Fully Physical Time-Dependent Compact Thermal Modelling of Complex Non Linear 3-Dimensional Systems for Device and Circuit Level Electro-Thermal CAD

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Abstract

An original, fully analytical, spectral domain decomposition approach to compact solution of the non linear time-dependent heat diffusion equation in complex volumes, is introduced. Its application to device and circuit level electro-thermal simulation on CAD timescales is illustrated. The first full treatment in coupled electro-thermal CAD, of thermal non linearity due to temperature dependent diffusivity, is described. Original thermal solutions are presented in the form of analytically exact thermal impedance matrix expressions for thermal subsystems. These include double Fourier series solutions for rectangular multilayers, which are an order of magnitude faster to evaluate than existing semi-analytical Fourier solutions based on DFT-FFT. They also include double Fourier series solutions for arbitrarily distributed volume heat sources and sinks, constructed without the use of Green's function techniques, and for rectangular volumes with prescribed fluxes on all faces. These analytical solutions allow treatment of arbitrary device structures without invoking conventional numerical methods. They provide minimal boundary condition independent compact thermal models, allowing CAD timescale coupled electro-thermal solution for complex systems, without requiring lumped element RC network extraction or node reduction. The time-independent thermal resistance matrix description of detailed device structure is illustrated by a fully physical, coupled electro-thermal study of the interaction of substrate thickness and surface convection in power HEMTs. The thermal time-dependent implementation is illustrated by circuit level harmonic balance simulation of a 3×3 MMIC amplifier array.

Keywords: thermal, electrothermal, compact modelling, non linear CAD, power FETs, circuits.

I. Introduction

Solutions of the heat diffusion equation for complex 3-dimensional systems are commonly based on finite volume, finite element, finite difference, boundary element or transmission line methods. All of these approaches require construction of a volume or surface mesh. They are computationally intensive and therefore generally too slow for direct coupling to electronic device and circuit simulators in the necessarily iterative solution of intrinsically non linear coupled electro-thermal problems. Even numerical solutions optimised for thermal treatment of electronic devices and circuits, *e.g.*, [1], which is based on hierarchical nesting to treat the wide range of length and time scales inherent in the coupled electro-thermal problem, or [2], based on successive node reduction for complex inhomogeneous 3-dimensional structures, are not fast enough for directly coupled electrical and thermal solution. Thus a number of faster thermal descriptions have been developed.

Until recently, the state of the art in time-independent thermal simulation of heatsink mounted power FETs and MMICs, for coupled electro-thermal CAD, has been represented by the hybrid finite element Green's function approach of Bonani *et al.* [3]. This thermal resistance matrix approach treats device structure such as surface metallisation, vias and partial substrate thinning. In time-dependent coupled electro-thermal CAD, fast 3-dimensional thermal descriptions have been more basic, and limited to simple rectangular multilayers. In particular, Szekely *et al.* have employed a Fourier series method for over 20 years, providing solutions for a variety of ICs, microsystem elements and MCMs [4]. For circuit level electro-thermal simulations, thermal model reduction techniques have been widely employed. Work in this area includes that of Sabry [5], Napieralski [6] and Szekely [7], and Szekely has combined fast sparse finite difference methods, [2], with lumped element RC network extraction, [7], to provide coupled electro-thermal simulation of complex 3-dimensional systems. The problem of compact model development is currently an active area of research [8], [9].

The aim of this paper is to describe a new, fully physical and analytical, approach to the non linear time-dependent thermal problem in complex 3-dimensional systems, suitable for directly coupled electro-thermal device and circuit simulation on CAD timescales, requiring no explicit model reduction. To illustrate the advantages of this approach, particular comparison is made with the comprehensive circuit level electro-thermal modelling capability of Szekely *et al.* Fully physical, coupled electro-thermal device simulations for the thermal time-independent and time-dependent cases, have been described by the authors in [10]–[15]. These were based on coupling of the thermal model presented here, to the quasi-2-dimensional Leeds Physical Model of MESFETs and HEMTs [16]–[19]. Coupling to a microwave circuit simulator Transim (NCSU) [20], has been introduced in [21]–[23].

Generically, the thermal approach presented in this paper is a fully analytical spectral domain decomposition technique. Simple composite systems have been treated previously by the Unsteady Surface Element (USE) method of Beck *et al.*, [24], and this approach has the advantage that, unlike conventional numerical methods, it only discretises interfaces between subsystems. Like the USE method, the approach presented here discretises only interfaces (along with power dissipating and temperature sensitive elements). It constructs solutions for thermal subvolumes which are fully analytical, with develop-

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ment of double Fourier series solutions for thermal subvolumes by explicit construction of series expansion coefficients. Thus it differs from semi-analytical Fourier approaches for simple rectangular multilayers [4], based on collocation or function sampling, which require numerical manipulation such as DFT-FFT to generate expansion coefficients. As solutions for subvolumes are fully analytical, no volume or surface mesh is required. The method described is a thermal impedance matrix approach. This time-dependent thermal impedance matrix formulation is a natural development of the fully analytical thermal resistance matrix approach for the time-independent case, introduced by the authors in [10], [11] and developed fully in [12]. It is shown that, in contrast to previous thermal resistance and impedance approaches, this thermal impedance matrix method can be formulated to provide an essentially exact solution of the heat diffusion equation in complex 3-dimensional systems. It therefore removes the need to utilise computationally intensive numerical techniques in order to treat complex structures, *e.g.* [5], [6], [25]. Previous Green's function or Fourier approaches have been restricted to simple rectangular homogeneous volumes and multilayers, *e.g.* [4],[26]–[28]. The fully analytical model presented here can describe simultaneously all device detail, from surface metallisation, vias and substrate thinning, in power FETs and MMICs, through (actively cooled) MMIC on substrate arrays, up to MCMs and circuit board level. It does this by providing double Fourier series solutions of the heat diffusion equation in thermal subsystems, and then constructing global solutions for complex systems by matching temperature and flux at subsystem interfaces. As the subsystem solutions are matrix expressions, an explicit matrix representation can be obtained for the global thermal impedance matrix of the complex device structure. A particular intended application is the treatment of MMIC arrays for spatial power combining at millimeter wavelengths.

Importantly, this modular thermal solution is constructed to be immediately compatible with directly coupled electro-thermal device and circuit simulation on CAD timescales. This is achieved by formulating the analytically exact subsystem solutions in terms of thermal impedance matrices which describe temperature variation with time, only in the vicinity of the power dissipating, temperature sensitive and interface regions required for coupling of the electrical and thermal problems. No redundant temperature information is generated on the surface or in the body of the subsystem volumes. As these minimal thermal solutions are generated analytically, the thermal impedance matrices are all precomputed, prior to the coupled electro-thermal simulation, purely from structural information. Thermal updates in the coupled electro-thermal problem are therefore rapid.

Fully coupled device level simulation can be implemented by combination of the Leeds thermal impedance matrix model with any thermally self-consistent device model. If the device model includes self-heating effects, then the global thermal impedance matrix will provide an accurate, CAD timescale, description of mutual thermal interaction between power dissipating and temperature sensitive elements, however complex the thermal system. Coupled electro-thermal solution is achieved by iterative solution of the electrical and thermal problems, with thermal updates provided by small matrix multiplications, and thermal non linearity transferred to the already non linear active device model.

Circuit implementation of this thermal solution, exploits the ability of network based microwave circuit simulators to describe multiport non linear elements in the time domain, and to treat linear distributed electromagnetic systems in terms of multiport network parameters [29]. After appropriate exact transformation of thermal non linearity, the initially non linear thermal problem is immediately compatible with network based microwave circuit simulation engines, by interpretation of thermal impedance matrices, for distributed thermal subsystems, in terms of generalised multiport network parameters. Analytical, s-space solution for thermal subsystems, means that no numerical identification of thermal networks, such as that provided by the NID method [30], is required. It also means that each thermal subsystem can be described in either the time domain or in the frequency domain. In the time domain, the thermal subsystem is treated as a non linear multiport element, which readily allows non linear matching of transformed temperatures at thermal subsystem interfaces [11], [12]. In the frequency domain, the thermal subsystem is represented by a matrix of complex phasors inserted into the modified nodal admittance matrix (MNAM) for the microwave system, and thermal non linearity is again transferred to the already non linear active device model. This gives coupled electro-thermal harmonic balance and transient solutions on CAD timescales. Coupled electro-thermal circuit level CAD generally requires thermal model reduction, *e.g.*, [5]–[7]. Rapidly convergent, fully analytical and minimal thermal impedance matrix expressions, in both the time and frequency domains, mean that no reduced, lumped element, RC network description, is required in the multiport network parameter approach. The thermal impedance matrices represent minimal boundary condition independent compact models of thermal subsystems [8], [9] for the time-dependent case, and with full treatment of thermal non linearity. Analytical expressions for the multiport network parameters of all thermal subvolumes mean that no distinct thermal simulator, separate from the coupled electro-thermal simulation engine, is required to characterise the complex thermal system.

A key aspect of the thermal solution presented here, is application of a generalised 'radiation' boundary condition, on the top and bottom surfaces of all thermal subvolumes, in the analytical subsystem solutions. This boundary condition allows analytical subsystem solutions with interface discretisation, and construction of global thermal solutions by vertical matching of temperatures and fluxes at subsystem interfaces. The boundary condition also allows integral treatment of surface radiation and convection in large area systems without approximation such as that invoked in [4]. (Radiation boundary conditions have also been applied, for example, in finite difference solutions for electro-thermal simulation [25], and in analytical solutions at the circuit board level [28].) One aim of this paper is to present explicit analytical solutions for thermal subsystem impedance matrices, allowing global solution for complex systems. Such fully analytical solutions treat arbitrarily complex 3-dimensional thermal systems without the use of volume meshes, uniform or non uniform, thus avoiding all problems with the wide range of length and time scales inherent in the coupled electro-thermal problem [31]. Generation of such solutions requires treatment

of thermal non linearity inherent in temperature dependent material parameters. An original treatment of this non linearity, for device and circuit level electro-thermal CAD, is presented first [32]. This is followed by derivation of thermal impedance matrix solutions for a homogeneous MMIC, and for an N-level rectangular multilayer. It is shown how the time-domain thermal impedance matrix can be expressed in rapidly convergent forms for all time, t. This is followed by presentation of an original double Fourier series solution to the time-dependent heat diffusion equation with arbitrarily distributed volume heat sources and sinks. This goes beyond previous solutions in the literature, which treat heat dissipating sources as planar, either at the surface or interfaces of rectangular multilayers [4],[11], [12]. Description of complex 3-dimensional structure, and construction of global impedance matrices, are outlined next, followed by discussion of the Leeds thermal impedance matrix approach as a minimal compact model. Use of the thermal resistance matrix approach to describe detailed device structure in the thermal time-independent case is then indicated by an illustrative, fully physical, electro-thermal device study of the relation between substrate thinning and the magnitude of surface convection in power HEMTs. Finally, implementation of the time-dependent thermal impedance matrix approach, in circuit level CAD, is illustrated by harmonic balance simulation of a 3×3 MMIC amplifier array.

II. Thermal Non Linearity

The time dependent heat diffusion equation is given by,

$$\nabla . [\kappa(T) \nabla T] + g = \rho C \frac{\partial T}{\partial t}, \tag{1}$$

where T is temperature, t is time, $\kappa(T)$ is temperature dependent thermal conductivity, $g(x, y, z, t)$ is rate of heat generation, ρ is density and C is specific heat. This equation is non linear through the temperature dependence of $\kappa(T)$ (and possibly of ρ and C). To linearise the equation, the Kirchhoff transformation is performed [33],

$$\theta = T_S + \frac{1}{\kappa_S} \int_{T_S}^{T} \kappa(T) dT, \tag{2}$$

where $\kappa_S = \kappa(T_S)$ and T_S is the heatsink mount temperature. The importance of performing the Kirchhoff transformation has been illustrated, *e.g.*, by Webb [34]. The inverse Kirchhoff transformation is trivial to impose *a postiori* to solution of the linear heat diffusion equation, by application of a simple analytical formula to the solution temperatures [12]. The equation for transformed temperature θ becomes,

$$\nabla^2 \theta - \frac{1}{k(\theta)} \frac{\partial \theta}{\partial t} = -\frac{g}{\kappa_S}, \tag{3}$$

where diffusivity $k = \kappa / \rho C$. k is now a function of θ so the equation is still non linear.

At this stage it is conventional, in electro-thermal simulations employing the Kirchhoff transformation, to assume that $k(\theta)$ is approximately constant, thus fully linearising the time-dependent heat diffusion equation. However, for typical semiconductor systems this assumption requires further examination and has been discussed by the authors in [32]. It is shown there, that the Kirchhoff transformation does not remove the temperature sensitivity of the material parameters for the time-dependent case.

A further (much less well known) transformation should therefore be applied to fully linearise the heat diffusion equation, by defining a new time variable, τ [35], [36],

$$k_S \tau = \int_0^t k(\theta) dt. \tag{4}$$

The time-dependent heat diffusion equation becomes finally,

$$\nabla^2 \theta - \frac{1}{k_S} \frac{\partial \theta}{\partial \tau} = -\frac{g}{\kappa_S}. \tag{5}$$

The fully linearised equation, Eq. (5), can now be solved exactly with general linear boundary conditions. To illustrate the significance of the time variable transformation, Eq. (4), for electro-thermal response [32], an analytical thermal impedance matrix is constructed to describe the response to step power input of 0.4 W, over a central square $0.1L \times 0.1L$, at the surface of a cubic GaAs die, side $L = 400$ μm. Such a configuration is illustrative of, for example, a multi-finger power FET. It is found that total neglect of thermal non linearity leads to a $\sim$30 K underestimate of the steady-state temperature rise of $\sim$140 K. Including the inverse Kirchhoff transformation, but neglecting the inverse time variable transformation is found to overestimate the temperature rise by $\sim$4 % at any given instant, or equivalently and more importantly, to underestimate the rise time required to reach a given temperature by as much as $\sim$35 %.

Another sometimes used approximation, is that of effectively linearising the time-dependent heat diffusion equation about a typical operating point, without employing either the Kirchhoff or the time variable transformation. The error in this approach corresponds to $\sim$6 % overestimate of temperature rise, or underestimate of rise time by as much as $\sim$60 %. In addition, simply guessing a suitable operating point for linearisation is highly subjective, and for the case of transient thermal variation of large amplitude, easily leads to large errors in the calculated steady-state operating temperatures. In Si systems, temperature dependence of material parameters is even more pronounced than in GaAs [37]. Full linearisation of the time-dependent heat diffusion equation should therefore be implemented to obtain sufficient accuracy in electro-thermal simulations.

III. Analytical Solutions

Having described the exact (not small signal) transformation of the non linear time-dependent heat diffusion equation, to produce a fully linear problem, analytical solution of the transformed problem in terms of thermal impedance matrices is now described. The thermal impedance matrix approach reduces to construction of global heat flow functions, for power dissipating and temperature sensitive elements in semiconductor integrated circuits, in the form

$$\overline{\Delta \theta_i} = \sum_j R_{TH_{ij}}(s) \overline{P_j} \tag{6}$$

where $\overline{\Delta \theta_i}$ is the Laplace transformed temperature rise of element i above its initial temperature, $R_{TH_{ij}}(s)$ is the

thermal impedance matrix in Laplace s-space and the $\overline{P_j}$ are the transformed time-dependent fluxes due to power dissipation in elements, $j = 1, ..., i, ...M$.

Formulation of the thermal impedance matrix approach in Laplace transform s-space, rather than in the time domain, is chosen for a number of reasons. Firstly, the s-space formulation is a natural development of the thermal resistance matrix approach for the time-independent case, described by the authors previously [10]-[12]. All of the advantageous features of the thermal resistance matrix approach for the coupled electro-thermal description of complex systems, carry over to the time-dependent case in s-space. Secondly, the s-space formulation of the thermal impedance matrix allows immediate incorporation as a multi-port distributed thermal network in circuit level harmonic balance simulators. Finally, Laplace inversion also allows use in circuit level transient simulations, and analytical inversion of s-space expressions readily gives rise to both small-time and large-time results for the thermal response, which are not easily obtained using a direct time domain formulation. However, the thermal impedance matrix approach can also be developed in the time domain using Green's function techniques, as described in [14].

In the thermal impedance matrix approach presented here, $R_{TH_{ij}}(s)$ is determined in explicit analytical form, purely from structural information. It is independent of temperature and power dissipation, and hence of device bias. Its order is determined only by the number of heat dissipating and temperature sensitive elements, independent of the level of the complexity of the device structure, so is already minimal without any explicit model reduction.

A. The Homogeneous Thermal Subsystem

An analytical solution to the linearised heat diffusion equation, Eq. (5), is constructed for the case of a rectangular, homogeneous, generic thermal subvolume, $0 \le x \le L$, $0 \le y \le W$, $0 \le z \le D$, with device elements $i = 1, ..., M$ described by surface elementary areas, D_i, and base discretised into elementary areas, D_j. Adiabatic boundary conditions are assumed on the side faces and a generalised 'radiation' boundary condition is imposed on the top and bottom faces, $z = 0, D$. This can be written,

$$\alpha_{0,D}\kappa_S \frac{\partial \theta}{\partial z} + H_{0,D}\left(\theta - \theta_{0,D}(x,y,\tau)\right) + p_{0,D}(x,y,\tau) = 0. \tag{7}$$

Non linear surface flux boundary conditions can be treated in the limit of a sequence of such fully linear problems [11], [38]. Here, imposed flux densities $p_{0,D}(x,y,\tau)$ are time dependent. Coefficients $H_{0,D}$ describe surfaces fluxes due to radiation and convection. The $\alpha_{0,D}$ equal zero for imposed temperature boundary conditions and unity for imposed flux boundary conditions. The respective ambient temperatures ($\alpha_{0,D} \neq 0$), or heatsink mount temperatures ($\alpha_{0,D} = 0$), are also dependent on time, $\theta_{0,D}(x,y,\tau)$. The generality of this boundary condition allows vertical matching of thermal subsystems, by interface discretisation and thermal impedance matrix manipulation, as well as integral treatment of surface fluxes.

To solve this problem, the Laplace transform is constructed giving,

$$\nabla^2\overline{\theta} - \frac{1}{k_S}\left[s\overline{\theta} - \theta(\tau = 0)\right] = 0, \tag{8}$$

assuming no volume sources or sinks, and describing surface fluxes by imposed boundary conditions, Eq. (7).

For the case of a uniform initial temperature distribution equal to uniform and time independent ambient temperature, and by separation of variables, the general solution is of the form,

$$\overline{\theta}(s) = \frac{\theta(\tau = 0)}{s} + \sum_{mn} \cos\lambda_m x \cos\mu_n y \times \left(C_{mn}\cosh\gamma_{mn}z + S_{mn}\sinh\gamma_{mn}z\right)\frac{1}{s} \tag{9}$$

where $m, n = 0, 1, 2, ...$, and

$$\lambda_m = \frac{m\pi}{L}, \quad \mu_n = \frac{n\pi}{W}, \quad \gamma_{mn}^2 = \lambda_m^2 + \mu_n^2 + \frac{s}{k_S}. \tag{10}$$

The time-dependent problem then resembles very closely the time-independent problem [11], [12], and explicit forms for the expansion coefficients are obtained from,

$$H_0 C_{mn} = -\gamma_{mn}S_{mn}\alpha_0\kappa_S - \frac{\int_0^L\int_0^W \cos(\lambda_m x)\cos(\mu_n y)s\overline{p}_0(x,y;s)dxdy}{\frac{LW}{4}(1+\delta_{m0})(1+\delta_{n0})} \tag{11}$$

and

$$C_{mn}\left[\alpha_D\kappa_S\gamma_{mn}\sinh(\gamma_{mn}D) + H_D\cosh(\gamma_{mn}D)\right] + S_{mn}\left[\alpha_D\kappa_S\gamma_{mn}\cosh(\gamma_{mn}D) + H_D\sinh(\gamma_{mn}D)\right] = -\frac{\int_0^L\int_0^W \cos(\lambda_m x)\cos(\mu_n y)s\overline{p}_D(x,y;s)dxdy}{\frac{LW}{4}(1+\delta_{m0})(1+\delta_{n0})}. \tag{12}$$

Here δ_{mn} is the Kronecker delta function.

Such fully analytical double Fourier series solutions in Laplace transform s-space have been described previously [26]. They are to be distinguished from semi-analytical Fourier solutions in frequency space, which are based on collocation or function sampling, and require numerical manipulation such as DFT-FFT to obtain expansion coefficients [4].

As in the time-independent case for the homogeneous MMIC [11], [12], to illustrate a particular time-dependent form of the thermal impedance matrix, put $\alpha_0 = 1$, $H_0 = 0$ (no radiation from the top surface, $z = 0$) and $\alpha_D = 0$, $H_D = 1$, $p_D(x,y,\tau) = 0$, $\theta_D(x,y,\tau) = \theta(\tau = 0)$ (uniform temperature on the bottom surface, $z = D$, corresponding to heat sink mounting at ambient temperature). Assume a surface power density of the form,

$$p_0(x,y,\tau) = \sum_i S_i(x,y)P_i(\tau), \tag{13}$$

where $S_i(x,y) = 1$ in elementary areas D_i, and $S_i(x,y) = 0$ otherwise. Then the corresponding temperature distribution is given by

$$\overline{\theta}(s) = \frac{\theta(\tau = 0)}{s} - \sum_{mn}\cos\lambda_m x\cos\mu_n y \times \frac{1}{\kappa_S LW}\frac{4}{(1+\delta_{m0})(1+\delta_{n0})}\sum_i I_{mn}^i\frac{1}{\gamma_{mn}} \times \left(\sinh\gamma_{mn}z - \tanh\gamma_{mn}D\cosh\gamma_{mn}z\right)\overline{P_i}, \tag{14}$$

with area integrals I^i_{mn} defined by

$$I^i_{mn} = \iint_{D_i} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{W}\right) dxdy. \quad (15)$$

Constructing the surface temperatures averaged over elementary areas, D_i, immediately gives the defining equation of the thermal impedance matrix approach, Eq. (6), in the form,

$$\overline{\theta}_{av_i} - \frac{\theta(t=0)}{s} = \sum_j R_{TH_{ij}}(s)\overline{P_j}, \quad (16)$$

where,

$$R_{TH_{ij}}(s) = \frac{1}{\kappa_S LW} \sum_{mn} \frac{4\tanh(\gamma_{mn}D)}{\gamma_{mn}(1+\delta_{m0})(1+\delta_{n0})} \frac{I^i_{mn}I^j_{mn}}{I^i_{00}}. \quad (17)$$

Extension to treat other realisations of the radiative boundary condition, Eq. (7), is immediate. This allows construction of solutions for large area substrates, with radiation and convection, and generation of series solutions for thermal subsystems with discretised interfaces, permitting vertical matching of thermal subvolume solutions in complex 3-dimensional systems. The expression for $R_{TH_{ij}}(s)$, Eq. (17), can be written in alternative equivalent forms [26], and is readily extended to treat N-level multilayers [26]. The temperature distribution of Eq. (14), and the corresponding thermal impedance matrix of Eq. (17), reduce to the respective time-independent forms [11], [12], in the limit $s/k \to 0$, giving the thermal resistance matrix,

$$R_{TH_{ij}} = \frac{1}{\kappa_S LW}\left[DI^j_{00} + \sum_{mn}{}' \frac{4\tanh(\Gamma_{mn}D)}{\Gamma_{mn}(1+\delta_{m0})(1+\delta_{n0})} \frac{I^i_{mn}I^j_{mn}}{I^i_{00}}\right] \quad (18)$$

where now, $\Gamma^2_{mn} = \lambda^2_m + \mu^2_n$, and the sum $\sum'_{mn}$ is over all $m, n = 0, 1, 2, ...$ excluding $(m, n) = (0, 0)$.

The thermal impedance matrix approach as described here means that generally, temperature will only be calculated in the vicinity of power dissipating and temperature sensitive elements, as required for the coupled electro-thermal solution. No redundant temperature information will be generated on the surface or in the body of the die. However, the solution of the heat diffusion equation just described, provides analytical expressions for both the thermal impedance matrix and for the corresponding temperature distribution throughout the body of the MMIC. This means that once power dissipations, $\overline{P_i}$, have been obtained self-consistently, by employing the thermal impedance matrix in the coupled electro-thermal implementation, temperature can be obtained essentially exactly, if required, at any point within the body or on the surface of the MMIC. This is of value for model validation against measured thermal images.

The matrix given by Eq. (17) represents an exact analytical solution for time-dependent 3-dimensional heat flow in a MMIC bearing an arbitrary distribution of power dissipating and temperature sensitive elements. These elements could be transistor fingers or finger subsections, grouped in any fashion, or could represent heat dissipating passive elements or effective thermocouples at metal-substrate contacts [39]. These analytical expressions describe exactly the finite volume of the die and the finite extent of transistor fingers, without making any approximations for infinite volume or finite end effects. If a simpler, single thermal impedance description of device heating is required, the elements of the matrix can be appropriately summed to give the total, area averaged, temperature rise.

The analytical solution, Eq. (17), represents the thermal impulse response of the MMIC. It is frequency dependent as characteristic of distributed systems and contains an infinite number of poles and zeros. It corresponds to a multiport thermal network which cannot be represented exactly by a finite network of frequency independent primitives, (such as thermal networks generated by numerical mesh descriptions, *e.g.* [5], [6], [25], which only give an exact thermal description in the limit of infinitely fine mesh discretisation). The multiport network is already minimal, in that it describes nodes corresponding only to power dissipating or temperature sensitive surface elements (or discretised interface elements). It therefore constitutes a boundary condition independent compact model for the non linear thermal time-dependent case, with 3-dimensional heat flow described exactly by a small number of thermal impedances. The multiport network parameter interpretation presented here, makes the thermal impedance matrix approach immediately compatible with network based electromagnetic and electrical circuit solvers [29], without the need for any model reduction beyond that implicit in summation of infinite series to just a finite number of terms.

The thermal impedance matrix, Eq. (17), can either be used directly in frequency space, for instance in harmonic balance simulations, or Laplace inverted to describe thermal time dependence directly in transient simulations. For the harmonic balance case, the solution for the thermal impedance matrix is just of the s-dependent form, Eq. (17), with $s \to i\omega$. It takes the form of an array of frequency dependent complex phasors containing phase and amplitude information for the (asymptotic) sinusoidal response to harmonic forcing. This matrix then corresponds to the network parameters of a distributed (originally non linear) multi-port thermal network.

Having obtained transformed temperature in s-space, $\overline{\theta}(s)$, and assuming $\overline{P_j}$ corresponding to simple step inputs of magnitudes P_j, analytical inversion gives the corresponding time domain thermal impedance matrix, $R_{TH_{ij}}(\tau)$, corresponding to step input,

$$R_{TH_{ij}}(\tau) = \mathcal{L}^{-1}\left\{R_{TH_{ij}}(s)\frac{1}{s}\right\} = \frac{1}{\kappa_S LW}\frac{2}{D}\sum_{lmn} \frac{4}{(1+\delta_{m0})(1+\delta_{n0})} \frac{I^i_{mn}I^j_{mn}}{I^i_{00}} \times \frac{1-\exp\left\{-\pi^2\left[\left(\frac{l+1/2}{D}\right)^2 + \left(\frac{m}{L}\right)^2 + \left(\frac{n}{W}\right)^2\right]k\tau\right\}}{\pi^2\left[\left(\frac{l+1/2}{D}\right)^2 + \left(\frac{m}{L}\right)^2 + \left(\frac{n}{W}\right)^2\right]} \quad (19)$$

with $l, m, n = 0, 1, 2, ...$ Taking the limit $\tau \to \infty$ and performing the l summation explicitly, the time-independent result is recovered [11], [12]. Using the Watson transformation and the Poisson summation formula, series solutions such as Eqns. (17)–(19) can be partially summed explicitly in closed form, and partially accelerated to give even more

rapidly evaluated expressions. These results will be presented elsewhere. Such treatment can be particularly important for the description of small elements on large die, which can require summation of large numbers of terms for sufficient resolution. Averaging power dissipations over larger areas can give differences of several decades in calculated time constants, compared to accurate representation of highly localised heating [40]. Accuracy in such descriptions is relevant, for instance, in studies of thermal intermodulation in power HEMTs [41].

Combining tables of standard integrals, *e.g.*, [24], with expressions for the inverse Laplace transform (using properties of theta functions), gives the equivalent time-domain form of the thermal impedance matrix,

$$
\begin{aligned}
&R_{TH_{ij}}(\tau) \\
&= \tfrac{2}{\kappa_S LW} I^j_{00} \left\{ \sqrt{\tfrac{k\tau}{\pi}} + 2\sum_{l=1}^{\infty}(-1)^l \right. \\
&\qquad \left. \times \left[\sqrt{\tfrac{k\tau}{\pi}} e^{-D^2l^2/(k\tau)} - Dl\,\mathrm{erfc}\left(\tfrac{Dl}{\sqrt{k\tau}}\right) \right] \right\} \\
&+ \tfrac{1}{\kappa_S LW} \sum'_{mn} \tfrac{4}{(1+\delta_{m0})(1+\delta_{n0})} \tfrac{I^i_{mn} I^j_{mn}}{I^j_{00}} \tfrac{1}{\Gamma_{mn}} \times \\
&\left\{ \mathrm{erf}\left(\Gamma_{mn}\sqrt{k\tau}\right) - \sum_{l=1}^{\infty}(-1)^l \times \right. \\
&\left. \left[\exp\left\{ \begin{matrix} 2Dl\Gamma_{mn} + \\ \ln\left[\mathrm{erfc}\left(\frac{Dl}{\sqrt{k\tau}} + \Gamma_{mn}\sqrt{k\tau}\right)\right] \end{matrix} \right\} \right. \right. \\
&\left. \left. - \exp\left\{ \begin{matrix} -2Dl\Gamma_{mn} + \\ \ln\left[\mathrm{erfc}\left(\frac{Dl}{\sqrt{k\tau}} - \Gamma_{mn}\sqrt{k\tau}\right)\right] \end{matrix} \right\} \right] \right\}.
\end{aligned}
\tag{20}
$$

This form of the time domain thermal impedance matrix is found to be far more rapidly convergent at very small times. It is an alternative to the explicit time constant form, Eq. (19).

Even though analytical inversion is readily achieved, numerical inversion is highly accurate and algorithmically simple to implement. It requires only evaluation of the Laplace transform and a corresponding weight function, at a small number of real or complex s-points [42]–[45],

$$
\mathcal{L}^{-1}\{f(s)\}_{\tau=\tau_p} = \sum_{\mu} w_\mu f(s_\mu), \tag{21}
$$

with w_μ and s_μ determined uniquely for a given τ_p. Typically 5 or 6 s-points are adequate so this approach can be computationally much cheaper than analytical inversion.

Fig. 1 shows temperature rise with time at turn-on, calculated using the thermal impedance matrix approach, for cubic GaAs die of side $L = 300, 400$ and 500μm, dissipating respectively 0.3, 0.4 and 0.5 W over a central square element of side $0.1L$ on the die surface [46].

The observed trend in the calculated time constant, with variation in die size, could not be predicted on the basis of commonly used thermal models, which invoke an infinite or semi-infinite substrate approximation. The significance of these results is discussed more fully in [46]. The differences in results obtained by analytical, Eq. (20), and numerical, Eqns. (17) and (21), Laplace inversion respectively, are indistinguishable on the scale of this plot. Numerical Laplace inversion gives by far the fastest solution, and the explicit time constant form, Eq. (19), requires summation of an impractical number of terms for sufficient accuracy at very small times.

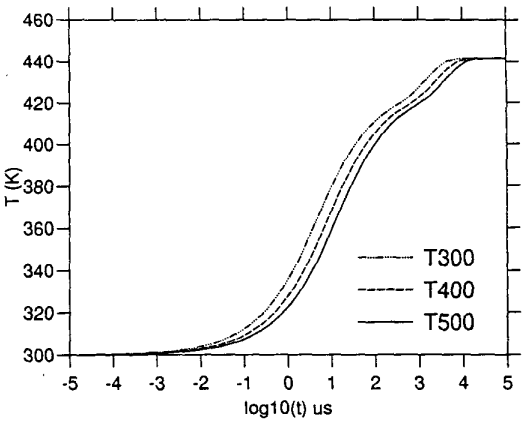

Fig. 1. Temperature rise with time at turn-on, in the immediate vicinity of the device active region in a central square, $0.1L \times 0.1L$, on the surface of cubic die, side $L = 300, 400, 500\ \mu$m, dissipating 0.3, 0.4, 0.5 W.

B. Volume Sources

To construct the time dependent thermal solution with volume heat sources/sinks and arbitrary initial conditions, requires the solution of Poisson's equation in Laplace transform s-space. This section presents an original technique for the generation of double Fourier series solutions, describing arbitrarily distributed volume heat sources and sinks, without the use of Green's functions. It therefore greatly extends the descriptive power of the Fourier approach beyond the surface and interface source terms that have been treated previously [4],[11], [12].

Writing the time-dependent heat diffusion equation with volume heat source in s-space,

$$
\nabla^2\bar{\theta} - \frac{s}{k_S}\bar{\theta} = -\left[\frac{1}{\kappa_S}\bar{g}(x,y,z;s) + \frac{\theta(\tau=0)}{k_S}\right], \tag{22}
$$

and assuming a generalised double Fourier series solution of the form,

$$
\bar{\theta}(s) = \sum_{mn} \cos\lambda_m x \cos\mu_n y Z_{mn}(z), \tag{23}
$$

gives,

$$
\frac{d^2 Z_{mn}}{dz^2} - \gamma^2_{mn} Z_{mn} = G_{mn}(z), \tag{24}
$$

where,

$$
\begin{aligned}
&G_{mn}(z) = \frac{-4}{(1+\delta_{m0})(1+\delta_{n0})LW} \times \\
&\int_0^L \int_0^W \cos\lambda_m x \cos\mu_n y \left[\frac{1}{\kappa_S}\bar{g}(x,y,z;s) + \frac{\theta(\tau=0)}{k_S}\right] \\
&\qquad dydx.
\end{aligned}
\tag{25}
$$

To solve Eq. (24) define,

$$
Z_{mn} = e^{\gamma_{mn} z} z_{mn}, \tag{26}
$$

and make the substitution,

$$
\zeta_{mn} = \frac{dz_{mn}}{dz}, \tag{27}
$$

to reduce Eq. (24) to an equation of 1^{st}-order in ζ_{mn}. This linear 1^{st}-order equation can be solved by use of a simple integrating factor, giving the general solution,

$$Z_{mn} = e^{-\gamma_{mn}z}\int_0^z e^{2\gamma_{mn}z'}\int_0^{z'} e^{-\gamma_{mn}z''}G_{mn}(z'')dz''dz' + \frac{c_{1mn}}{2\gamma_{mn}}e^{\gamma_{mn}z} + c_{2mn}e^{-\gamma_{mn}z}, \quad \text{for } (m,n)\neq(0,0), \quad (28)$$

(and $c_{100} + c_{200}\,z$ for $(m,n) = (0,0)$.) This solution of Eq. (24) contains two arbitrary constants so is a general solution, valid for all boundary conditions. It is to be distinguished from a Green's function solution constructed for a δ-function source, with in-built boundary conditions.

These solution techniques are individually implicit in standard texts. However, the authors believe that this double Fourier series method, for treatment of arbitrary volume sources or sinks without use of Green's function techniques, represents an original approach to solution of the time-independent and time-dependent heat diffusion equations. The double series solution, Eq. (23), is to be compared with much more computationally expensive triple series solutions obtained using Green's functions. This approach is not discussed in heat conduction textbooks such as [24], [47], [48].

Using this approach, the thermal impedance matrix for power dissipating volumes, distributed arbitrarily through the body of a MMIC, is given by

$$R_{TH_{ij}}(s) = \frac{1}{\kappa LW}\sum_{mn}\frac{I^i_{mn}I^j_{mn}}{I^i_{00}}\frac{4}{(1+\delta_{m0})(1+\delta_{n0})}\frac{1}{\gamma^2_{mn}} \times \begin{bmatrix} \frac{\sinh\gamma_{mn}z_{i2}-\sinh\gamma_{mn}z_{i1}}{\gamma_{mn}(z_{i2}-z_{i1})}\times \\ \frac{[\cosh\gamma_{mn}(D-z_{i1})-\cosh\gamma_{mn}(D-z_{i2})]}{\cosh\gamma_{mn}D} \\ +1-\frac{\sinh\gamma_{mn}(z_{i2}-z_{i1})}{\gamma_{mn}(z_{i2}-z_{i1})} \end{bmatrix}. \quad (29)$$

Here, z_{i1}, z_{i2} are the z-coordinates of the planes bounding heat dissipating volume, i, in the z-direction, and the I^i_{mn} are the area integrals over the $x-y$ cross-sections, D_i, of heat dissipating volumes, i, Eq. (15). This expression is to be compared with the thermal impedance matrix for power dissipating surface areas, Eq. (17). Taking the limit, $z_{i2} \to z_{i1}$, gives the solution for a die with dissipating areas distributed arbitrarily throughout its volume, of value for instance in describing the buried channels below the semiconductor surface of a multi-gate power FET. Taking the further limit, $z_{i2}, z_{i1} \to 0$, reproduces Eq. (17).

This solution also makes possible treatment of the time-dependent problem for other than homogeneous initial conditions. It therefore allows construction of a time-stepping thermal impedance matrix formulation for transient electro-thermal simulations, with repeated analytical resetting of initial conditions for the whole distributed thermal volume. Details will be presented elsewhere.

C. Rectangular N-Layer

The simple descriptions of the homogeneous MMIC, presented above, are readily generalised to treat multi-layer systems by use of a transfer matrix, or two-port network, approach [49]. This is based on matching of Fourier components at interfaces, and corresponds to use of the double cosine transform to convert the 3-dimensional partial differential equation, Eq. (5), into a 1-dimensional ordinary differential equation for the z-dependent double Fourier series coefficients. Matching of linearised temperature and flux at the interfaces of a multi-layer structure can then be imposed by use of a 2×2 transfer matrix on the Fourier series coefficients and their derivatives. Arbitrary N-level structures can be treated. Different thermal conductivities can be assumed in each layer allowing treatment of composites like Cu on AlN (both having temperature independent thermal conductivities) and MMICs with conductivities varying from layer to layer due to differences in doping levels (all layers having the same functional form for the temperature dependence of the conductivity).

The corresponding form for the impedance matrix is,

$$R_{TH_{ij}}(s) = \sum_{mn}\cos\lambda_m x_j \cos\mu_n y_j\,(A_{mn}/B_{mn}) \times \frac{-4}{\kappa_1 LW(1+\delta_{m0})(1+\delta_{n0})\gamma^{(1)}_{mn}} I^i_{mn}, \quad (30)$$

where,

$$\begin{pmatrix} A_{mn} \\ B_{mn} \end{pmatrix} = \underline{\underline{M}}^{(1)}\underline{\underline{M}}^{(2)}\ldots\underline{\underline{M}}^{(N-1)}\begin{pmatrix} 1 \\ -\coth\gamma^{(N)}_{mn}D_N \end{pmatrix}, \quad (31)$$

with,

$$\gamma^{(r)}_{mn} = \left(\lambda^2_m + \mu^2_n + \frac{s}{k_r}\right)^{1/2}, \quad (32)$$

and the layers have thickness, D_r, thermal conductivity, κ_r, and diffusivity k_r, $r = 1,\ldots,N$, respectively. The $\underline{\underline{M}}^{(r)}$ are analytically obtained 2×2 matrices, explicitly determined entirely by $\kappa_r, \kappa_{r+1}, \gamma^{(r)}_{mn}, \gamma^{(r+1)}_{mn}$ and D_r. These transfer matrices become nearly singular at high frequencies, simplifying construction of the multi-layer response.

To illustrate the accuracy and speed of this method [21], the above analytical solution for an N-level multi-layer, with $s \to i\omega$, is used to plot the complex locus of the thermal transfer impedance in Fig. 2. The 4-layer, heatsink mounted device considered, is a structure examined by Szekely *et al.*, ([7] Figs. 5 and 6; [30] Fig. 17). Agreement with the calculations of Szekely seems good. The data for this figure took less than 1s to generate on a 500 MHz Pentium processor and consists of 65 frequency points. This can be compared with Szekely's published

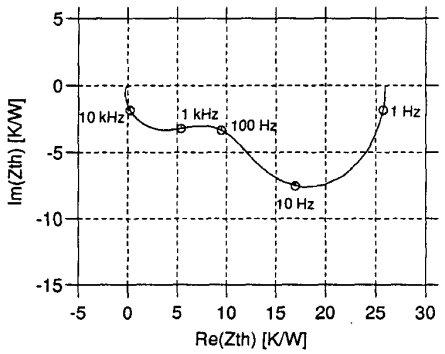

Fig. 2. Complex locus of the thermal transfer impedance, calculated using an analytical series expression, for a 4-layer, heatsink mounted structure examined by Szekely *et al.*

steady-state simulation times, [50]. The comparison suggests that despite the speed of the FFT, the need to generate redundant temperature information on a surface mesh in Szekely's semi-analytical function sampling or collocation approach, makes the fully analytical double Fourier series transfer matrix method at least 10× faster for the same number of basis states.

The method can be generalised further, by imposing specified flux discontinuities at the interfaces. The solution then represents, for instance, the case of a MMIC with active device channel buried by a thin layer of semiconductor, as described by Eq. (29) (with $z_{i2} \rightarrow z_{i1}$), but distinguishing the thermal conductivities of the various semiconductor layers. This transfer matrix approach can also be combined with the volume heat source solution of Sec. III-B, to describe rectangular N-layers containing an arbitrary distribution of power dissipating volumes, without the need to introduce artificial interfaces.

Digele has stated [25] that the Kirchhoff transformation is of no value for N-layer structures. However, this is incorrect. The Kirchhoff transformation is exact for multilayers with the same functional form (but different values) for the temperature dependent thermal conductivity in each layer. A single, global Kirchhoff transformation is also a good approximation for multilayers in which the functional form of $\kappa(T)$ differs between layers, so long as an appropriately modified value for κ_S is chosen in each layer for which the global transformation is not exact [51], [52].

D. MMIC Superstructure

It has been demonstrated that inclusion of surface metallisation is essential for accurate description of thermal effects in power devices [34],[52]. Comparison with experiment for multi-finger power HBTs shows that the simple thermal description corresponding to the resistance matrix of Eq. (18) is highly accurate when combined with a simple model of heat shunting by an air bridge [53]. The analytical thermal resistance and impedance matrix approach presented here, has been designed to allow descriptions of surface metallisation and air bridges, and other vertical geometries such as flip chips and solder bumps, as well as MMIC arrays, as outlined below.

The extension to include complex 3-dimensional structure is achieved by solving the heat diffusion equation analytically for thermal sub elements, then combining thermal impedance matrices for subsystems by matching of temperature and flux at discretised interfaces. For illustration, specifying flux on top and bottom surfaces, $z = 0, D$, and assuming no radiative or convective surface losses, ($\alpha_{0,D} = 1$, $H_{0,D} = 0$), the following relations are obtained for temperatures, $\overline{\theta}_{0\,av_i}$ and $\overline{\theta}_{D\,av_j}$, averaged over areas, D_i, and D_j, on faces $z = 0$ and $z = D$, respectively,

$$\begin{aligned} \overline{\theta}_{0\,av_i} - \frac{\theta(\tau=0)}{s} &= \sum_{i'} R^{00}_{TH_{ii'}} \overline{P}_{0i'} + \sum_{j} R^{0D}_{TH_{ij}} \overline{P}_{Dj}, \\ \overline{\theta}_{D\,av_j} - \frac{\theta(\tau=0)}{s} &= \sum_{i} R^{D0}_{TH_{ji}} \overline{P}_{0i} + \sum_{j'} R^{DD}_{TH_{jj'}} \overline{P}_{Dj'}. \end{aligned} \tag{33}$$

Here, $\overline{P}_{0i}$ and $\overline{P}_{Dj}$ are respective imposed fluxes in elementary areas, D_i and D_j, on faces $z = 0$ and $z = D$. The thermal impedance matrices are obtained in the form [22],

$$\begin{aligned} R^{00}_{TH_{ii'}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4\coth\gamma_{mn}D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{0i}_{mn} I^{0i'}_{mn}}{I^{0i}_{00}}, \\ R^{0D}_{TH_{ij}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4\,\mathrm{cosech}\gamma_{mn}D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{0i}_{mn} I^{Dj}_{mn}}{I^{0i}_{00}}, \\ R^{D0}_{TH_{ji}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4\,\mathrm{cosech}\gamma_{mn}D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{Dj}_{mn} I^{0i}_{mn}}{I^{Dj}_{00}}, \\ R^{DD}_{TH_{jj'}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4\coth\gamma_{mn}D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{Dj}_{mn} I^{Dj'}_{mn}}{I^{Dj}_{00}}, \end{aligned} \tag{34}$$

where the I^{0i}_{mn} and I^{Dj}_{mn} are area integrals of the form, Eq. (15), over elementary areas D_i and D_j, on faces $z = 0$ and $z = D$, respectively. These expressions are readily inverted analytically or numerically, as described in Sec. III-A, to give (fully parameterised) time domain results. As described in [21], analytical inversion gives rise to explicit expressions for the individual terms in pole-zero or time constant representations and allows direct construction of time constant spectra.

(Resistance matrix expressions for the time-independent case have been presented in [12]. Alternative forms for the series constructions given there, can be obtained using closed form summation and series acceleration.)

These series expressions represent generalised multiport Z-parameters for the distributed thermal subsystems. Network parameter descriptions for thermal systems have been described previously, [5]. However, the construction of S-parameters described in [5] was neither analytical, nor simple, and required complex numerical manipulations based on the boundary element method. This model also did not treat thermal non linearity and involved explicit model reduction and approximate fitting of S-parameters.

Combining the thermal impedance matrices for individual subvolumes, a global thermal impedance matrix for complex 3-dimensional systems can be obtained. This is illustrated in the next section [21] for the case of a metallised MMIC. More generally, thermal subsystems can be represented individually by netlist elements in circuit simulation. Expressing the thermal impedance matrices as non linear elements in the time domain, then allows non linear matching of interface temperatures at subsystem interfaces, in those cases where the functional form of the Kirchhoff transformation differs between subvolumes.

The s-space formulation means that no artificial piecewise constant time dependence is assumed for interface fluxes, in contrast to the time-domain USE method [24]. However, the thermal impedance matrix approach can be developed within the USE framework [14], with direct time domain interface matching, where it avoids repeated matrix inversion.

E. Global Impedance Matrices

Construction of thermal impedance matrices is now described for more complex systems, such as MMICs with surface metallisation.

To illustrate the interface matching approach, the global thermal impedance matrix is constructed for the case of N pieces of rectangular, but otherwise arbitrary, metallisation on the surface of an otherwise homogeneous heatsink

mounted MMIC. Matching flux and (linearised) temperature at the interface between metal and MMIC die, the following relation is obtained

$$\underline{\Delta\theta}^a = \underline{\underline{R}}_{TH}^{glob}\underline{P}^a, \tag{35}$$

where $\underline{P}^a$ is the vector of MMIC active device power dissipations, $\underline{\underline{R}}_{TH}^{glob}$ is the global thermal impedance matrix for the coupled GaAs and metal system, and $\underline{\Delta\theta}^a$ is the vector of MMIC active device temperature rises.

The global impedance matrix is given explicitly by

$$\underline{\underline{R}}_{TH}^{glob} = \underline{\underline{R}}_{TH}^{aa} + \underline{\underline{R}}_{TH}^{ai}\underline{\underline{R}}^{-1}\underline{\underline{R}}_{TH}^{ia}, \tag{36}$$

$$\underline{\underline{R}} = \mathrm{diag}\left(\underline{\underline{R}}_{TH}^{DD(1)}, ..., \underline{\underline{R}}_{TH}^{DD(n)}, ..., \underline{\underline{R}}_{TH}^{DD(N)}\right) - \underline{\underline{R}}_{TH}^{ii}. \tag{37}$$

Here, $\underline{\underline{R}}_{TH}$ of Eq. (17) for the MMIC die has been partitioned by active device elementary areas, a, and interface elementary areas between MMIC die and metal, i, and the $\underline{\underline{R}}_{TH}^{DD(n)}$ are thermal impedance matrices for each piece of metallisation, Eq. (34).

Thus, by simple matrix manipulation, the global thermal impedance matrix for the metallised MMIC can be obtained as an explicit matrix expression for any given value of Laplace transform variable, s. Also, using the simple algorithm for the numerical Laplace inverse, Eq. (21), the value of the global thermal impedance matrix can be evaluated at any time step, $n\delta\tau$, in the time domain. The s-space formulation means that when power dissipation is known *a priori*, temperature can be obtained directly at any required instant, without the need to take consecutive timesteps from $\tau = 0$. In cases where non linear interface matching cannot be neglected, the thermal impedance matrix approach allows formulation of a non linear system of equations for the correctly matched temperatures [11].

The significance of the relation, Eq. (36), should be stressed. It represents an explicit analytical expression for the solution of the time-dependent heat diffusion equation in an arbitrarily complex 3-dimensional volume. In contrast to conventional numerical techniques, such as FDTD or FETD, it requires no volume mesh, discretising only interfaces (and power dissipating and temperature sensitive elements). It is therefore extremely simple to formulate and implement, avoiding the large preparation times of FE simulations, as well as the intricacies of FDTD and FETD code for complex structures. The solution is modular and hierarchical, so once the global impedance matrix has been constructed for a single metallised MMIC, this could then, for example, be used to describe each MMIC in an $N \times N$ MMIC array. The global impedance matrix for the metallised MMIC only has to be constructed once, to describe all N^2 identical MMICs. It could also be stored for re-use in later coupled electro-thermal simulations, cutting later precomputation time effectively to zero. Finally, there is no restriction on heat loss mechanisms involved in this solution, and for instance, ultimate heat loss from the system could be purely by radiation and convection from the grid array substrate, without any heatsink mounting.

This method therefore avoids all the previous limitations of fully analytical approaches, as listed for instance in [31], and provides a natural solution to the problem of variation in length scale over the whole of an electro-thermal system. Resolution of temperature in each thermal subsystem is defined by its local coordinate system and the corresponding double Fourier series expansion. There is no need for any sort of uniform mesh resolving the finest detail at all length scales, or for imposed non uniform grid construction. Also, by development of closed form and accelerated expressions for the thermal impedance matrices, as indicated earlier and to be presented elsewhere, all series convergence rates are fast and resolution limits are removed within any single thermal subsystem.

Finally, the method presented here is immediately compatible with direct, explicitly coupled electro-thermal device and circuit level simulation of complex structures on CAD timescales. The directly coupled thermal impedance matrix approach represents 'near exact' solution of the non linear time dependent heat diffusion equation for the complex 3-dimensional system, at points, or averaged over regions, corresponding to power dissipating and temperature sensitive elements. The only approximations are finite interface discretisation between thermal subsystems; in the time domain, the assumption of piecewise constant time variation and numerical Laplace inversion if employed; and use of a single global Kirchhoff transformation if non linear interface matching between subsystems in not imposed.

F. Inhomogeneous Thermal Conductivity

The analytical double Fourier series solution for the thermal impedance matrix can be further generalised to treat, essentially exactly, piecewise uniform, but otherwise arbitrarily inhomogeneous thermal conductivity, such as full and partial thickness vias, and partial substrate thinning in power transistors and MMICs. A computationally much cheaper, but approximate, treatment of vias, based on the the simple equivalence principle method of Bonani *et al.*, [3], has also been implemented within the thermal resistance matrix approach. Construction of such solutions for the time-independent case is described in [12].

Vertical matching obtainable by use of the 'radiation' boundary condition, Eq. (7), can be extended by removal of the adiabatic side wall assumption. This allows horizontal matching of rectangular subvolumes for which flux boundary conditions are prescribed on all faces. The corresponding double Fourier series solution takes the form,

$$\bar{\theta}(s) = \frac{\theta(\tau=0)}{s} + \frac{1}{s}\times \sum_{mn}\left\{\begin{array}{ll} & \cos\lambda_m x\cos\mu_n y \\ & \times(A_{mn}\cosh\gamma_{mn}z + B_{mn}\sinh\gamma_{mn}z) \\ + & \cos\mu_m y\cos\gamma_n z \\ & \times(C_{mn}\cosh\lambda_{mn}x + D_{mn}\sinh\lambda_{mn}x) \\ + & \cos\lambda_m x\cos\gamma_n z \\ & \times(E_{mn}\cosh\mu_{mn}y + F_{mn}\sinh\mu_{mn}y) \end{array}\right\}, \tag{38}$$

where,

$$\lambda_n = \frac{n\pi}{L},\quad \mu_n = \frac{n\pi}{W},\quad \gamma_n = \frac{n\pi}{D},$$
$$\lambda_{mn}^2 = \mu_m^2 + \gamma_n^2 + \frac{s}{k_S}\quad,\quad \gamma_{mn}^2 = \lambda_m^2 + \mu_n^2 + \frac{s}{k_S},$$
$$\text{and}\quad \mu_{mn}^2 = \lambda_m^2 + \gamma_n^2 + \frac{s}{k_S}. \tag{39}$$

The expansion coefficients $A_{mn}, B_{mn}, ...$, are all obtained as explicit analytical expressions.

Combining the above solution with that for arbitrary distributions of heat sources described in Sec. III-B, gives a fully analytical description of arbitrarily inhomogeneous structures based on small dense matrix manipulation.

IV. Coupled Electro-Thermal Transient

The thermal impedance matrix in s-space can be used directly in coupled electro-thermal harmonic balance simulations. In this case, the matrix of frequency dependent complex phasors corresponds to the network parameters of the distributed multi-port thermal network. It is inserted directly into the MNAM for the microwave system and so does not increase the number of non linear equations describing the coupled solution.

In the coupled electro-thermal transient problem, Laplace transformed active power dissipations, $\overline{P}_j(s)$, are not known explicitly and must be obtained by self-consistent solution. To combine the electrical and thermal descriptions, the corresponding $P_j(\tau)$ must therefore be discretised in time. Dividing the time interval of interest into equal subintervals of length $\delta\tau$, with the $P_j(\tau)$ taking the piecewise constant form (for illustration)

$$P_j(\tau) = P_j^{(n)} \quad \text{for} \quad (n-1)\delta\tau < \tau \leq n\delta\tau, n = 1, ..., N \quad (40)$$

then gives

$$\overline{P}_j(s) = \sum_n \frac{1}{s}(1 - e^{-s\delta\tau})e^{-(n-1)s\delta\tau}P_j^{(n)}. \quad (41)$$

Laplace inverting the impedance matrix equation, Eq (16), the temperature rise of element i at time $t = m\delta\tau$, $\Delta\theta_i^{(m)}$, is obtained as a function of the $P_j^{(n)}$. Writing $\Delta\theta_i^{(m)} = \Delta\theta_i^{(m)}(P_i^{(m)})$ from the electrical model then gives,

$$\Delta\theta_i^{(m)}(P_i^{(m)}) = \mathcal{L}^{-1}\left\{R_{TH_{ij}}(s)\overline{P}_j(s)\right\}_{\tau = m\delta\tau}, \quad (42)$$

$$= \sum_n \sum_j \left[u(m-n+1)R_{TH_{ij}}((m-n+1)\delta\tau) - u(m-n)R_{TH_{ij}}((m-n)\delta\tau)\right] P_j^{(n)}, \quad (43)$$

where $u(\tau)$ is the unit step function.

This corresponds to N systems of equations in M unknowns, where N is the number of discretised time points in the time interval under consideration, and M is the number of power dissipating or temperature sensitive elements. The Laplace inversion, with piecewise constant power dissipation, avoids any explicit convolution operation.

The entire thermal description can therefore be obtained by precomputation of $R_{TH_{ij}}(\tau)$ at timesteps, $\tau = n\delta\tau$, $n = 0, ..., N$. These precomputed values can be stored for repeated re-use in different electro-thermal simulations. For reduction of precomputation time, the $R_{TH_{ij}}(\tau)$ can be generated at intervals, and interior points obtained accurately by interpolation. This is a time-domain approach equivalent to representation of a frequency space transfer function by a polynomial fit.

Extension to linear, quadratic or higher order interpolation of the active device power dissipations in each subinterval, $\delta\tau$, is immediate, and for sufficiently short step lengths, low orders of interpolation should be required.

After self-consistent electro-thermal solution, and inversion of the Kirchhoff and time variable transformations, physical active device temperatures are finally obtained as a function of physical time, $T_i(t)$, and electrical solutions, DC or RF, are determined.

V. Compact Models

In the fully analytical approach described above, the global thermal impedance matrix describing a complex 3-dimensional system, such as a packaged electronic component, consists of a minimal number of thermal impedances describing self-heating and mutual thermal interaction at only those sites chosen to be of interest for electro-thermal simulation. These thermal impedances do not have a direct interpretation in terms of discretised physical layout, but constitute 'thermal links' as defined in [9]. The general form of the equivalent circuit, corresponding to thermal resistance matrix $\underline{\underline{R}}_{TH}$ for the time independent case, is shown in Fig. 3 [10], [53], and has been employed successfully in SPICE-like circuit simulation [53].

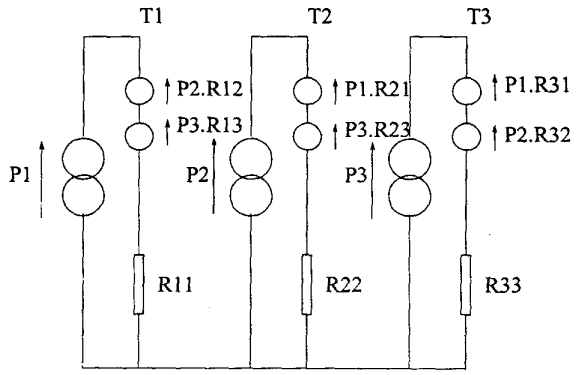

Fig. 3. Thermal circuit corresponding to resistance matrix Rij.

This basic form generalises readily to arbitrary numbers of nodes, unlike thermal networks based on direct physical discretisation of the thermal system, which can grow rapidly more convoluted with increase in size. Generalisation to the time-dependent thermal impedance matrix case is immediate. Implementation of the thermal multiport network in electro-thermal CAD can be achieved in both the time and frequency domains, as described above. Direct use of the solution of the heat diffusion equation, in the form of explicit double Fourier series expressions for thermal impedance matrices describing thermal multiports, avoids the need for lumped element RC network generation, and is already minimal without any node reduction such as that described in [54].

The thermal impedance matrix, which is generated analytically based on the imposed boundary condition, Eq. (7), constitutes a boundary condition independent compact model, as defined broadly by Lasance [8], [9]. Treatment of the time-dependent case, with description of thermal non linearity, represents a generalisation of the time-independent thermal resistance networks generally defining compact models. The analytically imposed 'radiation' boundary condition, Eq. (7), is sufficiently general to include a wide range of boundary condition sets, including free and forced convection, heat sink, cold plate and fluid bath, as well as unbalanced ambient temperatures.

Where full layout details are not available, or deviate from nominally specified values, global thermal impedance

matrix expressions, such as Eq. (36), provide a fully parameterised expression for compact model optimisation.

VI. Device Simulation

To illustrate the predictive value of the analytical thermal impedance matrix approach for description of detailed device structure in coupled electro-thermal simulations, the fully physical simulation of power FETs is now described for the time-independent case. The thermal resistance matrix model is coupled to the Leeds Physical Model (LPM). This is a fast, quasi-2-dimensional model of MESFETs and HEMTs [16]–[19]. It makes fully physical prediction of device performance based solely on specified layer compositions and doping levels and details of the device cross-section. It is thermally self-consistent, with device self-heating described by a temperature dependent mobility. The LPM requires no prior experimental device characterisation.

For the coupled electro-thermal solution, transistor action described by the thermally self-consistent device model gives the non linear relation,

$$\Delta\theta_i = \Delta\theta_i(P_i), \tag{44}$$

where each active element, i, corresponds to an individual transistor finger. Here, the Kirchhoff transformation has been applied to obtain the function $\Delta\theta_i(P_i)$ from the physical temperature dependence of the model, so thermal non linearity has been shifted from the thermal model to the already non linear active device model. Combining the active device model for each individual source-gate-drain sub-unit, Eq. (44), with the global thermal description gives

$$\Delta\theta_i(P_i) = \sum_j R_{TH_{ij}} P_j \tag{45}$$

which is a small, simple, non linear system to be solved self-consistently for the power densities, P_j. Having obtained the P_j at each bias point, from solution of the coupled electro-thermal problems, Eq. (45), by a simple relaxation algorithm, the full electrical solution is obtained and I-V curves are plotted. The temperature over the surface of the die, at a specified bias point, is obtained analytically once individual finger power dissipations have been obtained self-consistently.

Simulations are described of a 10-finger power HEMT, FP4000, provided by Filtronic plc. [13]. In particular, these calculations are used to ascertain the minimum physical description compatible with accurate construction of the thermal resistance matrix, e.g. inclusion of surface metallisation, air bridges, vias or surface flux losses.

Flux losses from the surface of a die can, in principle, act to reduce the thermal resistance. Radiative losses are easy to estimate and are orders of magnitude too small to have any significant impact [11]. If convective losses are of the same order of magnitude as radiative losses, then these too are insignificant. However, the magnitude of convective losses from small areas with fine surface structure are not easily estimated. Standard correlations from the literature tend to be for large area substrates. In the absence of a detailed model of fluid flow, significant convective losses from the die surface could not be totally discounted, however these effects have not been suggested in the literature as significant at the scale of the FP4000 die.

The effect of substrate thinning on device performance was examined, and two sets of simulations were performed for dies of differing substrate thicknesses, one set including convective losses, one set excluding convective losses. The results of these simulations are described below. Figure 4 shows I-V curves calculated on the assumption of a 6 μm thick uniform layer of metallisation covering the whole of the power FET surface. This is a reasonable approximation in the case of the heavily metallised FP4000. Zero surface convection and a 75 μm thick substrate were assumed. A full suite of I-V curves took around 30 minutes to produce on a 500 MHz Pentium processor.

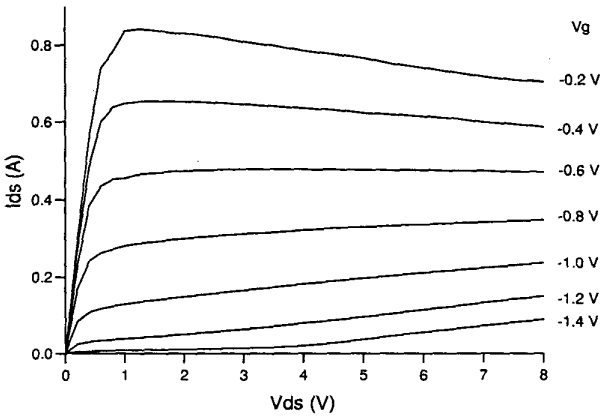

Fig. 4. Electro-thermally simulated I-V characteristics of the 10-finger Filtronic power HEMT with 6 μm surface metallisation and adiabatic surface boundary conditions. Finger width 400 μm; substrate thickness 75 μm; heatsink temperature 300 K.

Power dissipation at a bias point $(V_{DS}, V_G) = (3.5V, -0.2V)$ was calculated to be 2.8 W, and temperature at the metal-GaAs interface was calculated to vary from 27°C to 67°C. The simulation was repeated for a 400 μm thick substrate. At the same bias point, power dissipation was then found to be 2.1 W and interface temperature varied from 71°C to 127°C.

Figure 5 shows simulated temperature at the GaAs/metal interface, i.e. in the layer of the active device channels, for FP4000 with substrate thickness of 75 μm, but now assuming strong surface convection. Power dissipation is 3.0 W and temperature varies from 27 °C to 49 °C.

A further simulation of temperature at the GaAs/metal interface for FP4000 with substrate thickness of 400 μm and strong surface convection gave very similar results. Power dissipation at the same bias point was calculated to be 2.9 W and temperature varied from 27 °C to 54 °C. The effect of including a strong convective surface flux is to make active device temperature largely independent of substrate thickness, in contrast to the zero flux case.

Simulations constructed thermal resistance matrices for a bare GaAs die, a die with surface metallisation, a metallised die with vias, and a metallised die with strong surface convection. Vias were calculated to have negligible impact on thermal calculations of device performance. Surface metallisation was shown to have a heat spreading effect, reducing die peak temperatures and the corresponding thermal droop in I-V curves. These electro-thermal results are in agreement with previous thermal calculations, *e.g.*, [3], [34]. Increased substrate thickness, in the ab-

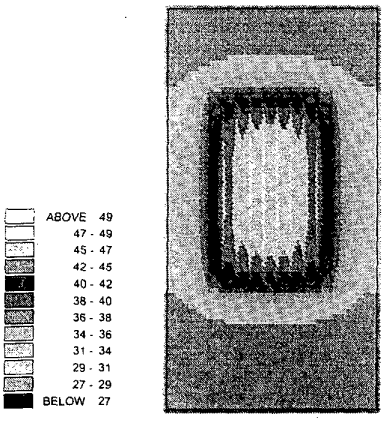

Fig. 5. Electro-thermally simulated interface temperature plot of the 10-finger Filtronic power HEMT with 6 μm surface metallisation and convective surface fluxes. Finger width 400 μm; substrate thickness 75 μm; heatsink temperature 300 K. Bias point (V_{DS}, V_G) = (3.5 V, -0.2 V); power dissipation 3.0 W. Temperature varies from 27 °C to 49 °C.

sence of convection, was seen to imply increased surface temperature. Imposition of large surface convective fluxes appeared unphysical as they implied surface temperature profiles largely independent of substrate thickness, in contrast to results obtained experimentally.

These results illustrate the power of coupling the fast, fully physical thermal model to the fast, fully physical electrical model. Such coupling provides accurate, CAD timescale prediction of trends in device performance with variation in detailed device structure, without the need for prior experimental characterisation.

VII. Circuit Simulation

Fully physical device level simulation based on relaxation, using the thermal resistance matrix approach for the time-independent case, has been described. Circuit level simulation based on simultaneous iteration, using the time-dependent thermal impedance matrix in microwave circuit simulator, Transim (NCSU) [20], is now illustrated.

A. Transim (NCSU)

Transim has an input format that is similar to the SPICE format with extensions for variables, sweeps, user defined models, and repetitive simulation. The program provides a variety of output data and plots. Transim allows the addition or removal of new circuit elements in a very simple way. It is designed so that new circuit elements can be coded and incorporated into the program without modification to the high-level simulator. It is also quite simple to add a new analysis type. Some insight into the program architecture is given in [20]. Simulations were performed using state variable harmonic balance [55] and convolution transient [56] methods.

Thermal effects were incorporated into the circuit simulator engine by making the thermal model look like an electrical circuit [57], [58], specifically a multi-port network described in either the time or frequency domain. As a result of the fully analytical description of thermal subsystems, no separate thermal simulator is required to characterise the thermal system, prior to coupled simulation by the electro-thermal simulation engine.

B. 3×3 MMIC Array

Fig. 6 illustrates calculated temperature response of a power FET array (inset) obtained from self-consistent electro-thermal, single-tone harmonic balance simulation, by implementing the thermal impedance matrix approach in Transim. The thermal description includes non linear-

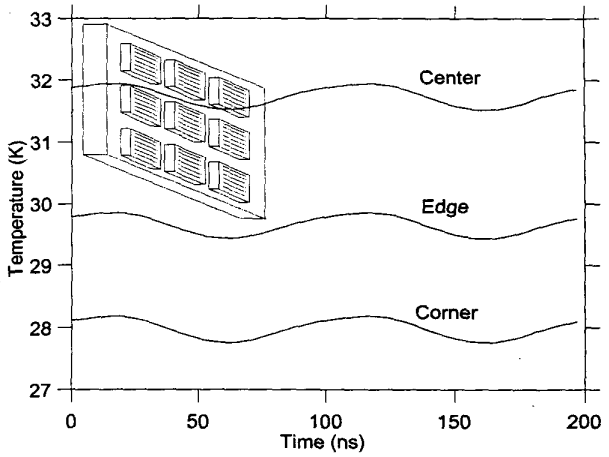

Fig. 6. Single-tone harmonic balance simulation of a power FET array (inset) representative of one kind of spatial power combining architecture. The global simulation is electro-thermally self-consistent. Heat loss is entirely by linearised radiation and convection into free space, with no heatsink mounting.

ity due to temperature dependence of material parameters. Heat loss is purely by linearised radiation and convection from the substrate, with no heatsink mounting. The thermally self-consistent electrical description is provided by the Curtice Ettemberg cubic model of the MESFET, with symmetric diodes and capacitances, embedded in a simple amplifier circuit, as illustrated in [21].

VIII. Conclusion

An original, fully analytical, spectral domain decomposition approach to the solution of the non linear time-dependent heat diffusion equation in complex 3-dimensional systems, has been described. This approach is immediately compatible with coupled electro-thermal device and circuit level simulation, on CAD timescales. This compatibility is achieved by implementation of the thermal solution as a minimal compact model in the form of analytically obtained thermal impedance matrices, describing temperature response only at power dissipating and temperature sensitive elements.

This fully analytical thermal model is observed to be at least 10× faster than corresponding semi-analytical Fourier solutions for N-level multilayers, and also treats arbitrarily complex 3-dimensional systems without invoking conventional numerical methods. The minimal thermal impedance matrices have immediate interpretation in terms of generalised multi-port thermal network parameters, in both the time and frequency domains, avoiding the need for approximation of the distributed thermal system by a lumped element RC network requiring node reduction.

The problem of thermal non linearity, due to temperature dependent thermal diffusivity, has been treated fully for the first time in electro-thermal CAD. This required application of a time variable transformation, in addition

to the well known Kirchhoff transformation for treatment of temperature dependent thermal conductivity. In contrast with many electro-thermal CAD models, which neglect thermal non linearity in order to generate a linear thermal network, the model presented here can treat fully thermal non linearity due to temperature dependence of material parameters, as well as that due to non linear surface fluxes in large area systems.

A range of original thermal solutions have been presented in the form of thermal impedance matrices for electro-thermal subsystems. In particular, an original, Green's function free, approach to the double Fourier series solution of problems with arbitrarily distributed volume heat sources and sinks, has been described. A double Fourier series solution for prescribed flux on all faces of a rectangular volume has also been presented. The construction of global thermal solutions for complex 3-dimensional systems, based on the thermal impedance matrix approach, has been outlined.

Use of the thermal impedance matrix approach in electro-thermal CAD has been illustrated by combination with the Leeds Physical Model of MESFETs and HEMTs, and by implementation in microwave circuit simulator, Transim (NCSU).

The ability to calculate trends in electro-thermal device performance, with variation of detailed device structure, has been indicated by fully coupled, physical electro-thermal simulation of power HEMT FP4000. These results illustrate the power of modelling the temperature rise physically. The effect of vias, surface metallisation, substrate thinning and surface fluxes can be predicted on the basis of device geometry and material constants.

A circuit level calculation of thermal response in a spatial power combining MMIC grid array has been presented and represents a fully coupled, electro-thermal simulation, of a highly complex and non linear system.

Future development of the Leeds thermal impedance matrix model in Transim will include extension of the approach to generate explicitly, heat transfer coefficient, H, describing surface flux losses. It will explore wavelet techniques for reduction of large, $N \times N$, dense matrix eigenvalue and inversion problems, from $O(N^3)$ operation to $O(N)$ operation processes, with particular application to the economical description of inhomogeneous thermal subsystems. It will include integration of the rapid Leeds Physical Model of MESFETs and HEMTs, into circuit simulator, Transim, to produce fully physical, coupled electro-thermal, circuit level CAD. Finally, it will include development of circuit simulation techniques to treat more fully the huge range of time constants inherent in fully coupled electro-thermal simulations of large systems.

The modelling capability described here will be applied to the study and design of spatial power combiners for use as high power sources at millimeter wavelengths.

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