Comparison of Wavelet- and Time-Marching-Based Microwave Circuit Transient Analyses

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Abstract— **A general reduced state variable circuit formulation is developed and implemented using wavelets in a circuit simulator for the first time. The same formulation can be used with non-wavelet transformations or to implement implicit time marching methods. This formulation is particularly well suited to modeling RF and microwave circuits. This paper reviews the formulation and presents the simulation of a nonlinear transmission line and a 2 by 2 quasi-optical grid amplifier using wavelets and a time marching method.**

I. Introduction

The most widespread method of nonlinear circuit analysis is time-domain analysis (also called transient analysis) using programs like Spice. Such programs use numerical integration to convert a nonlinear system of coupled algebraic and *ordinary differential equations* (ODEs) into a nonlinear algebraic system of equations. The number of nonlinear unknowns is approximately equal to the number of nodes in the circuit.

This paper outlines a method (originally developed to be used with wavelet transformations) of analyzing circuits with the minimum number of unknowns and error functions starting from a *modified nodal admittance matrix* (MNAM) of the linear part of the circuit. This approach has several advantages. The resulting system of nonlinear equations is generally much smaller than the nonlinear system resulting from applying conventional formulations. Also the flexibility of the modified nodal admittance matrix is kept as well as the robustness provided by the state variable approach [6]. This formulation is used to derive circuit transient analyses based on wavelets and the backward Euler numerical integration.

Wavelet basis functions are ideally suited to expanding a response with an overall coarse response but fine behavior in some regions as higher order and more localized basis functions can be concentrated on the regions where the response varies rapidly. Multiresolution analysis has been used with a wide variety of modeling problems including signal processing and electromagnetics. It is important to know where wavelet analysis is applicable as this guides future development. In circuits, voltage and current changes vary with time and location (*e.g.* node index) and so they can be modeled with few state variables by using variable resolution. In contrast, in conventional transient simulation the same fine time step is used at every node. Zhou *et al.* presented the pseudo-wavelet collocation method for simple networks in [3] and [4]. The state-variable-based wavelet transient analysis used in this work was presented in Reference [1]. This circuit analysis technique was implemented in an object-oriented circuit simulator (*Transim*, [5]).

Section II reviews and expands the formulation of the transient analysis originally presented in [1]. It is shown that a state variable transient analysis using the backward Euler method can also be derived by just replacing two matrices in the wavelet formulation. Section III presents the simulation of a nonlinear transmission line (NLTL) and a 2 by 2 quasi-optical grid amplifier using both analysis techniques. The results are discussed. Finally, the conclusions are given in Section IV.

II. Formulation of the Transient Analysis

We begin by reviewing the error function formulation presented in [1]. Wavelets are introduced by considering the function $g(t)$ defined in I. The following square matrices $\mathbf{W}_J$ and $\mathbf{W}'_J$ can be defined:

$$\mathbf{g} = \mathbf{W}_J \hat{\mathbf{g}}_J, \quad \mathbf{g}' = \mathbf{W}'_J \hat{\mathbf{g}}_J \qquad (1)$$

where $\mathbf{g}$, $\mathbf{g}'$ are vectors whose elements are the function and derivatives values, respectively, at the collocation

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points and $\hat{\mathbf{g}}_J$ is the vector of the corresponding coefficients. J is the maximum subspace level being considered.

The final linear circuit equation from [1] is

$$\mathbf{M}_J \hat{\mathbf{u}}_J = \mathbf{s}_{f,J} + \mathbf{T}_{1,J} \mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) \qquad (2)$$

where $\hat{\mathbf{u}}_J$ is the vector of wavelet coefficients of the nodal voltages and selected currents, $\hat{\mathbf{x}}_J$ is the vector with wavelet coefficients of the state variables of the nonlinear devices, $\mathbf{s}_{f,J}$ is the vector of independent sources, $\mathbf{T}_{1,J}$ is defined in [1] and $\mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J)$ is the vector of currents at the nonlinear device ports. The error function $\mathbf{F}(\hat{\mathbf{x}}_J)$ is defined as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{T}_{2,J} \hat{\mathbf{u}}_J - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0$$

where $\mathbf{T}_{2,J}$ is a matrix defined in [1] and $\mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J)$ is the vector of voltages at the nonlinear device ports. Combining the preceding with Eq. (2),

$$\begin{aligned} \mathbf{F}(\hat{\mathbf{x}}_J) &= \mathbf{T}_{2,J} \mathbf{M}_J^{-1} \mathbf{s}_{f,J} \\ &\quad + \mathbf{T}_{2,J} \mathbf{M}_J^{-1} \mathbf{T}_{1,J} \mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) \end{aligned}$$

which can be expressed as

$$\mathbf{F}(\hat{\mathbf{x}}_J) = \mathbf{s}_{sv,J} + \mathbf{M}_{sv,J} \mathbf{i}_{NL,J}(\hat{\mathbf{x}}_J) - \mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J) = 0 \quad (3)$$

Here $\mathbf{s}_{sv,J}$ is the compressed source vector (the initial conditions of the entire linear subcircuit are embedded in it) and $\mathbf{M}_{sv,J}$ is the compressed impedance matrix. They are defined as

$$\begin{aligned} \mathbf{s}_{sv,J} &= \mathbf{T}_{2,J} \mathbf{M}_J^{-1} \mathbf{s}_{f,J} \\ \mathbf{M}_{sv,J} &= \mathbf{T}_{2,J} \mathbf{M}_J^{-1} \mathbf{T}_{1,J} \end{aligned}$$

The system of nonlinear algebraic equations (3) is solved using globally convergent quasi-Newton methods. The size of $\mathbf{M}_{sv,J}$ is $(m-1)n_s \times (m-1)n_s$, where m is the number of collocation points. If the time interval to be simulated requires many collocation points, the nonlinear system to be solved becomes very large. One way to overcome this problem is to divide the total simulation time interval into smaller *windows*. Then solve one time window at a time. The final time sample at each window becomes the initial condition for the next and the method is applied for all windows. The approach becomes thus an hybrid between collocation and time marching methods. There are two main issues in solving the nonlinear system. Obtaining a good initial guess and reducing the number of unknowns. If the circuit being simulated has a periodic excitation, the time window size can be chosen equal to the period. Then the solution for a given time window can be used as a good guess for the solution at the next window. Unfortunately, for some circuits this would imply a time window too large to be to be handled efficiently. An adaptive scheme could be used to somewhat reduce the number of unknowns, but this increases the implementation complexity and only attenuates the problem of having to solve for many unknowns at a time.

This wavelet transient formulation (Eq. (3) could easily be modified to produce a formulation to find the periodic steady-state of a circuit. The only modification is that the equations relating the initial conditions are replaced by boundary condition equations. Another alternative formulation is to express the nonlinear error function in terms of the wavelet coefficients of the port voltages. This yields a similar error function where $\mathbf{v}_{NL,J}(\hat{\mathbf{x}}_J)$ must be transformed from the physical to the coefficient space (using $\mathbf{W}_J^{-1}$). At first, this approach would seem to be less efficient, since the resulting compressed matrix is not sparse in general and it requires the implementation of 'inverse' transformation. Nevertheless, this approach would allow the reduction of the linear system size if some coefficients of the port voltages are known to be zero, which can not be done in the physical space. These alternatives will not be developed here.

A. Initial Conditions in the State Variables

For each state variable, the wavelet coefficients are not completely independent. There is a constraint imposed by the initial condition. Therefore, the first transform coefficient is excluded from the unknowns. Given the initial condition x_0 and the remaining coefficients $\hat{\mathbf{x}}$, it is possible to obtain the rest of the samples $\mathbf{x}$ as follows

$$\mathbf{x} = (\mathbf{W}_r - \frac{\mathbf{w}_{c0}}{w_{0,0}} \mathbf{w}_{r0}) \hat{\mathbf{x}} + \frac{\mathbf{w}_{c0}}{w_{0,0}} x_0 \qquad (4)$$

where $\mathbf{W}_r$ is equal to $\mathbf{W}$ reduced by the first row and column, $\mathbf{w}_{c0}$ and $\mathbf{w}_{r0}$ are the first column and row of $\mathbf{W}$, respectively, excluding the first element $w_{0,0}$.

A similar expression can be obtained for $\mathbf{x}'$, namely

$$\mathbf{x}' = (\mathbf{W}'_r - \frac{\mathbf{w}'_{c0}}{w_{0,0}} \mathbf{w}_{r0}) \hat{\mathbf{x}} + \frac{\mathbf{w}'_{c0}}{w_{0,0}} x_0 \qquad (5)$$

Higher order derivatives were not used in the present work.

B. Other Transformation Types

The formulation in Eq. (3) is quite general, and can be applied not only with wavelet transformations, but with other transformations as well. In particular, some implicit time marching methods can be implemented. As an example, when using the following set of matrices,

$$\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{6}$$

and

$$\mathbf{W}' = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{7}$$

the formulation uses the backward Euler method. In this case the linear system to be solved is twice as large as the original MNAM (because the transformations matrices are 2×2) and the size of the nonlinear system resulting from Eq. (3) is equal to the number of state variables. Multi-step time-marching schemes can also be implemented by choosing the adequate $\mathbf{W}$ and $\mathbf{W}'$ and introducing some minor variations in the formulation. However, it is in general not practical to use the formulation of Eq. (3) in those cases because simpler and more efficient formulations exist as it will be shown elsewhere.

For comparison purposes, the wavelet transient analysis in Transim was modified to use $\mathbf{W}$ and $\mathbf{W}'$ as defined in Equations (6) and (7), respectively. The resulting analysis is the backward Euler state variable transient used to perform the simulations presented in the following section.

III. Simulation Results

A. Nonlinear Transmission Line

Consider the modeling of the 47-section nonlinear transmission line described in [1]. Since the method considered here is a time domain method, the transmission lines are modeled using RLGC sections. The attenuation factor in the following simulations is thus frequency-independent.

Figure 1 compare the simulated voltage of the diode near the load using wavelet transient, backward Euler and Spice3f5. The small difference in the response between wavelets and Spice is due to slightly different treatments of the diode model in Transim and Spice. The difference between the solution obtained using wavelets and backward Euler integration is due to the numerical attenuation introduced by the integration method. A large number of time windows was needed in the wavelet simulation to reduce the number of unknowns to be solved simultaneously. Since the results using wavelets and Spice are very close, we can conclude the wavelet analysis (at least in this simulation) behaves like a time-marching scheme, possibly due to fact that the circuit was analyzed using small time intervals at a time.

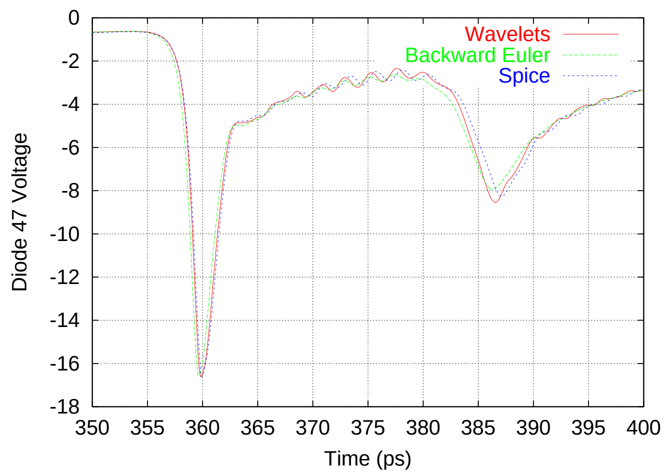

Fig. 1. Comparison of the voltage of the diode close to the load (diode 47) of the nonlinear transmission line.

B. Grid Amplifier

Transim is used in this section to model the nonlinear performance of a 2 by 2 quasi-optical grid amplifier system described in Reference [7]. The grid structure was modeled using a MOM field simulator [8] to generate the multi-port admittance matrix and excitation currents for the grid structure. Further processing is required to use this data in a time domain analysis such as wavelet or backward Euler transients using a Pole-Zero approximation.

The transient simulation of the grid amplifier using convolution was presented in Reference [2]. Even though it is time consuming, the simulation of this initial power-on transient is essential to ensure initial stability. The initial transient is given in Fig. 2. The microwave excitation is applied at $t = 2$ ns. Note the two different convolution results. If the convolution analysis is performed using the pole-zero model of the grid, the agreement with the other transient simulations is much better. We can conclude that the pole zero modeling must be improved.

A comparison of the run times for this and other simulations of the grid amplifier is given in Table I. Con-

TABLE I

Comparison times of the different simulation methods

Description	Convolution (h:m:s)	Wavelets (h:m:s)	Backward Euler (h:m:s)
Bias-on (12 μs)	16:15:00	00:00:37	00:00:20
Bias + Excitation (4 μs)	-	18:24:00	01:55:00
Bias + Excitation (4 ns)	00:09:34	00:04:40	00:00:10

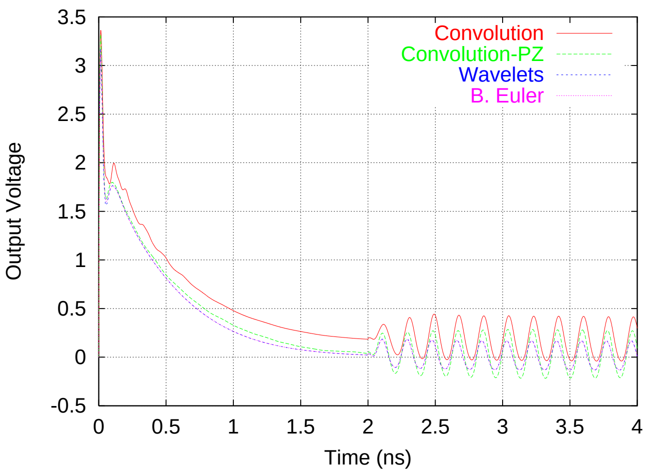

Fig. 2. Transient response at one of the MMIC output

volution transient is always the slowest method. Nevertheless, this method can potentially achieve the best accuracy. Transient based on wavelets is faster than convolution, but it is always slower than backward Euler transient. For very long transient simulations, the only viable alternative to perform the transient analysis seems to be the approximation of the grid network parameters using rational functions.

IV. Conclusions

Transim is, to the knowledge of the authors, the most advanced wavelet-based nonlinear circuit simulator developed to date. The simulation examples presented here are the most complex circuits ever simulated using wavelets. The simulation results show that time marching transient is faster than wavelet transient with fixed resolution. There are clear tradeoffs involved. In wavelet transient analysis the error is minimized over a time interval and there are many more unknowns than when the error is minimized at a single time point. However since error is minimized over a range and because of the $O(h^4)$ convergence rate of the wavelet basis used, where h is the time step, there are generally fewer time points than required in a Spice-like analysis. Our final conclusion is that circuit simulation techniques using wavelets still require more research before they can achieve the same efficiency as time marching techniques. In particular the implementation of dynamic variation of resolution including variable resolution at different circuit nodes is required.

References

[1] C. E. Christoffersen and M. B. Steer, "State-variable microwave circuit simulation using wavelets," submitted to the *IEEE Microwave and Guided Waves Letters*, September 2000.

[2] C. E. Christoffersen, S. Nakazawa, M. A. Summers, and M. B. Steer, "Transient analysis of a spatial power combining amplifier", *1999 IEEE MTT-S Int. Microwave Symp. Dig.*, June 1999, pp. 791-794.

[3] D. Zhou, N. Chen and W. Cai, "A fast wavelet collocation method for high-speed VLSI circuit simulation," *1995 IEEE/ACM ICCAD Symp. Digest*, pp 115-122, 1995.

[4] D. Zhou, X. Li, W. Zhang and W. Cai, "Nonlinear circuit simulation based on adaptive wavelet method," *1997 ISCAS Symp. Digest*, Vol. 3, pp. 1720-1723, 1997.

[5] C. E. Christoffersen, U. A. Mughal and M. B. Steer, "Object Oriented Microwave Circuit Simulation," *Int. J. of RF and Microwave Computer-Aided Engineering*, Vol. 10, Issue 3, 2000, pp. 164–182.

[6] V. Rizzoli, A. Lipparini, A. Costanzo, F. Mastri, C. Ceccetti, A. Neri and D. Masotti, *State-of-the-Art Harmonic-Balance Simulation of Forced Nonlinear Microwave Circuits by the Piecewise Technique*, IEEE Trans. on Microwave Theory and Techniques, Vol. 40, No. 1, Jan 1992.

[7] M. A. Summers, C. E. Christoffersen, A. I. Khalil, S. Nakazawa, T. W. Nuteson, M. B. Steer and J. W. Mink, "An integrated electromagnetic and nonlinear circuit simulation environment for spatial power combining systems," 1998 IEEE MTT-S Int. Microwave Symp. Dig., June 1998, pp. 1473-1476.

[8] T. W. Nuteson, H. Hwang, M. B. Steer, K. Naishadham, J.W.Mink, and J. Harvey, "Analysis of finite grid structures with lenses in quasi-optical systems," IEEE Trans. Microwave Theory Techniques, pp. 666-672, May 1997.