Characterization of Nonlinear Device Capacitance in Frequency Domain

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***Abstract*-- This paper formulates a simple matrix-based solution to estimate the voltage-dependent capacitance of a weakly nonlinear device at microwave and millimeter wave frequencies. Unlike other approaches, which rely on the direct capacitance measurement at different bias voltages, the proposed technique generates a large signal model for the capacitance directly from the frequency domain measurement and the known I-V characteristic. The resulting capacitance versus voltage characteristic can be incorporated as a simulation model into a microwave system design for harmonic balance simulation. Measurement of the capacitance of a highly nonlinear device using this approach agrees with the results of harmonic balance simulation for microwave and millimeter wave frequencies.**

I. INTRODUCTION

At an operating frequency above W-band, the nonlinear capacitance of a device can critically affect the system performance. For example, a nonlinear device with a zero-biased capacitance of 1pF will not be operational in the absence of any bias. Knowledge of the capacitance-versus-voltage characteristic of a device can help in understanding the correct bias voltage that makes the device functional at certain frequencies.

The frequency dependent behavior of the device is normally characterized as a nonlinear capacitance or nonlinear inductance connected in parallel with the nonlinear conductance. Rough measurement of capacitance-versus-voltage characteristic can be conducted with a commercial C-V meter at a frequency below 1 Ghz. In most situations, however, the zero-biased capacitance can be no more than a hundred femto-farads. Data obtained from the C-V meter will not yield any reliable model for high frequency circuit analysis. In this paper, a novel matrix-based method is derived to characterize the nonlinear capacitance directly from the frequency domain. The proposed method of solution is based on the theories discussed in [1,2,3], but the algorithm proposed in this presentation is tailored for nonlinear capacitance modeling. Unlike other approaches, which rely on direct capacitance measurement at different bias voltages, the proposed technique generates a large signal model for the device capacitance directly from the harmonic distribution and the known I-V characteristic. Experimental characterization of a highly abrupt device is then given to substantiate the presentation

II. ANALYTICAL TREATMENT

Fig. 1 illustrates the schematic of an experimental setup for measurement of a nonlinear device. The nonlinear device is excited by a single tone signal V_s through the source impedance R_s and the transmission line T_1. Multiple harmonics generated by the nonlinear device DUT are measured at the output using a spectrum analyzer or power meter. Measurement of the device nonlinearity involves two steps: a) resolving the spectral balances of the device by de-embedding the measurements taken from the spectrum analyzer; b) resolving the nonlinearity of the device with the given harmonic voltages and currents. What follows is an analytical description of these two steps.

c) Resolving the spectral balances of the device: By de-embedding the circuit shown in Fig. 1, the mth harmonic of the device current can be expressed as follows:

$$i_m = \frac{\left(V_{s_m} + (Z_s I_{outk} + V_{out_m})\cos(b\,l_{in} - b\,l_{out}) + j(Z_o I_{out_m} + \frac{Z_s}{Z_o} V_{out_m})\sin(b\,l_{in} - b\,l_{out})\right)}{\left(Z_s\cos(b\,l_{in}) + jZ_o\sin(b\,l_{out})\right)} \quad (1)$$

where m is the index representing the harmonic number and V_{s_m} is zero for $m \neq 1$. Similarly, the mth harmonic of the voltage across the device, v_{d_m}, can be expressed as:

$$v_{d_m} = V_{out_m}\cos(b\,l_{out}) - jZ_o I_{out_m}\sin(b\,l_{out}) \quad (2)$$

For a device without any analytic region, the nonlinear current and capacitance can be respectively expressed as

$$I_g = \sum_{k=0}^{\infty} a_k V^k \text{, and } C_d = \sum_{k=0}^{\infty} C_k V^k \quad (3) \text{ and } (4)$$

, where V is the time-domain variable of the total voltage across the intrinsic nonlinear element of the device. When the device is excited by a very weak signal, with only the fundamental component being measurable at the output, it can be reasonably assumed that all the second or higher order effects can be truncated. Under this condition, the nonlinear intrinsic elements of the device can be approximated to the first order and readily extracted, i.e.

$$I_g \approx a_0 + a_1 V \text{, and } C_d \approx C_o \quad (5) \text{ and } (6)$$

for $V_k \to 0$ when $k \neq 0{,}1$. Then, the overall impedance of the DUT in Fig. 1 becomes:

$$\frac{1}{a_1 + jv\,C_o} + jw\,L_s = \frac{v_{d_1}}{i_{d_1}} \quad (7)$$

where v_{d_1} and i_{d_1}, as obtained from ~~equation~~Equation (1) and (2), respectively represents the fundamental voltage and current in the device. L_s from ~~equation~~Equation (7), which represents the parasitic series inductance attached to the bond pad of the device, can be simultaneously resolved together with the other variables C_0 and a_1, given that measurements for at least three different frequencies are available. In the forthcoming analysis, it is assumed that the parasitic such as the lead inductance L_s can be isolated using ~~equation~~Equation (7), when the device is under a sufficiently weak excitation.

b) Resolving the nonlinearity of the device: Differentiating the charge accumulated in the device with respect to time gives the reactive current as:

$$I_c = \frac{d}{dt}\left(C_d V\right) = \frac{d}{dt}\left(\sum_{k=0}^{\infty} C_k V^{k+1}\right)$$

(8)

The time domain expression of the total current I in the device can be obtained by adding (3) and (8) together. That is,

$$I = I_c + I_g = \sum_{k=0}^{\infty} (a_k + (k+1)C_k \frac{dV}{dt})V^k$$

(9)

The voltage V across the device is a time-domain superimposition of all the harmonic components across the device, ~~as~~ and can be expressed as follows:

$$V = \sum_{m=1}^{\infty} v_{d_m} \cos(m\omega t + \psi_m)$$

(10)

where v_{d_m} can be measured with help of ~~equation~~ Equation (2). The exponential form of ~~equation~~ Equation (10) can be rewritten as:

$$V = \sum_{m=-N}^{N} V_m \exp(jm\omega t)$$

(12)

with $V_m = v_{d_m} \exp(j\psi_m)/2$. In a similar manner, the exponential form of the current, for the rest of this presentation, can be $I = \sum_{m=-N}^{N} I_m \exp(jm\omega t)$, with $I_m = i_{d_m} \exp(j\psi_m)/2$. By multinomial expansion of ~~equation~~Equation (10), V^k becomes:

$$V^k = \sum_{\underbrace{n_{-N}......n_N}_{\sum_{i=-N}^{N} n_N = k}} k! \prod_{p=-N}^{N} \frac{V_p^{\,n_p} \exp(jpn_p w\, t)}{n_p!},$$

(13)

or,

$$V^k = \sum_{n=-kN}^{kN} A_{k,n} \exp(jnwt)$$

(14)

with the coefficients $A_{k,n}$ being complex in quantity and given as $A_{k,n} = \sum_{\underbrace{n_{-N}......n_N}_{\sum_{i=-N}^{N} n_N = k}} k! \prod_{p=-N}^{N} \frac{V_p^{\,n_p}}{n_p!}$.

Instead of using multinomial expansion to solve ~~equation~~Equation (14), however, $A_{k,n}$ can be more efficiently resolved ~~by~~ using the following algorithm~~routine~~:~~-~~

for (r=1; r<=N; r++)
for (i=-(r-1)*M; i<=(r-1)*M; i++)
for (j=-M;j<=M; j++)

$$A_{r,i+j} = A_{r,i+j} + A_{r-1,i} * V_j$$

(15)

Here, N stands for the maximum order of the nonlinearity and M the number of the data points. Substituting ~~equation~~Equation (14) into ~~equation~~Equation (9) gives the expression for the total current as:-

$$I = \sum_{k=0}^{\infty} \left(a_k + (k+1) C_k \sum_{m=-N}^{N} jmw\, V_m \exp(jmw\, t) \right) \sum_{n=-kN}^{kN} A_{k,n} \exp(jnwt)$$

(16)

Following some algebraic re-arrangements on ~~equation~~Equation (13), the expression for the total current can be written as:

$$I_n = \sum_{k=0}^{\infty} \left(a_k A_{k,n} + C_k jw\ (k+1) \sum_{m=-kN}^{kN} (n-m) V_{n-m} A_{k,m} \right)$$

(17)

Although the phase reference of each harmonic is not clear at this stage, each harmonic current must be 90^o leading its respective harmonic voltage vector. Hence, we can still use the magnitudes of each vector variable to resolve ~~equation~~Equation (17):

$$\sum_{m=0}^{M-1}\left|(k+1)\sum_{m=-kN}^{kN}(n-m)V_{n-m}A_{k,m}\right|^{2} w^{2}C_m^{2} = |I_k|^{2} - \sum_{n=0}^{N}|A_{k,n}|^{2}a_n^{2} \tag{19}$$

Equation (19) is valid regardless of the phase angle of each harmonic voltage or current. In other words, regardless of ~~whatever~~ the phase differences among the harmonic components, the magnitude of the harmonic current remains $|I_k|$ for the k th harmonic. It follows that each harmonic current/voltage pair can be related as:

$$|I_n|\exp(j\iota_n) = \sum_{k=0}^{\infty}\left(a_k|A_{k,n}| + C_k jw(k+1)\sum_{m=-kN}^{kN}(n-m)|V_{n-m}||A_{k,m}|\right) \tag{20}$$

where ι_n can be resolved by equating the real part of the ~~equation~~Equation. C_k's of ~~equation~~Equation (20 represent the coefficients of the power series (or ~~equation~~Equation 4) expressing the nonlinear capacitance, and can be resolved by gaussian elimination, as long as both the I-V characteristic and the frequency-domain measurements are given.

III. Measurements

To validate the frequency domain solution of voltage-dependent capacitance, a series of measurements based on the previously described method have been conducted on a highly nonlinear two-terminal resonant tunneling device. The I-V characteristic of this resonant tunneling device is given in Fig. 2 (a). The measurement setup was implemented according to Fig.1 and the whole characterization process carried out by a software written in C, which resolves the problem in two stages: a) de-embedding the linear element and the parasitic attached to the device using ~~equation~~Equation (7); b) application of ~~equation~~Equation (19) to derive the capacitance-versus-voltage model. Table 1 is an example set of data taken from frequency domain and the resulting capacitance model as characterized using the proposed method is graphically illustrated in Fig. 2 (b).

According to Fig. 2 (b), the zero-biased capacitance together with the parasitic capacitance to the ground is around 1.5 pF in this measurement, while the zero-biased capacitance by vector network analyzer measurement was around 1.36 pF. The tunneling device has the negative differential resistance regions at +/- 0.5 volts, corresponding to what the model-predicted thresholds +/- 0.5 volts in Fig. 2 (b), where the nonlinear capacitance rapidly drops with voltage. The capacitance model has been tested in harmonic balance simulation and was found to be accurate enough to predict the conversion performance of a microwave mixer operating at 20 Ghz and a fifth-order harmonic generator having output at 104 Ghz.

IV. Conclusions

This paper has presented a simple technique to estimate the nonlinear capacitance of a device from the frequency domain. The extracted C-V model of the device has been validated by experiment and by harmonic balance simulation at millimeter-wave frequencies.

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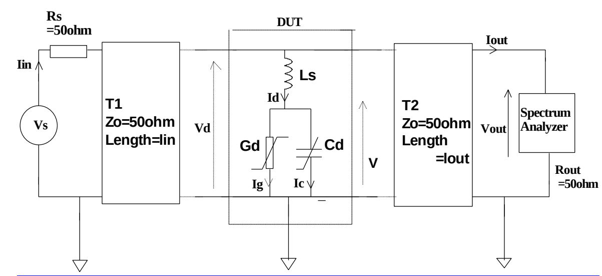

Fig 1. Experimental Setup For Nonlinear Measurement

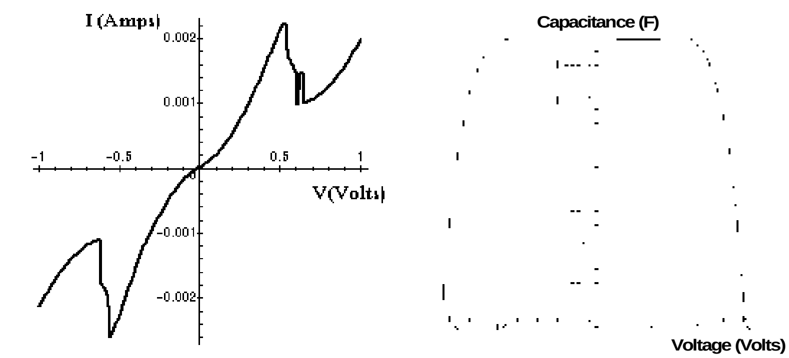

Fig. 2. a) I-V characteristic of the resonant tunneling device, with device diameter = 10 microns. b) Frequency-domain Modeled C-V characteristic

Frequency (Ghz)	Power (dbm)
2.054	2.13
4.160	-46.17
6.220	-32.00
8.240	-49.33
10.29	-37.83
12.34	-47.67
14.40	-42.50

Table 1. Frequency domain measurement taken from the spectrum analyzer. (Power Input from the source, v_s , = 5.77 dbm)