TE_{30}-mode interaction. In the exact case the analysis indicated an isolation >30 dB over the operating frequency band of 18.95–20.95 GHz. The same coupler was treated as an example in [10], which presents the theory for the analysis of the even-mode circuit to include the TE_{30} mode, followed by the generation of an equivalent circuit. Since couplers of coupling looser than 3 dB have slots of shorter length, the interaction effects of the TE_{30} mode are quite pronounced, even when there is no central matching element.

The equivalent circuit of the coupler takes the form shown in [10, Fig. 5]. Values of the elements in this equivalent circuit for several values of the half-slot width a_1 are shown in Fig. 6(a). It is seen that the series circuit elements X_1, X_2, and X_3 have linear frequency dependence over the 18.95–20.95-GHz band, while the shunt element B has smooth monotonic variation with frequency. The characteristic impedances Z_{02} and X_{03} shown in Fig. 6(b) are also monotonic, with X_{03} rapidly increasing at higher frequencies as the cutoff of the TE_{30} mode is approached. It is obvious from [10, Fig. 5] that the higher order mode has more effect at higher frequencies since then the attenuation of the parallel path is less.

The coupling obtained from multiplication of the individual scattering transfer matrices neglecting the TE_{30} mode, as shown in Fig. 5, demonstrates the same increasing deviation with increasing frequency (compared to the precise analysis) as the hybrid described in Section III. Calculation of the S_{21} phase error, as also described in Section III, gives a monotonic increase from 0.134° to 1.697° over the band, which agrees with the observed coupling differences. The coupling and isolation of the coupler derived from the equivalent circuit of [10, Fig. 5] is virtually identical to that obtained by direct analysis given in Fig. 5 here, confirming the validity of the equivalent circuit.

V. Conclusion

The flat coupling obtainable in the classic short-slot coupler is due to the effect of the evanescent TE_{30} mode in the coupling region, which has a major effect on the performance. Actually, it appears to be difficult or even impossible to obtain flat coupling unless this mode is significant, requiring a defined width of the coupling region so that the mode cutoff frequency is correctly located. The frequency variation of the phase "error" caused by the TE_{30} mode has an opposite slope to that of the dominant-mode phase, and the resulting phase compensation results in flatter coupling.

An equivalent circuit for the even mode, based on the formation of equivalent circuits from the generalized scattering matrix, has been derived, and all of the circuit elements have monotonic variation with frequency. The effect of the TE_{30} mode is clearly indicated as a parallel path connecting the input and output ports of the equivalent circuit.

Future work could be carried out to express the elements of the equivalent circuit in terms of dimensions, probably requiring an application of mode-matching theory. Operations on the equivalent circuit, such as new reference plane locations, may be required.

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Nonlinear Gain Compression in Microwave Amplifiers Using Generalized Power-Series Analysis and Transformation of Input Statistics

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***Abstract*—Two methods are presented for the estimation of gain compression generated by a digitally modulated carrier passed through a nonlinear RF circuit. The first one is based on developing an analytical expression for gain compression based on the transformation of input signal statistics. The second one is based on approximated expressions derived from generalized power-series analysis. The techniques are evaluated by comparing measured and predicted gain compression in a CDMA system.**

***Index Terms*—CDMA, gain compression, intermodulation distortion, microwave amplifiers, nonlinear power amplifiers.**

I. Introduction

Gain compression in nonlinear amplifiers with digitally modulated signals occurs at lower powers than that for one-tone signals. This is attributed to the nonunity peak-to-average ratio (PAR) of digitally modulated signals, with most of the compression being due to the peak signals. The PAR is a crude characterization of a digitally modulated signal, and a more accurate representation is to use the auto-correlation statistics of the signal or to treat the signal as being composed of a large number of uncorrelated tones. This paper develops expressions for gain compression with digitally modulated signals using both characterizations above. Gain compression is derived using a statistical approach previously used for arriving at the statistics of the signal at the output of a nonlinearity given the statistics of the input signal [1].

The alternative expression uses earlier research to evaluate the nonlinear response to a multitone signal [2], [3]. This expression has been

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modified to account for the tones being uncorrelated. Gain compression predicted using the two techniques is contrasted and compared to measurement.

II. Gain Compression of a Complex Gaussian Signal Passed Through a Nonlinear Device

A block diagram of the baseband equivalent quadrature modulator cascaded with a nonlinear element is shown in Fig. 1 [1], with the digital quadrature modulated signal $z(t)$ modeled as the quadrature addition of $x(t)$ and $y(t)$ as

$$\tilde{z}(t) = A(t)e^{j\theta(t)} = x(t) + jy(t). \quad (1)$$

The nonlinear device is represented by the complex power series

$$\tilde{G}(z) = \sum_{i=1}^{N} \tilde{a}_i \tilde{z}^i = \tilde{a}_1\tilde{z} + \tilde{a}_3\tilde{z}^3 + \tilde{a}_5\tilde{z}^5 + \ldots + \tilde{a}_N\tilde{z}^N \quad (2)$$

the coefficients of which are extracted from conveniently obtained AM–AM and AM–PM data, following conversion of an envelope characterization to a baseband equivalent characterization (see Section III). It is important to realize that (2) represents a general nonlinear transfer characteristic, not a gain expression. The procedure followed here is to calculate the auto-correlation function of the output of this system by applying an input signal, characterized by its complex auto-correlation function, to a model of the nonlinearity described by (2). This section derives a general gain expression for a modulated carrier passing through a memoryless bandpass nonlinearity, considering carrier effects. Following the development in [4], an amplitude- and phase-modulated carrier $w(t)$ with carrier frequency ω_c is modeled as

$$w(t) = A(t)\cos[\omega_c t + \theta(t)] = \frac{1}{2}\left[\tilde{z}(t)e^{j\omega_c t} + \tilde{z}^*(t)e^{-j\omega_c t}\right] \quad (3)$$

where $A(t)$ and $\theta(t)$ are the amplitude and phase modulation, and $z(t)$ is the quadrature signal (1). Using the binomial expansion, the nth power of $w(t)$ is

$$w^n(t) = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\left[\tilde{z}(t)\right]^k\left[\tilde{z}^*(t)\right]^{n-k} e^{j\omega_c(2k-n)t}. \quad (4)$$

Consider now only the frequency terms centered at the carrier frequency (this is usually referred as the first zonal filter at the output of the nonlinearity). This implies $2k - n = \pm 1$ for odd n only. Equation (4) then becomes

$$w^n(t) = \frac{1}{2^{n-1}}\binom{n}{\frac{n+1}{2}}\left[\tilde{z}(t)\tilde{z}^*(t)\right]^{((n-1)/2)}\tilde{z}(t). \quad (5)$$

Combining (4) with the complex transfer characteristic (2) yields the bandpass nonlinear gain expression

$$\tilde{G}(z) = \tilde{z}(t)\sum_{n=1}^{N}\frac{\tilde{a}_n}{2^{n-1}}\binom{n}{\frac{n+1}{2}}\left[\tilde{z}(t)\tilde{z}^*(t)\right]^{((n-1)/2)}. \quad (6)$$

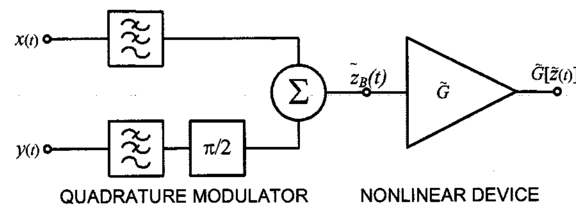

Fig. 1. Baseband equivalent quadrature modulated nonlinear amplifier.

The output auto correlation is then obtained by taking the moments of (6) as follows:

$$\begin{aligned}\tilde{R}_{gg}(\tau) &= E\left[\tilde{G}(\tilde{z}_1)\tilde{G}^*(\tilde{z}_2)\right] \\ &= \sum_{n=1}^{N}\sum_{m=1}^{N}\frac{\tilde{a}_n\tilde{a}_m^*}{2^{n+m+1}}\binom{n}{\frac{n+1}{2}}\binom{m}{\frac{m+1}{2}} \\ &\quad\times E\left[\tilde{z}_1\tilde{z}_2^*(\tilde{z}_1\tilde{z}_1^*)^{((n-1)/2)}(\tilde{z}_2\tilde{z}_2^*)^{((m-1)/2)}\right]\end{aligned} \quad (7)$$

where N is the order of the polynomial model and n and m are integer indexes. The evaluation of (7) requires finding a closed-form expression for the expected value

$$E\left[\tilde{z}_1^{(n+1/2)}(\tilde{z}_1^*)^{((n-1)/2)}\tilde{z}_2^{(m-1/2)}(\tilde{z}_2^*)^{((m+1)/2)}\right]. \quad (8)$$

A general expression for this expectation can be induced by expanding (8) for several values of n and m, and collecting terms of equal order. This yields a closed-form expression for gain compression

$$\begin{aligned}\tilde{R}_{gg}(\tau) &= \tilde{R}_{zz}(\tau)\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{n!m!}{2^{n+m-2}} \\ &\quad\times\binom{n}{\frac{n+1}{2}}\binom{m}{\frac{m+1}{2}}R_{zo}^{((n+m-2)/2)}\tilde{a}_n\tilde{a}_m^* \\ &= \tilde{R}_{zz}(\tau)\left[\sum_{n=1}^{N}\frac{n!}{2^{n-1}}\binom{n}{\frac{n+1}{2}}\tilde{a}_n R_{zo}^{((N-1)/2)}\right] \\ &\quad\times\left[\sum_{n=1}^{N}\frac{n!}{2^{n-1}}\binom{n}{\frac{n+1}{2}}\tilde{a}_n R_{zo}^{((N-1)/2)}\right]^*\end{aligned} \quad (9)$$

where $R_{zz}(\tau)$ and $R_{gg}(\tau)$ are the input and output auto-correlation functions, respectively, and $R_{zo} = R_{zz}(0)$ is a magnitude proportional to the average power of the input signal $z(t)$. This is a more general result than [1, eq. (9)] since none of the expected values of the cross products (8) are assumed to be zero.

III. Nonlinear Gain Compression Based on Generalized Power-Series Analysis

Previously, the authors have developed a formula for the phasors at the output of a nonlinear system described by a generalized power series (GPS) with an N-tone input [2], [3]. The GPS yields a deterministic expression for the output of a polynomial nonlinearity, considering a multitone input signal. When the input has a very large number of frequency components (such as in a CDMA signal), evaluation via power-series analysis might become computationally prohibitive, and a statistical description of the input–output transformation such as the one outlined in Section II is necessary.

A phasor component of the output Y_q with radian frequency f_q can be expressed as a sum of intermodulation products U_q [2], [3]

$$Y_q = \sum_{n=0}^{\infty} \sum_{\substack{n_1,\ldots,n_N \\ |n_1|+\ldots+|n_N|=n \\ n_1 f_1+\ldots+n_N f_N=f_q}} U_q(n_1,\ldots,n_N) \tag{10}$$

where a set of n_k's define an intermodulation product, n is the intermodulation order, and

$$U_q(n_1,\ldots,n_N) = K(n_1,\ldots,n_N)\Big[1+T'(n_1,\ldots,n_N)\Big] \tag{11}$$

is given in terms of the intermodulation term K, which, for $n \neq 0$, is

$$K(n_1,\ldots,n_N) = \tilde{a}_n \frac{n!}{2^n} \prod_{k=1}^{N} \frac{(X_k^+)^{|n_k|}}{|n_k|!} \tag{12}$$

where X_k is the phasor of the kth input tone, and the saturation term T'

$$T'(n_1,\ldots,n_N) = \sum_{\alpha=1}^{\infty} \sum_{\substack{S_1,\ldots,S_N \\ S_1+\ldots+S_N=\alpha}} \left(\frac{(n+2\alpha)!}{n! 2^{2\alpha}}\right) \frac{\tilde{a}_{n+2\alpha}}{\tilde{a}_n} \times \prod_{k=1}^{N} \frac{|X_k|^{2S_k} |n_k|!}{S_k!(|n_k|+S_k)!}. \tag{13}$$

In the above, $X_k^+ = X_k$ for $n_k \geq 0$, and $X_k^+ = X_k^*$, its complex conjugate, otherwise. The S_k's are integer sequences defined by the summation index α.

Considering only first-order intermodulation $(n = 1)$ enables gain compression to be described. The correlated component of the output at frequency f_q due to a multitone input signal (of uncorrelated components) of equal amplitude $|X_k| = |X_q|$, $k = 1,\ldots,N$ is

$$Y_{qc} = K_{qc}(1 + T'_{qc}) \tag{14}$$

where $K_{qc} = \tilde{a}_1 X_q$ and

$$T'_{qc} = \sum_{\alpha=1}^{\alpha_m} \frac{(1+2\alpha)!}{2^{2\alpha}} \frac{\tilde{a}_{1+2\alpha}}{\tilde{a}_1} \sum_{\substack{S_1,\ldots,S_N \\ S_1+\ldots+S_N=\alpha}} \prod_{k=1}^{N} \frac{X_q^{2S_k}}{(1+S_k)!}. \tag{15}$$

Now, if the input is a single tone $(q = N = 1)$, (14) and (15) reduce to

$$Y_1 = \tilde{a}_1 X_1 + \sum_{\alpha=1}^{\alpha_m} b_{1+2\alpha} |X_1|^{2\alpha} X_1 \tag{16}$$

where the $\tilde{a}$ and b coefficients are related by

$$\tilde{a}_{1+2\alpha} = b_{1+2\alpha} \frac{2^{2\alpha}\alpha!(1+\alpha)!}{(1+2\alpha)!}; \quad \alpha = 1,\ldots,\alpha_m. \tag{17}$$

Thus, (16) is an envelope behavioral model with the effect of the carrier embedded in the model coefficients. However, an instantaneous model or baseband equivalent behavioral model, with coefficients $\tilde{a}_n$ obtained using (17) is required to determine the effects of multitone input signals, including gain compression. The standard nonlinear gain expression (16) relates a phasor at the input to the phasor at the output, and corresponds to the complex gain measured in AM–AM and AM–PM characterization.

The evaluation of (13) in CDMA systems can be computationally prohibitive due to the large number of frequency components required to accurately represent the modulated signal. An asymptotic expression for the compression term T' can be obtained by assuming that the tones of the input signal are uncorrelated and the number of tones N is sufficiently large. Thus, the asymptotic saturation term T'_{qc} is

$$T'_{qc} = \frac{1}{\tilde{a}_1} \sum_{\alpha=1}^{\alpha_m} \tilde{a}_{1+2\alpha} \frac{(1+2\alpha)!}{\alpha! 2^{1+2\alpha}} |X_q|^{2\alpha} \tag{18}$$

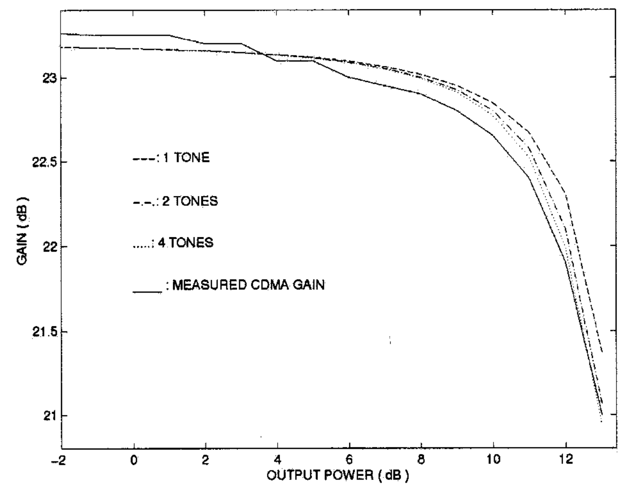

Fig. 2. Prediction of nonlinear compression based on GPS analysis of (13).

and is called the asymptotic gain-compression expression. This expression does not use the statistical properties of the phase since (18) is an approximation. On the other hand, the method outlined in Section II considers phase characteristics when computing the auto correlation of the output signal since phase is included in the signal model [see (3) and (4)].

IV. Results and Discussion

A. Behavioral Model

The AM–AM and AM–PM curves for a 900-MHz CDMA power amplifier with 23-dB gain were obtained using a single-frequency input power sweep and a vector network analyzer [1], [5]. A complex power series of order 13 was fitted to the AM–AM and AM–PM data from a 900-MHz CDMA power amplifier, using least-squares optimization. The resulting envelope behavioral model was then transformed into the baseband equivalent instantaneous behavioral model using (17). The power-series model was verified by comparing measured and predicted AM–AM and AM–PM data, as shown in [1].

B. Prediction of Gain Compression

The baseband equivalent behavioral model (1) was first used with the GPS model (13) to predict gain compression of the digitally modulated carrier, by simulating the expression for the saturation term (T') at different input power levels.

Due to the computational complexity involved in evaluating (13), only a few tones were used. Results are shown in Fig. 2, where the solid line represents measured CDMA gain, and the dashed and dotted lines correspond to compression factor [T' in (13)], calculated for several multifrequency inputs whose power match the measured available power, scaled by the linear gain coefficient $\tilde{a}_1$. The one-tone result corresponds identically to the measured AM–AM characteristic since it was measured using a single tone. The amplitudes of the phasors in (13) were derived from the input signal power, considering that tones were uncorrelated, so that the input power was divided equally among the tones. This assumption is not sufficient when predicting spectral regrowth [1]. Compression is expected to occur at lower output power levels in CDMA signals than in signals that only include a few tones. This effect is not properly predicted by this model, partly due to the least-squares process used in determining model coefficients, resulting in a small residual error in the first-order coefficient. The asymptotic expression that assumes a large number of input tones (18) was also

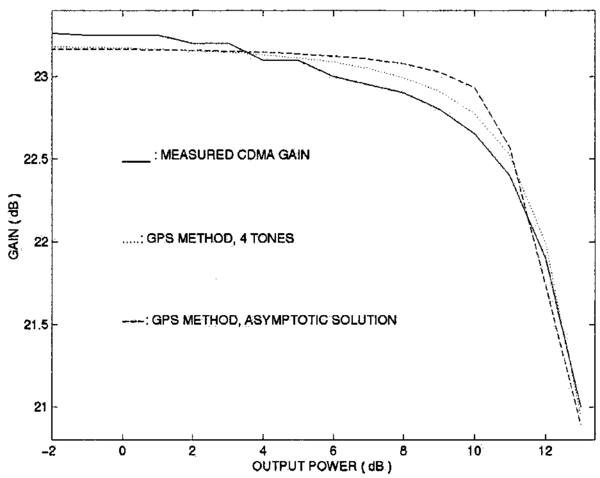

Fig. 3. Prediction of nonlinear compression based on GPS analysis. The asymptotic expression, i.e., (18), is compared to the exact solution of (13).

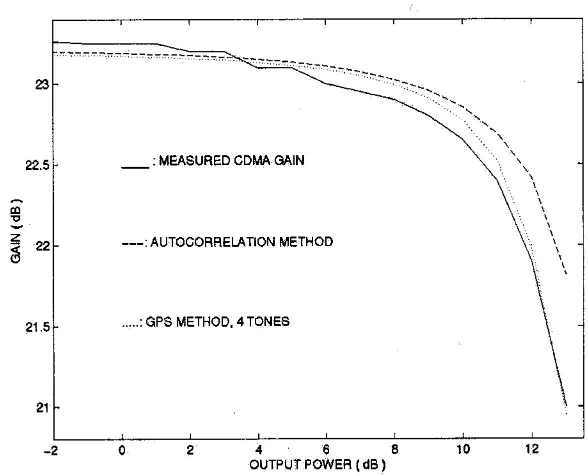

Fig. 4. Prediction of nonlinear compression. The exact solution of the GPS analysis of (13) is compared to the autorrelation transformation method of (9).

simulated and compared to the exact result for four tones, and to gain measurements. Results are shown in Fig. 3. The asymptotic gain compression (18), while computationally less demanding than the exact solution (13), yields less accurate estimates than the four-tone exact solution due to the approximations involved in (18).

The accuracy of the modeled gain-compression characteristic increases with the number of uncorrelated input tones considered. This provides important insight into why gain compression with digitally modulated signals occurs at lower power levels than with a single tone. With many uncorrelated input tones, gain compression is dominated by a saturation effect [T' in (13)]. Gain compression was also simulated using the auto-correlation method (9), and compared both to gain measurements and the four-tone simulation obtained from the exact GPS solution of (13). Results are shown in Fig. 4. The two methods track each other up to weak gain-compression levels. At high output power levels, the statistical method underestimates compression since the output auto-correlation expression [1] could only be expanded up to ninth order due to the computational complexity involved. Similarly, the GPS prediction tends to underestimate compression since higher order intermodulation components $(n > 1)$ in the output signal were neglected.

V. Conclusion

Two methods for the estimation of gain compression of digitally modulated signals passed through a nonlinear amplifier, modeled by a complex power series, have been presented in this paper. The first one uses statistical properties of the moments of the modulated signal to develop a simple expression for gain compression. The second, based on GPS analysis, provides two different expressions to predict gain compression, one based on the exact GPS solution (13), and another one based on assuming a very large number of tones [see (19)]. Both methods are qualitatively well suited to predict output power levels at which compression occurs, and the exact GPS solution can be used favorably even with relatively small numbers of tones. GPS analysis has also been used to derive an expression that relates an envelope behavioral model (for which parameters are readily available from sweep continuous wave (CW) tests) with an instantaneous power-series nonlinear transfer characteristic such as (2). These methods provide computational tools to the amplifier designer to predict gain compression given a polynomial model of the device that is readily available from AM–AM and AM–PM data.

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