Predictive microwave device design by coupled electro-thermal simulation based on a fully physical thermal model

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Abstract

Coupled electro-thermal simulations are performed to demonstrate predictive design of microwave devices. These simulations are based on an original, fully physical, thermal impedance matrix approach, capable of describing 'nearly exactly' time-dependent heat flow in complex 3-dimensional systems, whilst requiring no model reduction for electro-thermal CAD. This thermal model is validated by thermal imaging of passive grid arrays representative of spatial power combining architectures. Electro-thermal transient, single-tone, two-tone and multi-tone harmonic balance simulations are presented for a MESFET amplifier, by implementing the thermal impedance matrix approach in microwave circuit simulator, Transim (NCSU).

I. Introduction

The importance of self-heating, and of mutual thermal interaction between active elements, in power devices and circuits, is well known. Hence predictive design, based on electro-thermal simulation, has been pursued for over 30 years [1]–[3]. Until recently, the state of the art in time-independent thermal simulation of heatsink mounted power FETs and MMICs, for coupled electro-thermal CAD, has been represented by the hybrid finite element Green's function approach of Bonani *et al.* [4]. This approach treated device structure such as surface metallisation, vias and partial substrate thinning. In time-dependent coupled electro-thermal CAD, thermal descriptions have been more basic, and limited to simple rectangular multilayers. Work in this area is exemplified by that of Szekely *et al.* [5], who generated a large range of semi-analytical thermal solutions for ICs, MCMs and microsystem elements. A variety of thermal model reduction techniques have also been employed to make tractable the coupled electro-thermal simulation problem on CAD timescales [6], [7], and the problem of compact model development is currently a highly active area of research [8].

The authors have progressed the state of the art, by presenting a thermal resistance matrix model capable of describing, nearly exactly, 3-dimensional time-independent heat flow in complex device structures, in a fashion compatible with electro-thermal CAD [9], [10]. This model includes description of surface fluxes and treats all thermal non linearities. Device level electro-thermal simulation was obtained by iterative coupled solution of the thermal resistance matrix model with the quasi-2-dimensional, Leeds Physical Model (LPM) of MESFETs and HEMTs [11]–[14]. More recently, the authors have presented a thermal impedance matrix model, extending this description to treat the thermal time-dependent case, both steady-state and transient [15], [16]. This time-dependent formulation allows interpretation of non linear thermal subsystems in terms of generalised multi-port network parameters, without requiring model reduction. Circuit

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level electro-thermal simulation was achieved [16] by implementation of this thermal impedance matrix model in a network based microwave circuit simulator, Transim (NCSU) [17].

Fully physical electro-thermal device CAD, based on coupling of the thermal impedance matrix approach to the rapid LPM, has been described by the authors elsewhere [18]. Application of the resulting fully physical electro-thermal model to optimisation of power transistor and MMIC design, by transient simulation, is described by David *et al.*, (this conference, [19]). In contrast, the aim of this paper is to illustrate the potential of the thermal impedance matrix model for construction of design rules describing power devices embedded in amplifier circuits. This is achieved by electro-thermal transient and harmonic balance simulation, particularly demonstrating thermal effects on intermodulation distortion and spectral regrowth. These represent an essential aspect of device optimisation for narrowband digital modulation applications such as CDMA for mobile communications. The ultimate intended application of the modelling capability described here, is study and design of spatial power combining systems for use as high power sources at millimeter wavelengths. The thermal model is therefore validated by thermal imaging of passive grid arrays representative of one form of quasi-optical system architecture.

II. Thermal impedance matrix method

The time dependent heat diffusion equation is given by,

$$\nabla . [\kappa(T)\nabla T] + g = \rho C \frac{\partial T}{\partial t}, \tag{1}$$

where T is temperature, t is time, $\kappa(T)$ is temperature dependent thermal conductivity, $g(x, y, z, t)$ is rate of heat generation, ρ is density and C is specific heat. This equation is non linear through the temperature dependence of $\kappa(T)$ (and possibly of ρ and C). To partially linearise the equation, the well known Kirchhoff transformation is performed [20], to treat the temperature dependent thermal conductivity. A much less well known technique can then be applied to fully linearise the equation, by defining a new time variable, τ [21],

$$k_S \tau = \int_0^t k(\theta) dt. \tag{2}$$

Here, thermal diffusivity, k equals $\kappa/\rho C$ and θ is Kirchhoff transformed temperature. The time-dependent heat diffusion equation becomes finally,

$$\nabla^2 \theta - \frac{1}{k_S}\frac{\partial \theta}{\partial \tau} = -\frac{g}{\kappa_S}, \tag{3}$$

where k_S and κ_S are independent of temperature (and time). The fully linearised equation, Eq. (3), can now be solved exactly. The significance of the time variable transformation, Eq. (2), for treatment of temperature dependent diffusivity, is described elsewhere [22].

An analytical, double Fourier series, solution to the time-dependent heat diffusion equation is then constructed for the case of a generic rectangular thermal subvolume, $0 < x < L, 0 < y < W$, $0 < z < D$. Active device elements $i = 1, ..., M$ are described by surface elementary areas, D_i, and the base is discretised into elementary areas, D_j, to allow accurate matching of temperatures and fluxes, at interfaces between elementary subvolumes, in complex 3-dimensional systems. The method follows very closely the authors' full description for the time-independent case in [10]. Adiabatic boundary conditions are assumed on the MMIC side faces and the top and bottom faces, $z = 0, D$, are described by the generalised 'radiation' boundary condition,

$$\alpha_{0,D} \kappa_S \frac{\partial \theta}{\partial z} + H_{0,D} (\theta - \theta_{0,D}(x, y, t)) + p_{0,D}(x, y, t) = 0. \tag{4}$$

Non linear surface fluxes can be treated as the limit of a sequence of fully linear problems [9]. Imposed flux densities $p_{0,D}(x, y, t)$ are time dependent. Coefficients $H_{0,D}$ describe surface fluxes due to radiation and convection. The $\alpha_{0,D}$ equal zero for imposed temperature boundary conditions and unity for flux boundary conditions. The respective ambient temperatures ($\alpha_{0,D} \neq 0$), or heatsink mount temperatures ($\alpha_{0,D} = 0$), are also dependent on time, $\theta_{0,D}(x, y, t)$.

Constructing the Laplace transform for the case of a uniform initial temperature distribution equal to uniform and time independent ambient temperature, separation of variables gives the general solution, $\overline{\theta}(s)$. Explicit analytical expressions can be obtained for the Fourier series expansion coefficients, without the need for numerical manipulation such as DFT-FFT. Fully analytical Fourier solutions in Laplace s-space have been described previously [3]. For illustration, specifying flux on top and bottom surfaces, $z = 0, D$, and assuming no radiative or convective surface losses, ($\alpha_{0,D} = 1$, $H_{0,D} = 0$), the following relations are obtained for temperatures, $\overline{\theta}_{0\,av_i}$ and $\overline{\theta}_{D\,av_j}$, averaged over areas, D_i, and D_j, on faces $z = 0$ and $z = D$,

$$\begin{aligned} \overline{\theta}_{0\,av_i} - \frac{\theta(t=0)}{s} &= \sum_{i'} R^{00}_{TH_{ii'}} \overline{P}_{0i'} + \sum_{j} R^{0D}_{TH_{ij}} \overline{P}_{Dj}, \\ \overline{\theta}_{D\,av_j} - \frac{\theta(t=0)}{s} &= \sum_{i} R^{D0}_{TH_{ji}} \overline{P}_{0i} + \sum_{j'} R^{DD}_{TH_{jj'}} \overline{P}_{Dj'}. \end{aligned} \tag{5}$$

Here, $\overline{P}_{0i}$ and $\overline{P}_{Dj}$ are respective imposed fluxes in elementary areas, D_i and D_j, on faces $z = 0$ and $z = D$. The thermal impedance matrices are obtained in the explicit form,

$$\begin{aligned} R^{00}_{TH_{ii'}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4 \coth \gamma_{mn} D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{0i}_{mn} I^{0i'}_{mn}}{I^{0i}_{00}}, \\ R^{0D}_{TH_{ij}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4 \,\mathrm{cosech} \gamma_{mn} D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{0i}_{mn} I^{Dj}_{mn}}{I^{0i}_{00}}, \\ R^{D0}_{TH_{ji}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4 \,\mathrm{cosech} \gamma_{mn} D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{Dj}_{mn} I^{0i}_{mn}}{I^{Dj}_{00}}, \\ R^{DD}_{TH_{jj'}} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4 \coth \gamma_{mn} D}{(1+\delta_{m0})(1+\delta_{n0})\gamma_{mn}} \frac{I^{Dj}_{mn} I^{Dj'}_{mn}}{I^{Dj}_{00}}, \end{aligned} \tag{6}$$

where $m, n = 0, 1, 2, ...,$ and

$$\lambda_m = \frac{m\pi}{L}, \quad \mu_n = \frac{n\pi}{W}, \quad \gamma^2_{mn} = \lambda^2_m + \mu^2_n + \frac{s}{k_S}, \tag{7}$$

with the I^{0i}_{mn} and I^{Dj}_{mn} area integrals of the form,

$$I^i_{mn} = \iint_{D_i} \cos \lambda_m x \cos \mu_n y \; dx dy, \tag{8}$$

over elementary areas D_i and D_j, on faces $z = 0$ and $z = D$, respectively.

The matrix relations, Eqns. (5) and (6), constitute an analytically exact solution of the time-dependent heat diffusion equation for arbitrarily complex, 3-dimensional systems, so long as these can be adequately represented in terms of rectangular subvolumes. These thermal solutions describe arbitrary layouts of active device elements on semiconductor die, without any approximation for end effects, or for infinite or semi-infinite substrates. The relations return only temperatures in the vicinity of the device active regions, and at interfaces, as required for the coupled electro-thermal solution. The solution is mesh free, only active elements and interfaces are discretised, and no redundant temperature information is generated at nodal points within, or on the surfaces, of the rectangular subvolumes. However, once power dissipations have been obtained by self-consistent solution of the coupled electro-thermal problem, temperatures can be obtained for model validation purposes, at any point within the complex 3-dimensional system, from the corresponding full analytical solutions for thermal subvolumes.

The series expressions, Eq. (6), represent generalised multiport Z-parameters for the non linear, distributed thermal subsystems. Combining the thermal impedance matrices for individual subvolumes, a global thermal impedance matrix for complex 3-dimensional systems can be obtained. This is illustrated explicitly in [16] for the case of a metallised MMIC. More generally, thermal subsystems are represented individually by netlist elements in microwave circuit simulator, Transim (NCSU) [16], [17]. Expressing the thermal impedance matrices as non linear elements in the time domain, $R_{TH_{ij}}(t)$, by analytical or numerical Laplace inversion, then allows transient simulation with non linear matching of interface temperatures at subsystem interfaces, in those cases where the functional form of the Kirchhoff transformation differs between subvolumes. Alternatively, expressing the thermal impedance matrices in frequency space, $s \rightarrow i\omega$, as arrays of complex phasors, means that they can be included directly in the modified nodal admittance matrix for the microwave system, allowing harmonic balance (HB) analysis.

III. Results

Fig. 1 demonstrates validation of the thermal model, by comparison of simulation against thermal imaging, for a passive grid array representative of spatial power combining architectures. Fig. 2 depicts the simulated amplifier used for study of thermal effects on MESFET performance. Fig. 3 illustrates transient decay in drain-source current I_{ds}, as a result of thermal variation under the influence of a step input in drain-source voltage V_{ds}, for a multi-finger power transistor, calculated using the thermal impedance matrix approach implemented in Transim. Figs. 4 and 5 illustrate corresponding single-tone harmonic balance simulation. Figs. 6 and 7 illustrate two-tone HB, demonstrating intermodulation distortion due to amplifier non linearity. As a result of thermal inertia, thermal response is seen to be much greater at the 1 MHz difference frequency than at the ~1 GHz fundamental frequencies. Fig. 8 illustrates the potential of the model for prediction of thermal effects on spectral regrowth and ACPR, by means of multi-tone HB.

IV. Conclusion

An original spectral domain decomposition technique, for solution of the time-dependent heat diffusion equation in complex 3-dimensional systems, with full treatment of thermal non linearity, has been described. This thermal impedance matrix approach has direct interpretation in terms of analytically exact, generalised multi-port thermal network parameters, for non linear, distributed thermal subsystems. This interpretation allows implementation of the thermal model, as multi-port thermal elements, in circuit simulator Transim. Examples of the implementation of the thermal impedance matrix model in Transim have been presented, illustrating the potential for predictive microwave device design by coupled electro-thermal simulation.

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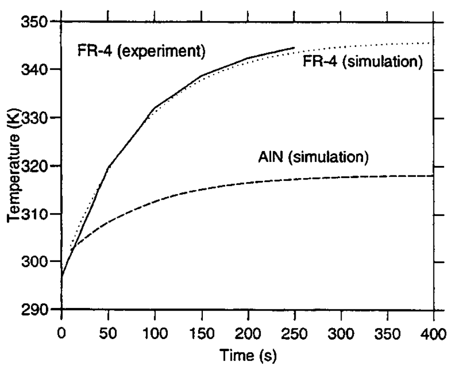

Fig. 1: Central temperature rise with time, of a 10×10 passive grid array dissipating 2 W, obtained from ThermaCAM measurements. Substrates: FR-4 (experiment and simulation) and AlN (simulation). Cooling is purely by radiation and convection (no heatsink).

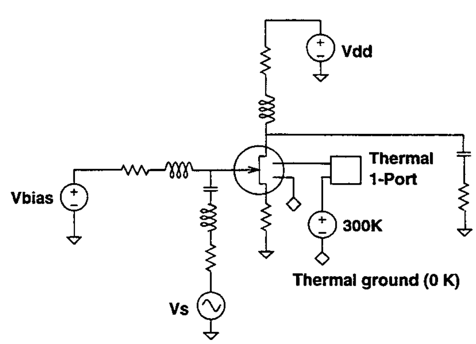

Fig. 2: Schematic of the simulated MESFET amplifier with thermal circuit. The MESFET is described by the Curtice-Ettemberg cubic model with symmetric diodes and capacitances. The MMIC die is described by analytically exact thermal 1-port network parameters.

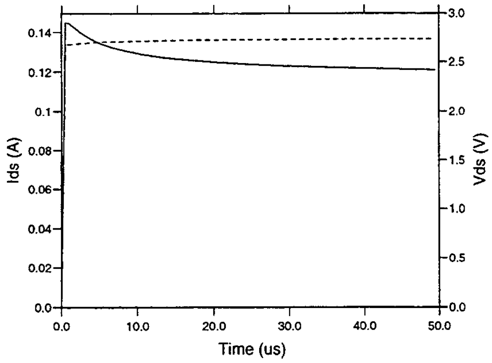

Fig. 3: Drain-source current I_{ds} (←, solid line) and drain-source voltage V_{ds} (dashed line, →) for a 5-finger power transistor, from transient electro-thermal analysis.

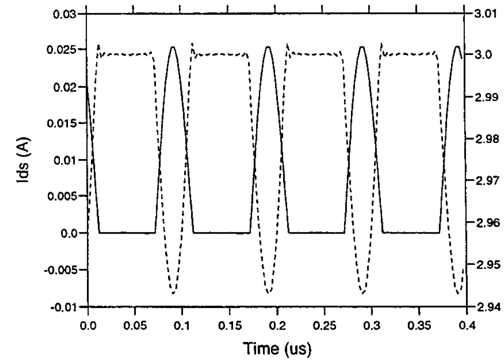

Fig. 4: Drain-source current i_{ds} (←, solid line) and drain-source voltage v_{ds} (dashed line, →) from single-tone HB analysis with fundamental frequency 10 MHz.

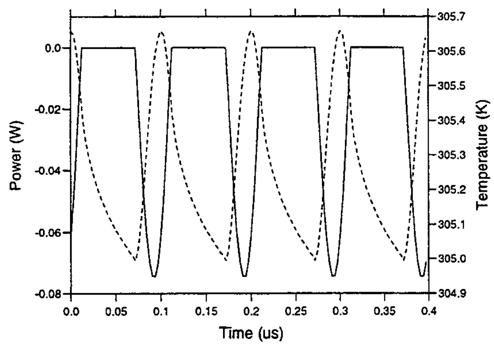

Fig. 5: Power dissipation (←, solid line) and temperature variation (dashed line, →) for the 5-finger power transistor, from single-tone HB analysis with fundamental frequency 10 MHz.

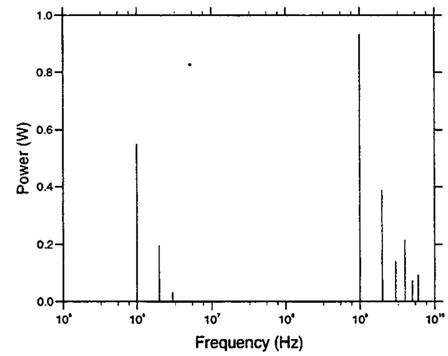

Fig. 6: Power dissipation in the frequency domain for a 50-finger power transistor, from two-tone HB analysis with fundamental frequency 1 GHz and difference frequency 1 MHz.

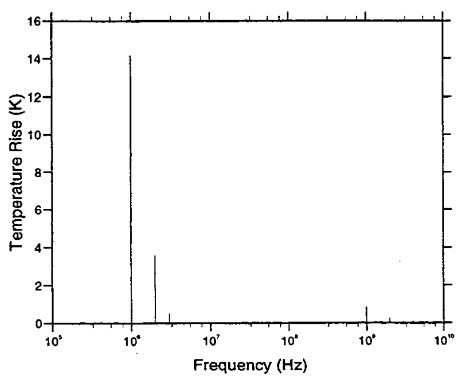

Fig. 7: Temperature variation in the frequency domain for the 50-finger power transistor, from two-tone HB analysis with fundamental frequency 1 GHz and difference frequency 1 MHz.

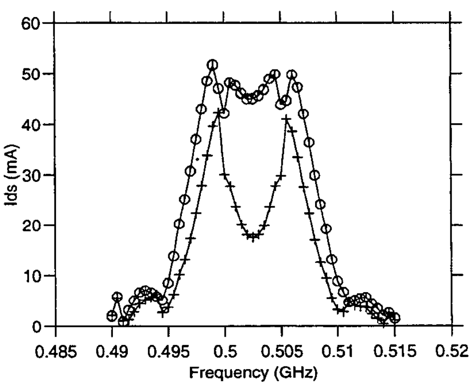

Fig. 8: Current response from multi-tone HB analysis with 11 fundamentals at frequency 0.5 GHz and difference frequency steps of 0.5 MHz. Circles: without thermal effects. Crosses: with thermal effects.