Electric Green's Dyadics for Modeling Resonance and Surface Wave Effects in a Waveguide-Based Aperture-Coupled Patch Array

Alexander B. Yakovlev[1*], Sean Ortiz[2], Mete Ozkar[2], Amir Mortazawi[2], and Michael B. Steer[2]

[1]Department of Electrical Engineering, The University of Mississippi, University, Mississippi 38677-1848
[2]Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695-7914

Abstract

Electric Green's dyadics for a semi-infinite partially filled rectangular waveguide are developed for the full-wave analysis of a waveguide-based aperture-coupled patch amplifier array. The Green's functions are derived in the form of a double series expansion over the complete system of eigenfunctions of the Helmholtz operator. In this representation, the one-dimensional characteristic Green's functions along the waveguide provide a physical insight on resonance and surface wave effects occurring in overmoded layered waveguide transitions. Particularly, this is related to the coupling of waveguide modes to surface waves propagating in the transverse direction. This is demonstrated for the example of an aperture-coupled patch amplifier array in the N-port waveguide transition, although the analysis is applicable to other waveguide-based antenna structures, which allow for the propagation of surface waves.

I. Introduction

Spatial power amplifiers are used to combine a power from an array of solid state devices at millimeter-wave frequencies resulting in increased output power and power combining efficiencies. A modeling environment for waveguide-based spatial power combining circuits has been recently developed in [1], [2]. The full-wave analysis of interacting electric- (patch, strip) and magnetic-type (slot, aperture) antennas is based on the integral equation formulation for the induced electric and magnetic currents discretized via the Method of Moments (MoM). The electric Green's dyadics in this formulation provide the necessary relationship between scattered fields and induced currents serving as kernels of the integral equations.

In this paper, we discuss a method of developing electric Green's dyadics of the third kind for a semi-infinite, partially filled rectangular waveguide terminated by a perfectly conducting ground plane. Components of the Green's dyadics are expressed in a double infinite series expansion over the complete system of orthonormal eigenfunctions of the transverse Helmholtz operator. The unknown coefficients in this expansion represent one-dimensional characteristic Green's functions along the waveguide. In this representation, transverse and longitudinal coordinates are functionally separated, which allows one to immediately reduce the three-dimensional problem to a one-dimensional Sturm-Liouville boundary-value problem for the unknown characteristic Green's functions. The characteristic Green's functions provide a physical insight on resonance and surface wave effects occurring in overmoded layered waveguide transitions. This is demonstrated for the example of a 2×3 aperture-coupled patch amplifier array analyzed in [2].

II. Electric Green's Dyadics for Layered Waveguides

Electric Green's dyadics of the third kind, $\underline{\mathbf{G}}_{e11}^{(1)}(\mathbf{r}, \mathbf{r}')$ and $\underline{\mathbf{G}}_{e21}^{(1)}(\mathbf{r}, \mathbf{r}')$, for a semi-infinite partially filled rectangular waveguide (Fig. 1) are obtained as the
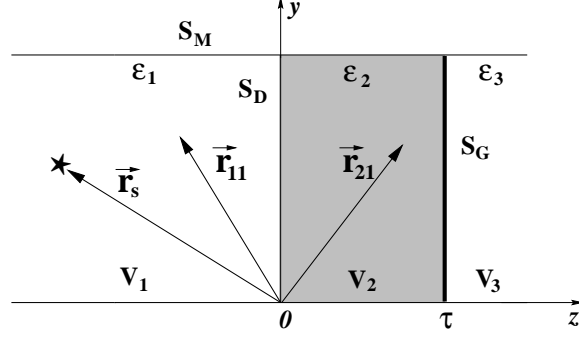
Fig. 1. Geometry of a semi-infinite partially filled waveguide with a point source (star) positioned in the region V_1.

solution of the system of dyadic differential equations subject to boundary conditions on the waveguiding surface S_M and on the surface of the ground plane S_G, and mixed continuity conditions across the dielectric interface S_D (a boundary-value problem is formulated in [1], Appendix A). The solution yields nine components of the electric Green's dyadics, which can be expressed as

$$
\left\{ \begin{array}{c} G_{e11}^{(1)\nu v} \\ G_{e21}^{(1)\nu v} \end{array} \right\} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \phi_{mn}^{(1)\nu}(x,y)\phi_{mn}^{(1)v}(x',y') \left\{ \begin{array}{c} f_{mn}^{(11)\nu v}(z,z') \\ f_{mn}^{(21)\nu v}(z,z') \end{array} \right\} \tag{1}
$$

for $\nu, v = x, y, z$, where $\phi_{mn}^{(1)\nu}(x,y)$ are the orthonormal eigenfunctions of the transverse Helmholtz operator, and $f_{mn}^{(11)\nu v}(z,z')$ and $f_{mn}^{(21)\nu v}(z,z')$ represent characteristic Green's functions.

Orthogonality and completeness of the eigenfunctions $\phi_{mn}^{(1)\nu}(x,y)$ in the double series expansion (1) allow for the reduction of the system of dyadic differential equations to a system of second-order differential equations for the one-dimensional characteristic Green's functions $f_{mn}^{(11)\nu v}(z,z')$ and $f_{mn}^{(21)\nu v}(z,z')$,

$$
\left(\frac{\partial^2}{\partial z^2} - \gamma_{mn}^{(1)2} \right) f_{mn}^{(11)\nu v}(z,z') = -\xi_{mn}^{\nu v}\delta(z-z') \tag{2}
$$

$$
\left(\frac{\partial^2}{\partial z^2} - \gamma_{mn}^{(2)2} \right) f_{mn}^{(21)\nu v}(z,z') = 0, \quad \nu, v = x, y
$$

where

$$
\xi_{mn}^{xx} = \frac{k_1^2 - \left(\frac{m\pi}{a}\right)^2}{k_1^2}, \quad \xi_{mn}^{xy} = \xi_{mn}^{yx} = -\frac{\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)}{k_1^2}, \quad \xi_{mn}^{yy} = \frac{k_1^2 - \left(\frac{n\pi}{b}\right)^2}{k_1^2}
$$

and $\gamma_{mn}^{(1,2)}$ is the propagation constant in the regions V_1 and V_2, respectively.

The boundary condition for the one-dimensional transverse characteristic Green's functions at $z = \tau$ and $z > z'$ is obtained as,

$$
f_{mn}^{(21)\nu v}(\tau, z') = 0 \tag{3}
$$

and the continuity conditions on the dielectric interface at $z = 0$ and $z > z'$ can be written in the form,

$$
f_{mn}^{(11)\nu v}(0, z') = f_{mn}^{(21)\nu v}(0, z') \tag{4}
$$

$$
\frac{\partial}{\partial \nu} f_{mn}^{(11)zv}(0, z') - \frac{\partial}{\partial z} f_{mn}^{(11)\nu v}(0, z') = \frac{\partial}{\partial \nu} f_{mn}^{(21)zv}(0, z') - \frac{\partial}{\partial z} f_{mn}^{(21)\nu v}(0, z').
$$

The z-directed Green's functions $f_{mn}^{(11)zv}$ and $f_{mn}^{(21)zv}$ are expressed in terms of transverse components.

The solution of the system of differential equations (2) is obtained as a superposition of primary and scattered parts, where the primary part is due to a point source positioned in Region V_1, and the scattered part is reflected from the interface $z = 0$, traveling in the negative z-direction. In the region V_2 we have only the scattered part which represents traveling backward and forward waves propagating between the interface at $z = 0$ and the ground plane at $z = \tau$.

Finally, this procedure results in the representation of the one-dimensional transverse characteristic Green's functions $f_{mn}^{(11)\nu v}(z, z')$ in terms of the primary and scattered parts,

$$
\begin{aligned}
f_{mn}^{(11)xx}(z, z') &= \xi_{mn}^{xx} \frac{e^{-\gamma_{mn}^{(1)}|z-z'|}}{2\gamma_{mn}^{(1)}} - e^{\gamma_{mn}^{(1)}(z+z')} \left(\frac{\xi_{mn}^{xx}}{2\gamma_{mn}^{(1)}} - \frac{1}{Z_o^{TE}} + \frac{(\frac{m\pi}{a})^2 Z_e^{TE}}{k_1^2 Z_o^{TE} Z_e^{TM}} \right) \\
f_{mn}^{(11)xy}(z, z') &= f_{mn}^{(11)yx}(z, z') \\
&= \xi_{mn}^{xy} \left(\frac{e^{-\gamma_{mn}^{(1)}|z-z'|}}{2\gamma_{mn}^{(1)}} - e^{\gamma_{mn}^{(1)}(z+z')} \left(\frac{1}{2\gamma_{mn}^{(1)}} - \frac{Z_e^{TE}}{Z_o^{TE} Z_e^{TM}} \right) \right) \\
f_{mn}^{(11)yy}(z, z') &= \xi_{mn}^{yy} \frac{e^{-\gamma_{mn}^{(1)}|z-z'|}}{2\gamma_{mn}^{(1)}} - e^{\gamma_{mn}^{(1)}(z+z')} \left(\frac{\xi_{mn}^{yy}}{2\gamma_{mn}^{(1)}} - \frac{1}{Z_o^{TE}} + \frac{(\frac{n\pi}{b})^2 Z_e^{TE}}{k_1^2 Z_o^{TE} Z_e^{TM}} \right)
\end{aligned}
\tag{5}
$$

and functions $f_{mn}^{(21)\nu v}(z, z')$ are obtained in terms of scattered waves,

$$
\begin{aligned}
f_{mn}^{(21)xx}(z, z') &= - \left(\frac{1}{Z_o^{TE}} - \frac{(\frac{m\pi}{a})^2 Z_e^{TE}}{k_1^2 Z_o^{TE} Z_e^{TM}} \right) \frac{e^{\gamma_{mn}^{(1)}z'} \sinh\gamma_{mn}^{(2)}(z-\tau)}{\sinh\gamma_{mn}^{(2)}\tau} \\
f_{mn}^{(21)xy}(z, z') &= f_{mn}^{(21)yx}(z, z') = -\xi_{mn}^{xy} \frac{Z_e^{TE}}{Z_o^{TE} Z_e^{TM}} \frac{e^{\gamma_{mn}^{(1)}z'} \sinh\gamma_{mn}^{(2)}(z-\tau)}{\sinh\gamma_{mn}^{(2)}\tau} \\
f_{mn}^{(21)yy}(z, z') &= - \left(\frac{1}{Z_o^{TE}} - \frac{(\frac{n\pi}{b})^2 Z_e^{TE}}{k_1^2 Z_o^{TE} Z_e^{TM}} \right) \frac{e^{\gamma_{mn}^{(1)}z'} \sinh\gamma_{mn}^{(2)}(z-\tau)}{\sinh\gamma_{mn}^{(2)}\tau}.
\end{aligned}
\tag{6}
$$

Here, Z_e^{TE}, Z_o^{TE}, and Z_e^{TM} are the characteristic functions of even and odd TE and TM modes, respectively, of a grounded dielectric slab of thickness τ bounded with electric walls at $x = 0, a$ and $y = 0, b$ (Region V_2), given as

$$
\begin{aligned}
Z_e^{TE} &= \gamma_{mn}^{(1)} + \gamma_{mn}^{(2)} \tanh\gamma_{mn}^{(2)}\tau \\
Z_o^{TE} &= \gamma_{mn}^{(1)} + \gamma_{mn}^{(2)} \coth\gamma_{mn}^{(2)}\tau \\
Z_e^{TM} &= \frac{\varepsilon_2}{\varepsilon_1}\gamma_{mn}^{(1)} + \gamma_{mn}^{(2)} \tanh\gamma_{mn}^{(2)}\tau
\end{aligned}
\tag{7}
$$

where the propagation constant $\gamma_{mn}^{(i)}$ for $i = 1, 2$ is given in terms of transverse wave numbers $k_{mn} = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}$. Zeros of the characteristic functions (7) represent resonance frequencies of TE and TM oscillations in the waveguide cross-section. Moreover, it can be seen that zeros of Z_o^{TE} and Z_e^{TM} for TE-odd and TM-even modes correspond to poles of the characteristic Green's functions (5) and (6). The characteristic functions (7) for an infinite grounded dielectric slab of thickness τ define even and odd TE and TM surface waves with the propagation constant $k_{sw} \equiv k_{mn}$.

At a resonance frequency corresponding to the transverse wavenumber k_{mn} of the shielded grounded dielectric slab, the value of k_{mn} is equal to the value of the
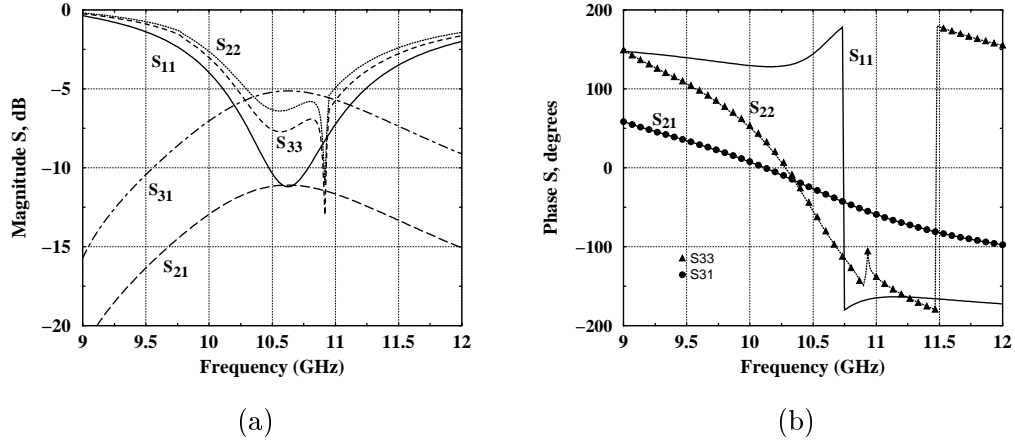
Fig. 2. Magnitude (a) and phase (b) of the S-parameters against frequency for the 2×3 aperture-coupled patch array in the rectangular waveguide transition.

propagation constant k_{sw} of a surface wave associated with an infinite dielectric slab. This is related to coupling of waves propagating along the waveguide in the z-direction with propagation constants $\gamma_{mn}^{(i)}$ to TE and TM surface waves propagating in an infinite grounded dielectric slab (associated with resonance wavenumbers k_{mn} in the waveguide cross-section).

III. NUMERICAL RESULTS

The numerical results for the S-parameters (magnitude and phase) of the dominant TE_{10} mode in the 2×3 aperture-coupled patch array waveguide transition operating at X-band (geometry of the N-port waveguide transition is shown in [2]) are demonstrated in Figure 2(a,b). The geometrical and material parameters of the 2×3 aperture-coupled patch array are given in [2]. In Figure 2(a,b), S_{11} is the reflection coefficient at the interface $z = 0$ in Region V_1 (with the excitation from V_1), and S_{22} and S_{33} are the reflection coefficients at $z = \tau$ (ground plane) in Regions V_s^2 and V_s^3 (with the excitation from V_s^2 and V_s^3, respectively, see Fig. 2 in [2]). The transmission coefficients from V_1 into V_s^2 and V_s^3 are denoted as S_{21} and S_{31}, respectively. The sharp resonance obtained at 10.8965 GHz corresponds to the occurrence of a transverse resonance, and is associated with the coupling of a mode propagating along the waveguide to the surface wave TM_0 of a dielectric slab (substrate) propagating in the transverse direction (waveguide cross-section). The normalized propagation constant of the surface wave at the transverse resonance frequency is equal to $k_{sw} = 1.00484$.

IV. ACKNOWLEDGMENTS

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