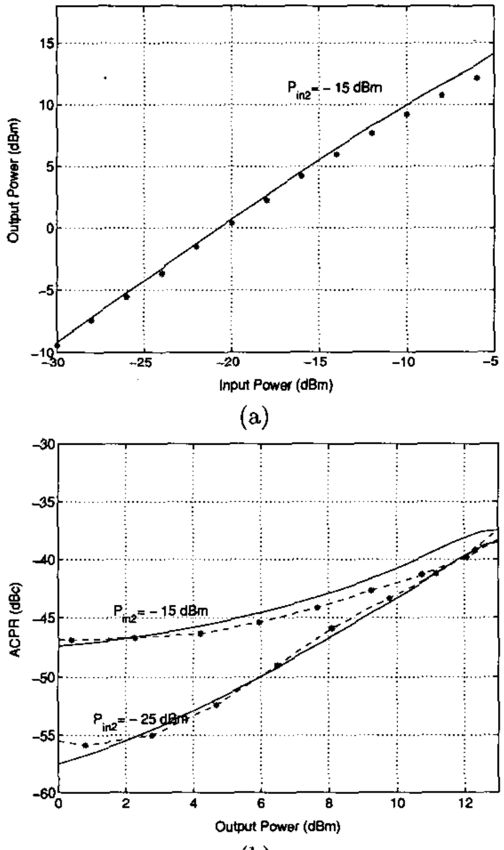


Figure 3: Output Spectrum divided into linear, intermodulation and cross modulation components.



(a)

(b)

Figure 4: (a) Gain compression at the first carrier, (b) ACPR of the first carrier (Solid: simulated, Dashed: measured).

V CONCLUSION

A new behavioral modeling technique for modeling cross modulation in a multichannel power amplifier has been presented and verified. The new technique for measuring the AM-AM and AM-PM characteristics along with our statistical approach enables the accurate estimation of cross modulation distortion which is a great concern of the multichannel PA designer.

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REFERENCES

[1] M. Johansson, T. Mattsson, L. Sundstrom, M. Faulkner, "Linearization of multi-carrier power amplifiers," *IEEE 43rd Conf. Veh. Tech.*, pp. 684–687, May 1993.

[2] J. S. Kenney and A.Leke, "Design considerations for multicarier CDMA base station power amplifiers," *Microwave J.*, vol.42, no.2, Feb. 1999.

[3] K. Gard, L. E. Larson and M. B. Steer, "Generalized autocorrelation analysis of spectral regrowth from bandpass nonlinear circuits," *2001 IEEE MTT-S Int. Microwave Symp. Digest.*, pp. 9–12, May 2001.

[4] K. Gard, H. Gutierrez, M. B. Steer, "Characterization of spectral regrowth in microwave amplifiers based on the nonlinear transformation of a complex Gaussian process," *IEEE Trans. Microwave Theory Techn.*, vol. 47, pp. 1059–1069, July 1999.

[5] V.Aparin, "Analysis of CDMA signal spectral regrowth and waveform quality," *IEEE Trans. Microwave Theory Techn.*, vol. 49, pp. 2306–2314, Dec. 2001.