Steady-State and Transient Electro-Thermal Simulation of Power Devices and Circuits Based on a Fully Physical Thermal Model

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Abstract

An original approach is presented to the time-dependent thermal modelling of complex 3-dimensional electronic systems, from metallised power FETs and MMICs, though MCMs, up to circuit board level. This method offers a powerful alternative to conventional numerical thermal simulation techniques. In contrast to semi-analytical Fourier approaches involving DFT-FFT, the method is based on explicit, fully analytical, double Fourier series expressions for thermal subsystem solutions in complex frequency space. It goes beyond previous rectangular multilayer solutions by treating complex geometries, and providing a full description of thermal material non linearities as well as explicit inclusion of surface fluxes. It provides a fully physical, and near exact, generalised multiport network parameter description of distributed thermal subsystems. In contrast to existing circuit level approaches, it requires no reduced model RC-network approximation for fully coupled, electro-thermal CAD. Implementation of the thermal model as N-port elements within a circuit simulation engine, Transim, is described and illustrative simulation results presented.

I. Introduction

The impact of self-heating and mutual thermal interaction on electronic device and integrated circuit performance is well known, and the electro-thermal simulation problem has been studied for at least 30 years [1]–[3]. The thermal modelling of complex 3-dimensional structures can be achieved by standard numerical techniques. However, these conventional numerical approaches are generally too slow to allow explicit coupling with electronic device and circuit simulators. Thus a number of faster thermal descriptions have been developed.

The simplest thermal models for the time-independent case are provided by analytical thermal resistance approaches of varying levels of complexity, *e.g.* [4], [5]. However, it has been stated repeatedly [6], [7] that the thermal resistance approach is fundamentally approximate and inadequate for detailed description of power devices. To describe the thermal effects of surface metallisation, vias and partial substrate thinning, in power FETs and MMICs, Bonani *et al* employed a hybrid finite element Green's function technique [6].

A few analytical thermal impedance expressions have also been presented for the thermal time-dependent case, for instance [8]. Veijola has implemented a simple, approximate thermal impedance description, based on analytical solution for heat-generating spheres, in circuit simulation programme APLAC [9]. Rizzoli has employed a Green's function construction of the thermal resistance matrix in a wide range of circuit level, harmonic balance and transient simulations, but with thermal capacitances described approximately based on an enthalpy formulation [10]. Analytical Green's function and Fourier solutions have been used to describe the time-dependent thermal problem by a number of authors. In particular, Szekely *et al* have employed a Fourier series method for over 20 years, providing solutions for a variety of ICs, microsystem elements and MCMs [11].

For circuit level electro-thermal simulations, a large number of thermal model reduction techniques have also been employed. Work in this area includes that of Sabry [12], Napieralski [13] and Szekely [14].

The aim of this paper is to describe a new, fully physical, approach to the time-dependent thermal problem in complex 3-dimensional systems, suitable for coupled electro-thermal device and circuit simulation on CAD timescales, with essentially no model reduction. The implementation of this thermal model, coupled to a fully physical electrical model (the Leeds Physical Model), has been presented elsewhere [15]. Here, coupling to a microwave circuit simulator Transim [16], is described.

The thermal model is based on the thermal impedance matrix approach. This time-dependent formulation is a natural development of the thermal resistance matrix approach for the time-independent case, described by the authors in [17] and developed fully in [18]. It is shown that, in contrast to previous thermal resistance and impedance approaches, this thermal impedance matrix method can be formulated to provide an essentially exact solution of the heat diffusion equation in complex 3-dimensional systems. It therefore removes the need to utilise computationally intensive numerical techniques in order to treat complex structures, *e.g.* [12], [13], [19]. Previous Green's function or Fourier approaches have been restricted to simple rectangular homogeneous vol-

umes and multilayers, *e.g.* [3], [10], [14], [20]. The model presented here can describe simultaneously all device detail, from surface metallisation, vias and substrate thinning, in power FETs and MMICs, to (actively cooled) MMIC on substrate arrays *e.g.* for spatial power combining applications, up to MCMs and circuit board level. It does this by providing fully analytical solutions of the heat diffusion equation in thermal subsystems, and then matching temperature and flux at subsystem interfaces. As the subsystem solutions are matrix expressions, explicit matrix representations can be obtained for the global thermal impedance matrices of the complex device structure. A distinguishing feature of the double Fourier series, thermal subsystem solutions, to be presented here, is that they are given by explicit analytical formulae. This is in contrast to the numerical manipulations required in the DFT-FFT approach of Szekely *et al*, who employ a collocation, or function sampling technique, to obtain the expansion coefficients in their Fourier formulation [11]. The analytical thermal subsystem description is formulated in Laplace transform s-space (complex frequency space) and so is shown to give rise to both frequency-domain and time-domain expressions, allowing treatment of both the harmonic steady-state, as well as the transient case. The fact that thermal subsystem solutions are fully analytical in s-space, also removes the need for any numerical approach to the identification of corresponding thermal networks [21]. This time-dependent thermal solution is formulated to provide full treatment of material non linearities due to temperature dependent thermal conductivity (and diffusivity [22]). It also employs a 'radiation' boundary condition that allows treatment of radiative and convective surface fluxes in large area systems without approximation such as that invoked in [11]. (Similar boundary conditions have also been applied, for example, in finite difference solutions for electro-thermal simulation [19], and in analytical solutions at the circuit board level [20].) Importantly, this approach allows essentially exact description of distributed thermal subsystems via generalised multiport network parameters in circuit level CAD, without any need for representation by a reduced, lumped-element, finite RC network.

II. Thermal impedance matrix

The thermal impedance matrix approach reduces to construction of global heat flow functions, for active elements in semiconductor integrated circuits, in the form

$$\overline{\Delta\theta_i} = \sum_j R_{TH_{ij}}(s)\overline{P_j} \qquad (1)$$

where $\overline{\Delta\theta_i}$ is the Laplace transformed temperature rise of active element i above its initial temperature, $R_{TH_{ij}}(s)$ is the thermal impedance matrix in Laplace s-space and the $\overline{P_j}$ are the transformed time-dependent fluxes due to power dissipation in active elements, $j = 1, ..., i, ...M$.

This linear form requires use of the Kirchhoff transformation to treat the non linear time-dependent heat diffusion equation [23]. The importance of performing the Kirchhoff transformation has been illustrated, *e.g.* by Webb [24], and for transient simulations involving $\sim$100 K temperature rises [15], there is no obvious operating point about which to make a simple linearisation of the temperature dependent thermal conductivity. This Kirchhoff transformation is trivial to impose *a postiori* to solution of the linear heat diffusion equation, by application of a simple analytical formula to the solution temperatures [18]. In the thermal impedance matrix approach presented here, $R_{TH_{ij}}(s)$ is then determined in explicit analytical form, purely from structural information. It is independent of temperature and power dissipation, and hence of device bias. Its order is determined only by the number of active device elements, independent of the level of the complexity of the device structure, so is already minimal without any model reduction. Thermal updates in the coupled electro-thermal problem therefore reduce to small matrix multiplications, Eq. (1). This approach offers orders of magnitude speed-up compared to numerical thermal solutions.

Construction of the thermal impedance matrix $R_{TH_{ij}}(s)$ is now described for a homogeneous MMIC, with simulation results presented for an N-level multilayer, and then for more complex structures such as a MMIC with surface metallisation, or a MMIC array.

A. Homogeneous MMIC and N-layer

An analytical, double Fourier series, solution to the time-dependent heat diffusion equation is constructed for the case of a homogeneous MMIC, $0 < x < L$, $0 < y < W$, $0 < z < D$, with active device elements $i = 1, ..., M$ described by surface elementary areas, D_i. The method follows very closely the authors' full description for the time-independent case in [18]. Adiabatic boundary conditions are assumed on the MMIC side faces and a generalised 'radiation' boundary condition is imposed on the top and bottom faces, $z = 0, D$,

$$\alpha_{0,D}\kappa_S\frac{\partial\theta}{\partial z} + H_{0,D}\left(\theta - \theta_{0,D}(x,y,t)\right) + p_{0,D}(x,y,t) = 0. \qquad (2)$$

Non linear surface fluxes can be treated as the limit of a sequence of such fully linear problems [17]. Here, imposed flux densities $p_{0,D}(x,y,t)$ are time dependent. Coefficients $H_{0,D}$ describe surfaces fluxes due to radiation and convection. The $\alpha_{0,D}$ equal zero for imposed temperature boundary conditions and unity for imposed flux boundary conditions. The respective ambient temperatures ($\alpha_{0,D} \neq 0$), or heatsink mount temperatures ($\alpha_{0,D} = 0$), are also dependent on time, $\theta_{0,D}(x,y,t)$.

To solve this problem, the Laplace transform, $\overline{\theta}(s)$, is constructed, and for the case of a uniform initial temperature distribution equal to uniform and time independent ambient temperature, separation of variables gives the general solution for $\overline{\theta}(s)$. Explicit analytical expressions can be obtained for the Fourier series expansion coefficients, without the need for numerical manipulation such as DFT-FFT. Fully analytical Fourier solutions in Laplace s-space have been described previously [3].

To demonstrate a definite time-dependent form of the thermal impedance matrix, assume, for instance, no flux from the MMIC top surface, and uniform constant temperature on the bottom surface, corresponding to heat sink mounting. Assuming a time-varying surface power density which is piecewise uniform, with values $P_i(t)$ in active device elementary areas D_i, then constructing the surface temperature rises, $\overline{\Delta\theta_i}$, averaged over elementary

areas D_i, Eq. (1) is obtained immediately with,

$$R_{TH_{ij}}(s) = \frac{1}{\kappa_S LW} \sum_{mn} \frac{4\tanh(\gamma_{mn} D)}{\gamma_{mn}(1+\delta_{m0})(1+\delta_{n0})} \frac{I^i_{mn} I^j_{mn}}{I^i_{00}}, \tag{3}$$

where $m, n = 0, 1, 2, ...,$

$$\lambda_m = \frac{m\pi}{L}, \quad \mu_n = \frac{n\pi}{W}, \quad \gamma^2_{mn} = \quad \lambda^2_m + \mu^2_n + \tfrac{s}{k}, \tag{4}$$

and area integrals I^i_{mn} have been defined by

$$I^i_{mn} = \iint_{D_i} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{W}\right) dx dy. \tag{5}$$

Diffusivity $k = \kappa_S/\rho C$, $\kappa(T)$ is temperature dependent thermal conductivity, T is physical temperature, ρ is density and C is specific heat. κ_S is $\kappa(T_S)$ where T_S is the Kirchhoff transformation temperature. δ_{mn} is the Kronecker delta function.

The thermal impedance matrix of Eq. (3), reduces to the respective time independent form [17], [18], in the limit $s/k \to 0$. Extension to treat other realisations of the 'radiative' boundary condition, Eq. (2), is immediate [18]. The expression for $R_{TH_{ij}}(s)$, Eq. (3), can be written in alternative equivalent forms [3], and is readily extended to treat N-level multilayers by means of a simple transfer matrix approach for the analytically obtained Fourier expansion coefficients [18].

To illustrate the accuracy and speed of this method, the analytical solution for an N-level multilayer is used to plot the complex locus of the thermal transfer impedance in Fig. 1. The 4-layer, heatsink mounted device considered, is a structure examined by Szekely *et al* ([14] Figs. 5 and 6; [21] Fig. 17). Agreement with the calculations of Szekely seems good. The data for this figure took less than 1s to generate on a 500 MHz Pentium processor and consists of 65 frequency points.

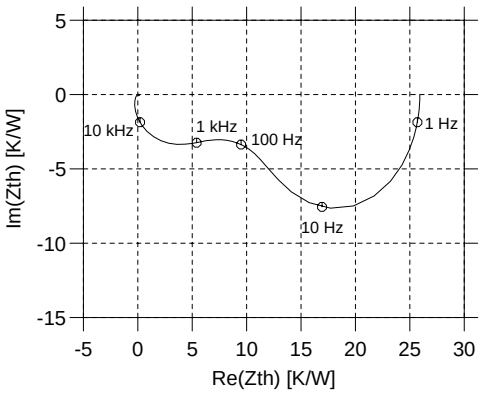

Fig. 1. Complex locus of the thermal transfer impedance, calculated using an analytical series expression, for a 4-layer, heatsink mounted structure examined by Szekely *et al.*

Digele has stated [19] that the Kirchhoff transformation is of no value for multilayer structures. However, this is incorrect. The Kirchhoff transformation is exact for N-level multilayers with the same functional form (but different values) for the temperature dependent thermal conductivity in each layer. A single, global Kirchhoff transformation is also a good approximation for multilayers in which the functional form of $\kappa(T)$ differs between layers, so long as an appropriately modified effective value for κ_S is chosen in each layer for which the global transformation is not exact [24], [25].

The thermal impedance matrix approach as described here means that generally, temperature will only be calculated in the vicinity of active elements, as required for the coupled electro-thermal solution. No redundant temperature information will be generated on the surface or in the body of the die. However, the solutions of the heat diffusion equation just described, provide analytical expressions for both the thermal impedance matrix and for the corresponding temperature distribution throughout the body of the MMIC. This means that once power dissipations, $\overline{P_i}$, have been obtained self-consistently, by employing the thermal impedance matrix in the coupled electro-thermal implementation, temperature can be obtained essentially exactly, if required, at any point within the body or on the surface of the MMIC. This is of value for model validation against thermal measurements.

The analytical, double Fourier series, derivation of the thermal impedance matrix can be extended to treat 3-dimensional volume heat sources, rather than just the planar heat sources that have been treated previously [11], [17], [18]. Details will be presented elsewhere. The double Fourier series approach also allows exact treatment of inhomogeneous structures, such as MMIC die with vias and partial substrate thinning [18].

The matrix given by Eq. (3) represents an exact analytical solution for time-dependent 3-dimensional heat flow in a MMIC bearing an arbitrary distribution of power dissipating elements. These elements could be transistor fingers or finger subsections, grouped in any fashion, or could represent heat dissipating passive elements. The thermal impedance matrix, Eq. (3), takes full account of all finite length and end effects, and simulates exactly the finite volume of the MMIC die.

The analytical solution, Eq. (3), represents the thermal impulse response of the MMIC. It is frequency dependent as characteristic of distributed systems and contains an infinite number of poles and zeros. It corresponds to a multiport thermal network which cannot be represented exactly by a finite network of frequency independent primitives, (such as thermal networks generated by numerical mesh descriptions, *e.g.* [12], [13], [19], which only give an exact thermal description in the limit of infinitely fine mesh discretisation). The multiport network is already minimal, in that it describes nodes corresponding only to surface heating elements (or discretised interface elements). The multiport network parameter interpretation presented here, makes the thermal impedance matrix approach immediately compatible with network based electromagnetic and electrical circuit solvers [26], without the need for any model reduction beyond that implicit in summation of infinite series to just a finite number of terms.

The thermal impedance matrix, Eq. (3), can either be used directly in frequency space, for instance in harmonic balance simulations, or Laplace inverted to describe thermal time dependence directly in transient simulations. For the harmonic balance case, the solution for the thermal impedance matrix is just of the s-dependent form, Eq. (3), with $s \to i\omega$. It takes the form of an array of frequency dependent complex phasors containing phase and amplitude information for the (asymptotic) si-

nusoidal response to harmonic forcing. This matrix then corresponds to the network parameters of a distributed multi-port thermal network.

Having obtained linearised temperature in s-space, $\overline{\theta}(s)$, and assuming $\overline{P_j}$ corresponding to simple step inputs of magnitudes P_j, analytical inversion gives the corresponding time domain thermal impedance matrix, $R_{TH_{ij}}(t)$, corresponding to step input,

$$R_{TH_{ij}}(t) = \mathcal{L}^{-1}\left\{R_{TH_{ij}}(s)\frac{1}{s}\right\}$$
$$= \frac{1}{\kappa_S LW}\frac{2}{D}\sum_{lmn}\frac{4}{(1+\delta_{m0})(1+\delta_{n0})}\frac{I^i_{mn}I^j_{mn}}{I^i_{00}}$$
$$\times\frac{1-\exp\left\{-\pi^2\left[\left(\frac{l+1/2}{D}\right)^2+\left(\frac{m}{L}\right)^2+\left(\frac{n}{W}\right)^2\right]kt\right\}}{\pi^2\left[\left(\frac{l+1/2}{D}\right)^2+\left(\frac{m}{L}\right)^2+\left(\frac{n}{W}\right)^2\right]} \quad (6)$$

with $l,m,n = 0,1,2,....$ Taking the limit $t \to \infty$ and performing the l summation explicitly, the time-independent result is recovered [17], [18]. Combining tables of standard integrals with alternative expressions for the inverse Laplace transform (using properties of theta functions), equivalent forms for the time-dependent thermal impedance matrix can be obtained which are far more rapidly convergent at very small times. Using the Watson transformation and the Poisson summation formula, series solutions such as Eqns. (3) and (6) can be partially summed explicitly in closed form, and partially accelerated to give even more rapidly evaluated expressions. These results will be presented elsewhere.

Eqns. (3) and (6) also give immediately pole-zero or time constant representations for the thermal impedance matrices. Writing [27],

$$R_{TH}(t) = \sum_i R_i\left(1-\exp(-t/\tau_i)\right) \quad (7)$$

with $i \equiv (l,m,n)$, it is apparent from Eq. (6) that R_i and τ_i are obtained in explicit analytical form. Retaining just those terms corresponding to the dominant time constants gives a representation similar to that abandoned by Napieralski *et al* [13] because it could not be obtained in a simple parameterised form.

Although it is distributed, the finite thermal system is seen not to be represented by a continuous time constant spectrum, but by a countably infinite number of time constants. However, summing all contributions within range ζ to $\zeta+\Delta\zeta$, where $\zeta = \ln\tau$ [27], continuous spectra are obtained, Fig. 2. These spectra are calculated for a silicon chip considered by Szekely *et al* [27] and agreement with the calculated results presented in Fig. 23 of that paper is good. Exact agreement is not to be expected, as details of the time constant spectrum depend on the magnitude and placement of intervals $\Delta\zeta$. The two curves shown correspond to division of the 8 decade logarithmic interval, into 40 (solid line) and 80 (dotted line) equal subdivisions, respectively.

B. Complex structures

Construction of thermal impedance matrices is now described for more complex systems, such as MMICs with surface metallisation, or MMIC arrays [15], [18].

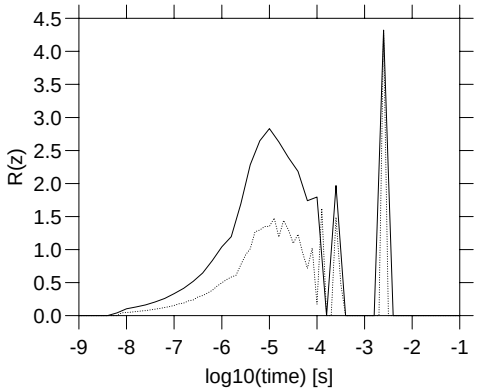

Fig. 2. Time constant spectra obtained from Eqns. (6) and (7) for a Si chip considered by Szekely *et al.*

To illustrate the interface matching approach, the global thermal impedance matrix is constructed for the case of N pieces of rectangular, but otherwise arbitrary, metallisation on the surface of an otherwise homogeneous heatsink mounted MMIC. Matching flux and (linearised) temperature at the interface between metal and MMIC die, the following relation is obtained

$$\underline{\Delta\theta^a} = \underline{\underline{R}}^{glob}_{TH}\underline{P^a}, \quad (8)$$

where $\underline{P^a}$ is the vector of MMIC active device power dissipations, $\underline{\underline{R}}^{glob}_{TH}$ is the global thermal impedance matrix for the coupled GaAs and metal system, and $\underline{\Delta\theta^a}$ is the vector of MMIC active device temperature rises.

The global impedance matrix is given explicitly by

$$\underline{\underline{R}}^{glob}_{TH} = \underline{\underline{R}}^{aa}_{TH} + \underline{\underline{R}}^{ai}_{TH}\underline{\underline{R}}\,\underline{\underline{R}}^{ia}_{TH}, \quad (9)$$

$$\underline{\underline{R}} = \mathrm{diag}\left(\underline{\underline{T}}^{(1)},...,\underline{\underline{T}}^{(n)},...,\underline{\underline{T}}^{(N)}\right)$$
$$\times\left[\underline{\underline{I}}^i - \underline{\underline{R}}^{ii}_{TH}\mathrm{diag}\left(\underline{\underline{T}}^{(1)},...,\underline{\underline{T}}^{(n)},...,\underline{\underline{T}}^{(N)}\right)\right]^{-1}. \quad (10)$$

Here, $\underline{\underline{I}}^i$ is the identity matrix, $\underline{\underline{R}}_{TH}$ of Eq. (3) for the MMIC die has been partitioned by active device elementary areas, a, and interface elementary areas between MMIC die and metal, i, and the $\underline{\underline{T}}^{(n)}$ are thermal impedance matrices for each piece of metallisation.

Hence, by simple matrix manipulation, the global thermal impedance matrix for the metallised MMIC can be obtained as an explicit matrix expression for any given value of Laplace transform variable, s. Also, using the simple algorithm for the numerical Laplace inverse, *e.g.* [28], the value of the global thermal impedance matrix can be evaluated at any time step, $n\delta t$, in the time domain. Direct time domain interface matching (which avoids repeated matrix inversion) is described in [15]. In cases where non linear interface matching cannot be neglected, the thermal impedance matrix approach allows formulation of a non linear system of equations for the correctly matched temperatures [17].

III. Circuit simulations

This section describes coupled simulations at the circuit level, based on integration of the thermal impedance

matrix model with the microwave EM/electrical circuit simulator, Transim. Some insight into the Transim program architecture is given in [16]. In this paper, simulations were performed using state variable harmonic balance [29] and convolution transient [30] methods.

One way of incorporating thermal effects in a circuit simulator [31] is to make the thermal model look like an electrical circuit. A problem with this strategy is to provide separate circuits for the electrical and thermal parts. This has been addressed by the concept of local reference nodes, initially developed for integrated circuit and EM field analysis of distributed microwave circuits [32]. The local reference nodes guarantee that there is no mixing of electric and thermal currents.

The circuit used in the simulations described below, is shown in Fig. 3. The MESFET was modelled using the

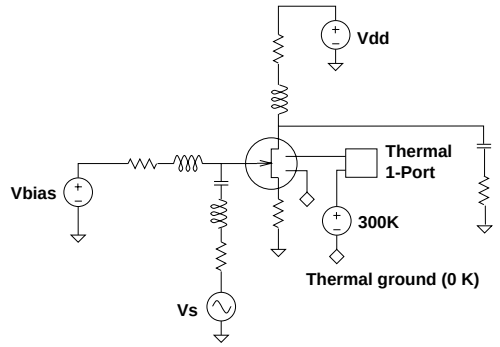

Fig. 3. Schematic of the simulated amplifier with thermal circuit.

Curtice-Ettemberg cubic model with symmetric diodes and capacitances [33]. The extra terminals in the MESFET schematic represent the thermal connections.

Transient analysis of distributed microwave circuits is complicated by the inability of frequency independent primitives to model distributed circuits. Generally, the linear part of a microwave circuit is described in the frequency domain by network parameters, especially where numerical field analysis is used to model a spatially distributed structure. Inverse Fourier transformation of these network parameters yields the impulse response of the linear circuit. This has been used with convolution to achieve transient analysis of distributed circuits. Transim uses a state variable approach for the convolution transient [30]. An algebraic nonlinear system is solved at each time step using a quasi-Newton method.

The thermal element is considered as a nonlinear element for the convolution transient analysis. Therefore it is treated in the time domain, Eq. (6). The chosen state variable is the input power to the thermal system. The thermal impedance matrix approach is used to calculate the resulting temperature at each iteration to solve the nonlinear system. The Kirchhoff transformation is implemented within the thermal element, so allowing non linear interface matching between thermal subsystems.

The harmonic balance (HB) technique uses a linear combination of sinusoids to approximate the periodic and quasi-periodic signals found in a time-dependent steady-state response. The system of nonlinear differential equations describing the circuit can then be transformed into a nonlinear algebraic system. Details of the implementation of HB in Transim are given in [29].

In this analysis, the thermal element is modelled in the frequency domain, Eq. (3). The elements of the thermal impedance matrix are entered directly into the modified nodal admittance matrix of the circuit at each frequency. The thermal element is then 'embedded' in the linear part of the HB formulation and does not increase the size of the nonlinear system of equations. The Kirchhoff transformation for the linear thermal system is transferred into the already non linear active device model by appropriate state variable definition.

No separate thermal simulation is required for the coupled electro-thermal calculation. All thermal impedance matrices are generated by multiport thermal network elements defined within the electro-thermal circuit simulation engine, Transim. Thermal impedance matrices (in either the time or frequency domain) are precomputed from fully analytical expressions, prior to the coupled electro-thermal simulation. They only have to be generated once, for any specified thermal subsystem, and can be stored for re-use in later simulations.

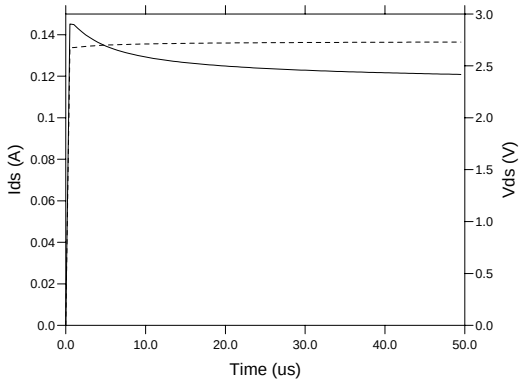

Fig. 4. Drain-source current I_{ds} (solid line) and drain-source voltage V_{ds} (dashed line) for a 5-finger power transistor, from transient electro-thermal analysis.

Fig. 4 illustrates transient decay in drain-source current I_{ds}, as a result of thermal variation under the influence of a step input in drain-source voltage V_{ds}, for a multi-finger power transistor, calculated using the thermal impedance matrix approach implemented in Transim. 2-tone HB simulations were also performed illustrating intermodulation distortion due to amplifier non linearity. These simulations used the simplest one-port thermal description of the power FET, returning surface average temperature rise as a function of surface average power dissipation over the active regions of the multigate device. N-port thermal elements have also been implemented in Transim. After precomputation of thermal impedances, the coupled electro-thermal simulations took a few seconds on a 500 MHz Pentium processor.

IV. Conclusion

The thermal impedance matrix method described here represents an original approach to global thermal modelling of complex device structures. It is based on fully analytical expressions for solution of the heat diffusion equation in rectangular thermal subvolumes (though other regular geometries, such as cylinders, are readily treated). The method describes arbitrarily complex 3-dimensional volumes without the need to invoke finite volume, finite element, finite difference or boundary el-

ement methods. It requires no volume or surface discretisation, discretising only the interfaces between thermal subsystems. It is compatible with network based EM/electrical circuit simulators via interpretation as a linear thermal network, with direct use of essentially exact, generalised multiport network parameters, in either frequency space or the time domain. This approach avoids the need for explicit model reduction, apart from that inherent in truncation of infinite series at a finite number of terms sufficient to ensure convergence (and in numerical Laplace inversion, when employed). The method has no power or temperature restrictions, so is not a small signal approximation.

This thermal impedance matrix method has been illustrated by generation of thermal responses for test systems in both the frequency and time domains, and compared against published results. Agreement was found to be good. Integration of the thermal impedance matrix model with the EM/electrical circuit simulator, Transim, has been demonstrated, and leads to explicit prediction of thermal effects at the circuit level. This modelling capability will be applied to the study and design of spatial power combining systems.

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