A Generalized Scattering Matrix Method Using the Method of Moments for Electromagnetic Analysis of Multilayered Structures in Waveguide

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***Abstract*—The method of moments (MoM) in conjunction with the generalized scattering matrix (GSM) approach is proposed to analyze transverse multilayered structures in a metal waveguide. The formulation incorporates ports as an integral part of the GSM formulation, thus, the resulting model can be integrated with circuit analysis. The proposed technique permits the modeling of interactive discontinuities due to the consideration of a large number of modes in the cascade. The GSM–MoM method can be successfully applied to the investigation of a variety of shielded multilayered structures, iris coupled filters, determining the input impedance of probe excited waveguides, and of waveguide-based spatial power combiners.**

***Index Terms*— Generalized scattering matrix, method of moments, numerical modeling.**

I. INTRODUCTION

THE generalized scattering matrix (GSM) method has been widely used to characterize waveguide junctions and discontinuities. The GSM is a matrix of coefficients of forward and backward traveling modes and describes all self and mutual interactions of scattering characteristics, including contributions from both propagating and evanescent modes. Thus structures of multiple discontinuities are modeled by cascading a number of GSM's. This paper was motivated by the need to globally model waveguide-based spatial power-combining systems [1]–[4]. In such systems, a large number of active cells radiate signals into a waveguide, and power is combined when the individual signals coalesce into a single propagating waveguide mode. Most spatial power combiners can be viewed as multiple layers of arbitrarily metalized planes transverse to the longitudinal direction of a metal waveguide. Active devices are inserted at ports in some of the metalized transverse planes. The contribution of the work presented in this paper is to introduce circuit ports (ports with voltages and currents) into the GSM formulation. This facilitates the incorporation of the electromagnetic model of a microwave structure into a nonlinear microwave-circuit simulator as required in computer-aided global modeling.

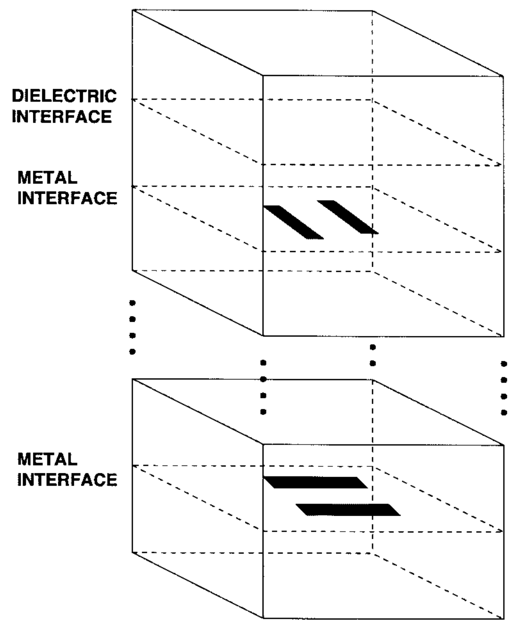

Fig. 1. Multilayer structure in metal waveguide showing cascaded blocks.

The problem of modeling multilayered structures with ports in a shielded environment can be analyzed by at least two approaches. In the first, a specific Green's function for the proposed structure is constructed and then the method of moments (MoM) [5] is directly applied to the entire structure. This results in severe computational and memory demands for electrically large structures. The second approach proposed here is to characterize each layer using a GSM with circuit ports and then cascade this matrix with its neighbors to obtain the composite GSM of a complete system, such as that shown in Fig. 1. Various formulations have been used in developing the standard GSM (without circuit ports). The mode-matching technique is the most widely used for waveguide junctions and discontinuities of simple geometries. The MoM has been used in developing the GSM of arbitrarily shaped dielectric discontinuities [6], metallic posts [7], waveguide junctions [8], and waveguide problems with probe excitation [9]. In

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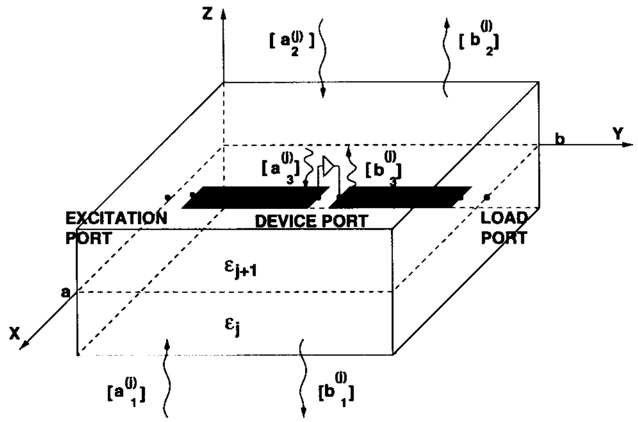

Fig. 2. Geometry of the j the GSM blocks. The four vertical walls are metal.

its common implementation, the MoM uses subdomain basis functions of current. This is used here to compute a port impedance matrix in the solution process [10]. As well as using subdomain current basis functions on the metallization, the MoM formulation implemented here uses delta gap voltages and, thus, the MoM characterization naturally employs port voltage and current variables. The ports are explicitly defined in the GSM and they are accessible after cascading. The method can address a wide class of problems such as a variety of shielded multilayered structures, iris coupled filters, input impedance for probe excited waveguides, and waveguide-based spatial power combiners. From this point on, we will refer to a circuit port as just a port, and an electromagnetic port, which is defined for incident and scattered modes, as a mode.

II. GSM for Building Blocks

The key concept in the method developed here is formulation of a GSM for one transverse layer at a time, and the GSM of individual blocks are cascaded to model a multilayer structure. The general building block is shown in Fig. 2. Here, an arbitrarily shaped metallization is located at the interface of two dielectric media with relative permittivities ϵ_j and $\epsilon_{(j+1)}$, respectively. For illustration purposes, an internal port is specified to show the location of a device, and an excitation port is defined in connection with the source or load, although the number of circuit ports is arbitrary. The vector of coefficients $[a_1^j]$ represents the coefficients of modes incident from medium j into medium $j+1$, $[a_2^j]$ represents the coefficients of modes incident from medium $j+1$ into medium j, and $[a_3^j]$ represents all coefficients of power waves incident from the circuit ports. Similarly, $[b_i^j]$ are the vectors of reflected mode or power wave coefficients corresponding to $[a_i^j]$, $i = 1, 2, 3$. The relations between $[b_i^j]$ and $[a_i^j]$ are determined in this section and the matrix relationship is the GSM.

The analysis begins by expressing the phasors of the electric- and magnetic-field vectors in terms of their eigenmode expansions [11]

$$\overline{E}^{+}(x,y,z) = \sum_{l=1}^{\infty} a_l \overline{E}_l^{+}(x,y,z) \tag{1}$$

$$\overline{H}^{+}(x,y,z) = \sum_{l=1}^{\infty} a_l \overline{H}_l^{+}(x,y,z) \tag{2}$$

$$\overline{E}^{-}(x,y,z) = \sum_{l=1}^{\infty} b_l \overline{E}_l^{-}(x,y,z) \tag{3}$$

$$\overline{H}^{-}(x,y,z) = \sum_{l=1}^{\infty} b_l \overline{H}_l^{-}(x,y,z) \tag{4}$$

where the individual electric and magnetic eigenmodes are

$$\overline{E}_l^{\pm}(x,y,z) = (\overline{e}_l \pm \overline{e}_{zl})\ \exp(\mp\Gamma_l z)$$
$$\overline{H}_l^{\pm}(x,y,z) = (\pm\overline{h}_l + \overline{h}_{zl})\ \exp(\mp\Gamma_l z).$$

The propagation constant of the lth mode is $\Gamma_l = \sqrt{k_{cl}^2 - k_j^2}$ with $k_{cl} = \sqrt{k_{xl}^2 + k_{yl}^2}$, $k_j = \omega\sqrt{\mu_0\epsilon_0\epsilon_j}$, $k_{xl} = m\pi/a$, and $k_{yl} = n\pi/b$. For simplicity, the index pair (m,n) has been replaced by a single index l. Note that all TE and TM waveguide modes are considered. The amplitude coefficients of mode l are denoted as a_l and b_l for waves propagating in the positive and negative z-directions, respectively. The "$\pm$" sign indicate propagation in the positive and negative z-directions. The electric and magnetic mode functions e_l and h_l are normalized using the normalization condition

$$\iint_A \left[\overline{e}_l \times \overline{h}_l\right] \cdot (\overline{z})\, ds = 1$$

where A is the waveguide cross section.

A. Metallization at Interface

The concept behind the procedure that follows is that distinct waveguide modes are coupled by irregular distributions of conductors at the dielectric/dielectric interface. The regions at the interface that are not metalized do not couple modes. The characterization of the metalized interface is developed by separately considering mode-to-mode, port-to-port, and port-to-mode interactions [12].

1) Mode-to-Mode Interaction: In this section, only the layer at the interface of the dielectric media is considered and the matrix model developed relates the variables at ports to the coefficients of the modes (in each dielectric medium) that are incident and reflected at the layer. First, the MoM is applied to the problem and then the GSM is calculated. The electric-field integral-equation formulation is obtained by enforcing the following impedance boundary condition on the metal surface:

$$\overline{E}^i + \overline{E}^s = Z_s \overline{J} \tag{5}$$

where $\overline{E}^i$ denotes the tangential incident field, $\overline{E}^s$ the tangential scattered field, Z_s the surface impedance, and $\overline{J}$ the unknown surface current density. Later on, the surface impedance is used to represent the lumped load impedances of the ports. The first step in the MoM formulation is to express the scattered field in terms of the electric dyadic Green's function $(\overline{\overline{G}}_e)$ as follows:

$$\overline{E}^s(r) = \iint_{S'} \overline{\overline{G}}_e(r,r') \cdot \overline{J}(r')\, ds'. \tag{6}$$

Here, primed coordinates denote the source location while unprimed coordinates denote the observation location. In solving for $\overline{E}^s(r)$, the surface current density $\overline{J}(r')$ is expanded as a set of subdomain basis functions as follows:

$$\overline{J}(r') = \sum_{i=1}^{N} I_i \overline{B}_i(r') \quad (7)$$

where $\overline{B}_i$ is the ith basis function and I_i is the unknown current amplitude at the ith basis. Each basis corresponds to one of N ports. A Galerkin procedure yields the discretization of the integral equation (5)

$$\iint_S \overline{B}_j(r) \cdot \overline{E}^i(r)\, ds = -\sum_{i=1}^{N} I_i \iint_S \iint_{S'} \overline{B}_j(r) \cdot \overline{\overline{G}}_e(r,r') \cdot \overline{B}_i(r')\, ds'\, ds + \sum_{j=1}^{N} I_j \iint_S Z_s(r) \overline{B}_j(r) \cdot \overline{B}_j(r)\, ds \quad (8)$$

leading to a matrix system for the unknown current coefficients $[I] = [I_1 \cdots I_i \cdots I_N]^T$ as follows:

$$[Z + Z_L][I] = [V] \quad (9)$$

where the jith element of the impedance matrix $[Z]$ is

$$Z_{ji} = -\iint_S \iint_{S'} \overline{B}_j(r) \cdot \overline{\overline{G}}_e(r,r') \cdot \overline{B}_i(r')\, ds'\, ds \quad (10)$$

the jth port voltage

$$V_j = \iint_S \overline{B}_j(r) \cdot \overline{E}^i(r)\, ds \quad (11)$$

and the load impedance

$$Z_L = \mathrm{diag}(Z_{L1} \cdots Z_{\mathrm{Li}} \cdots Z_{LN}) \quad (12)$$

with Z_{Li} being the loading impedance at port i. If port i is not loaded, then its corresponding entry is zero [14].

In order to construct the GSM efficiently, it is essential to treat the incident field as being composed of a summation of waveguide modes rather than considering a single mode one at a time [15]. For an incident field propagating in the positive z-direction from medium 1 into medium 2 at the interface

$$\overline{E}^i(r) = \sum_{l=1}^{L_{\max}} a_l^1\, (1 + R_l) \overline{e}_l^1\, \exp\left(-\Gamma_l^1 z\right) \quad (13)$$

where Γ_l^1, $\overline{e}_l^1$ are the propagation constant and the electric-mode function of mode l corresponding to medium 1, respectively. R_l is the reflection coefficient of mode l, defined so that the transverse-electric- and -magnetic mode reflection coefficients are

$$R_l^{\mathrm{TE}} = \frac{\Gamma_l^1 - \Gamma_l^2}{\Gamma_l^1 + \Gamma_l^2} \quad (14)$$

$$R_l^{\mathrm{TM}} = \frac{\Gamma_l^2 \epsilon_1 - \Gamma_l^1 \epsilon_2}{\Gamma_l^2 \epsilon_1 + \Gamma_l^1 \epsilon_2}. \quad (15)$$

Substituting (13) into (11) and, without loss of generality, assuming that the interface plane is located at $z = 0$

$$V_j = \sum_{l=1}^{L_{\max}} a_l^1\, (1 + R_l) \iint_S \overline{B}_j(r) \cdot \overline{e}_l^1\, ds. \quad (16)$$

Hence, the matrix form of (9) can be written as

$$[Z + Z_L][I] = \left[W^1\right][U + R]\left[a_1^1\right] \quad (17)$$

thus, the current vector $[I]$ is written in terms of the modal vector $[a_1^1] = [a_1^1 \cdots a_l^1 \cdots a_{L_{\max}}^1]^T$ as

$$[I] = [Y]\left[W^1\right][U + R]\left[a_1^1\right] \quad (18)$$

where the admittance matrix

$$[Y] = [Z + Z_L]^{-1}$$

and the elements of the $[W^q]$ matrix are given by

$$W_{ji}^q = \iint_S \overline{B}_j \cdot \overline{e}_i^q\, ds.$$

U is the identity matrix and R is a diagonal matrix with diagonal elements being the modal reflection coefficients. Scattering from both the metallization and dielectric interface leads to scattered fields with mode coefficients

$$b_l^1 = -\frac{(1 + R_l)}{2} \iint_S \overline{J} \cdot \overline{E}_l^{+} ds + R_l a_l^1, \qquad l = 1, \cdots, L_{\max}. \quad (19)$$

Using the current density expansion (7), the coefficients of the scattered modes $[b_1^1]$ can be written as

$$\left[b_1^1\right] = -\frac{1}{2}\, [U + R]\left[W^1\right]^T [I] + [R]\left[a_1^1\right] \quad (20)$$

where T indicates the transpose matrix operation. Substituting (18) into (20) results in the following representation:

$$[b_1^1] = \left(-\frac{1}{2}\, [U + R]\left[W^1\right]^T [Y]\left[W^1\right][U + R] + [R]\right)\left[a_1^1\right]. \quad (21)$$

Since $[b_1^1] = [S_{11}^1][a_1^1]$, we can readily write

$$\left[S_{11}^1\right] = -\frac{1}{2}\, [U + R]\left[W^1\right]^T [Y]\left[W^1\right][U + R] + [R] \quad (22)$$

and

$$\left[S_{21}^1\right] = -\frac{1}{2}\, [C]\left[W^1\right]^T [Y]\left[W^1\right][U + R] + [C] \quad (23)$$

where $[C]$ is a diagonal matrix representing the transmission coefficients.

Equations (13)–(23) are for an incident field traveling in the positive z-direction from layer 1 into layer 2. By symmetry, when the incident field is propagating in the negative z-direction from layer 2 into layer 1, we can write

$$\left[S_{22}^1\right] = -\frac{1}{2}\, [U - R]\left[W^2\right]^T [Y]\left[W^2\right][U - R] - [R] \quad (24)$$

$$\left[S_{12}^1\right] = -\frac{1}{2}\, [C]\left[W^2\right]^T [Y]\left[W^2\right][U - R] + [C]. \quad (25)$$

Equations (22)–(25) are a full representation of scattered modes due to incident modes on a loaded scatterer residing on the interface of two adjacent dielectrics.

2) Mode-to-Port Scattering: The interaction between an incident mode and a port can be described using the concept of generalized power waves [12]. First, assume that port k is terminated by an arbitrary impedance Z_{Lk}. Since the scattering parameters are normally given with reference to a 50-Ω system, it is appropriate to set Z_{Lk} to $R_0 = 50\ \Omega$. The generalized power waves at that port are then given by [13]

$$V_i = \frac{1}{2}\,(V_k + R_0 I_k) \tag{26}$$

$$V_r = \frac{1}{2}\,(V_k - R_0 I_k) \tag{27}$$

$$a_k = \frac{V_i}{\sqrt{R_0}} \tag{28}$$

$$b_k = \frac{V_r}{\sqrt{R_0}} \tag{29}$$

where V_i and V_r are the incident and reflected voltage waves. When there is no excitation at the port, $V_i = 0$ and $V_r = -R_0 I_k$. Hence, the scattered power wave coefficient at port k due to mode excitation is $b_k = -\sqrt{R_0} I_k$. Thus, the scattering coefficients at the ports due to incident modes from medium 1 can be written in matrix form

$$\left[b_3^1\right] = -[R_0]^{1/2}[I]. \tag{30}$$

Substituting for the current using (18) and recalling that $[b_3^1] = [S_{31}^1][a_1^1]$, the scattering submatrix

$$\left[S_{31}^1\right] = -[R_0]^{1/2}[Y]\left[W^1\right][U+R]. \tag{31}$$

Similarly, the scattering coefficients at the ports due to incident modes from medium 2 can be written as

$$\left[S_{32}^1\right] = -[R_0]^{1/2}[Y]\left[W^2\right][U-R]. \tag{32}$$

By reciprocity, the scattering matrix of modes due to port excitation is readily obtained as $[S_{13}^1] = [S_{31}^1]^T$ and $[S_{23}^1] = [S_{32}^1]^T$.

3) Port-to-Port Scattering: Port quantities are related by a scattering matrix, which relates port-to-port scattering [13]

$$\left[S_{33}^1\right] = [R_0]^{1/2}[Z_p + R_0]^{-1}[Z_p - R_0][R_0]^{-(1/2)} \tag{33}$$

where Z_p is the port impedance matrix.

B. Dielectric Interface

In the absence of metallization, there is no coupling of modes at the dielectric interface. Hence, the scattering matrix is diagonal. For a dielectric interface between mediums 1 and 2 with relative permittivities ϵ_1 and ϵ_2, respectively, the scattering parameters are given by

$$\begin{aligned}
[S_{11}] &= \mathrm{diag}(R_1 \cdots R_l \cdots R_{L\max}) \\
[S_{12}] &= \mathrm{diag}(C_1 \cdots C_l \cdots C_{L\max}) \\
[S_{21}] &= [S_{12}] \\
[S_{22}] &= \mathrm{diag}(-R_1 \cdots -R_l \cdots -R_{L\max})
\end{aligned}$$

where

$$\begin{aligned}
C_l^{\mathrm{TE}} &= \frac{2\sqrt{\Gamma_l^1 \Gamma_l^2}}{\Gamma_l^1 + \Gamma_l^2} \\
C_l^{\mathrm{TM}} &= \frac{2\sqrt{\Gamma_l^1 \epsilon_2 \Gamma_l^2 \epsilon_1}}{\Gamma_l^1 \epsilon_2 + \Gamma_l^2 \epsilon_1}.
\end{aligned}$$

As expected, $(R_l^{\mathrm{TE}})^2 + (C_l^{\mathrm{TE}})^2 = 1$ and $(R_l^{\mathrm{TM}})^2 + (C_l^{\mathrm{TM}})^2 = 1$, indicating conservation of power.

III. Cascade Connection

The technique of the previous section develops a GSM for a single interface at a transverse plane (with respect to the direction of propagation) in a metal waveguide. A multilayer structure, such as that shown in Fig. 1, is modeled by cascading the GSM's of individual layers and propagation matrices. Each propagation matrix describes translation of the mode coefficients from one transverse plane to another through a homogeneous medium.

The modeling of a two-layer structure with the layers separated by a waveguide section is illustrated in Fig. 3. The analysis proceeds by computing the GSM of the first layer $[S^{(1)}]$ and then evaluating a propagating matrix $[P]$ describing the waveguide section. Finally, computation of the GSM of the second layer $[S^{(2)}]$ enables cascading of $[S^{(1)}]$, $[P]$, and $[S^{(2)}]$ to obtain the composite GSM $[S^{(c)}]$.

Each block is represented by

$$\left[b^i\right] = \left[S^{(i)}\right]\left[a^i\right], \qquad i = 1, 2 \tag{34}$$

where

$$\left[S^{(i)}\right] = \begin{bmatrix} S_{11}^i & S_{12}^i & S_{13}^i \\ S_{21}^i & S_{22}^i & S_{23}^i \\ S_{31}^i & S_{32}^i & S_{33}^i \end{bmatrix}. \tag{35}$$

In calculating the composite GSM, the internal wave coefficients $[a_2^1]$, $[b_2^1]$, $[a_1^2]$, and $[b_1^2]$ must be translated through the waveguide section. This is achieved using the propagation matrix

$$[P] = \mathrm{diag}\Big(\exp(-\Gamma_1 d) \cdots \exp(-\Gamma_l d) \cdots\ \exp(-\Gamma_{L_{\max}} d)\Big)$$

where d is the waveguide section separating the two layers. $[P]$ is a diagonal matrix, as the modes do not couple in the waveguide section, and each is translated by its exponential propagation constant. Thus, the internal mode coefficients are

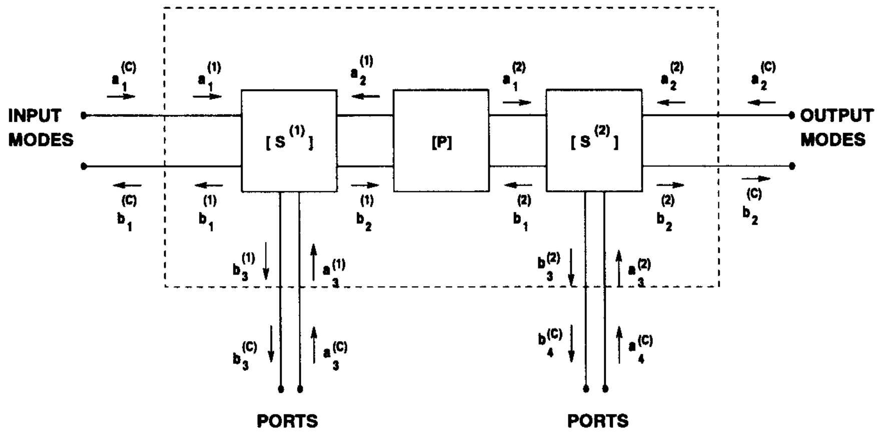

Fig. 3. Block diagram for cascading building blocks.

related by

$$\left[b_2^1\right] = [P]^{-1}\left[a_1^2\right] \tag{36}$$

$$\left[b_1^2\right] = [P]^{-1}\left[a_2^1\right]. \tag{37}$$

Thus, the internal mode coefficients $[a_2^1]$ and $[a_1^2]$ can be written in terms of the modes at the external interfaces as follows:

$$\left[a_2^1\right] = [H_2]\Big(\left[S_{11}^2\right][P]\left[S_{21}^1\right]\left[a_1^1\right] + \left[S_{11}^2\right][P]\left[S_{23}^1\right]\left[a_3^1\right] + \left[S_{12}^2\right]\left[a_2^2\right] + \left[S_{13}^2\right]\left[a_3^2\right]\Big) \tag{38}$$

and

$$\left[a_1^2\right] = [H_1]\Big(\left[S_{21}^1\right]\left[a_1^1\right] + \left[S_{22}^1\right][P]\left[S_{12}^2\right]\left[a_2^2\right] + \left[S_{22}^1\right][P]\left[S_{13}^2\right]\left[a_3^2\right] + \left[S_{23}^1\right]\left[a_3^1\right]\Big). \tag{39}$$

Here, the matrices $[H_1]$ and $[H_2]$ are given by

$$[H_1] = \Big([U] - [P]\left[S_{22}^1\right][P]\left[S_{11}^2\right]\Big)^{-1}[P]$$

and

$$[H_2] = \Big([U] - [P]\left[S_{11}^2\right][P]\left[S_{22}^1\right]\Big)^{-1}[P].$$

Combining (36)–(39) yields the composite scattering matrix

$$[S^{(c)}] = \begin{bmatrix} S_{11}^c & S_{12}^c & S_{13}^c & S_{14}^c \\ S_{21}^c & S_{22}^c & S_{23}^c & S_{24}^c \\ S_{31}^c & S_{32}^c & S_{33}^c & S_{34}^c \\ S_{41}^c & S_{42}^c & S_{43}^c & S_{44}^c \end{bmatrix} \tag{40}$$

with submatrices

$$\left[S_{11}^c\right] = \left[S_{11}^1\right] + \left[S_{12}^1\right][H_2]\left[S_{11}^2\right][P]\left[S_{21}^1\right]$$
$$\left[S_{12}^c\right] = \left[S_{12}^1\right][H_2]\left[S_{12}^2\right]$$
$$\left[S_{13}^c\right] = \left[S_{13}^1\right] + \left[S_{12}^1\right][H_2]\left[S_{11}^2\right][P]\left[S_{23}^1\right]$$
$$\left[S_{14}^c\right] = \left[S_{12}^1\right][H_2]\left[S_{13}^2\right]$$
$$\left[S_{21}^c\right] = \left[S_{21}^2\right][H_1]\left[S_{21}^1\right]$$
$$\left[S_{22}^c\right] = \left[S_{22}^2\right] + \left[S_{21}^2\right][H_1]\left[S_{22}^1\right][P]\left[S_{12}^2\right]$$
$$\left[S_{23}^c\right] = \left[S_{21}^2\right][H_1]\left[S_{23}^1\right]$$
$$\left[S_{24}^c\right] = \left[S_{23}^2\right] + \left[S_{21}^2\right][H_1]\left[S_{22}^1\right][P]\left[S_{13}^2\right]$$
$$\left[S_{31}^c\right] = \left[S_{31}^1\right] + \left[S_{32}^1\right][H_2]\left[S_{11}^2\right][P]\left[S_{21}^1\right]$$
$$\left[S_{32}^c\right] = \left[S_{32}^1\right][H_2]\left[S_{12}^2\right]$$
$$\left[S_{33}^c\right] = \left[S_{33}^1\right] + \left[S_{32}^1\right][H_2]\left[S_{11}^2\right][P]\left[S_{23}^1\right]$$
$$\left[S_{34}^c\right] = \left[S_{32}^1\right][H_2]\left[S_{13}^2\right]$$
$$\left[S_{41}^c\right] = \left[S_{31}^2\right][H_1]\left[S_{21}^1\right]$$
$$\left[S_{42}^c\right] = \left[S_{32}^2\right] + \left[S_{31}^2\right][H_1]\left[S_{22}^1\right][P]\left[S_{12}^2\right]$$
$$\left[S_{43}^c\right] = \left[S_{31}^2\right][H_1]\left[S_{23}^1\right]$$
$$\left[S_{44}^c\right] = \left[S_{33}^2\right] + \left[S_{31}^2\right][H_1]\left[S_{22}^1\right][P]\left[S_{13}^2\right].$$

IV. Results and Discussion

The GSM–MoM method developed here can be used for metal-waveguide-like structures with multiple layers of arbitrarily shaped metallization. It is not limited to structures in infinite waveguides, as is demonstrated by the shielded microstrip that follows. Numerical results have been obtained for the specific example of the shielded microstrip filter shown in Fig. 4. The filter is contained in a box of dimensions 92 × 92 × 11.4 mm $(a \times b \times c)$. The substrate height is 1.57 mm and it has a relative permittivity of 2.33. In analysis, the structure is

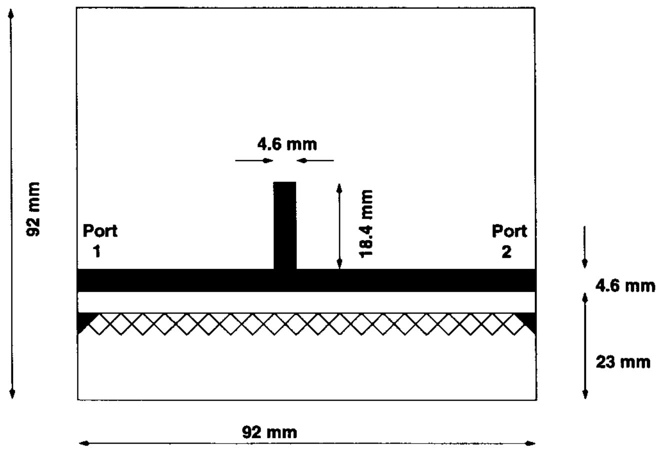

Fig. 4. Geometry of a microstrip stub filter showing the triangular bases functions used. Shaded basis indicate port locations.

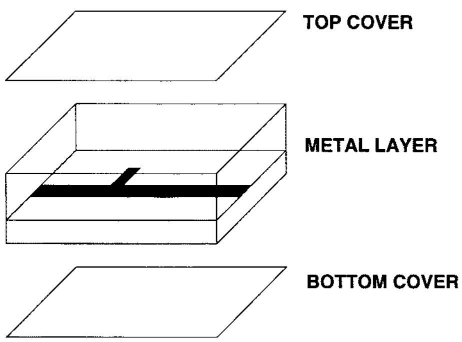

Fig. 5. Three-dimensional view illustrating the layers of the stub filter.

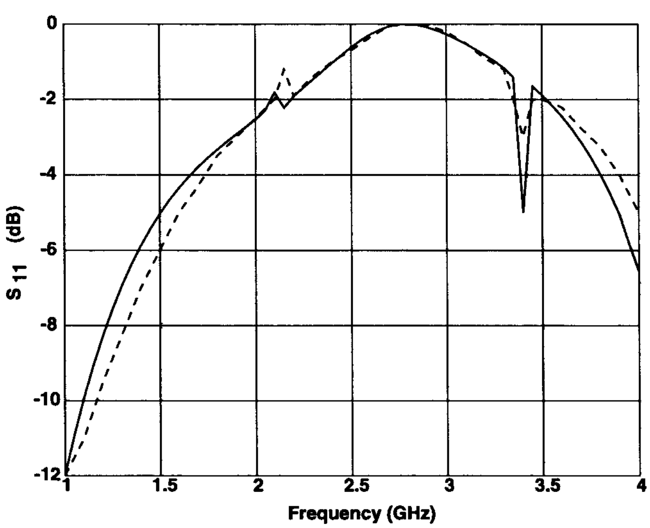

Fig. 6. Scattering parameter S_{11}. Solid line: GSM–MoM. Doted line from [16].

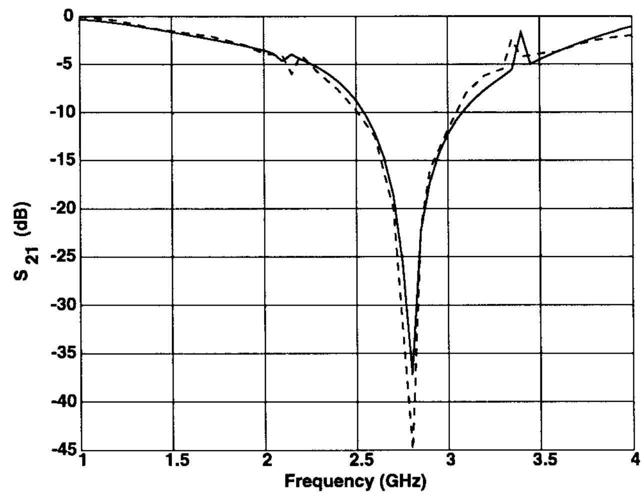

Fig. 7. Scattering parameter S_{21}. Solid line: GSM–MoM. Doted line from [16].

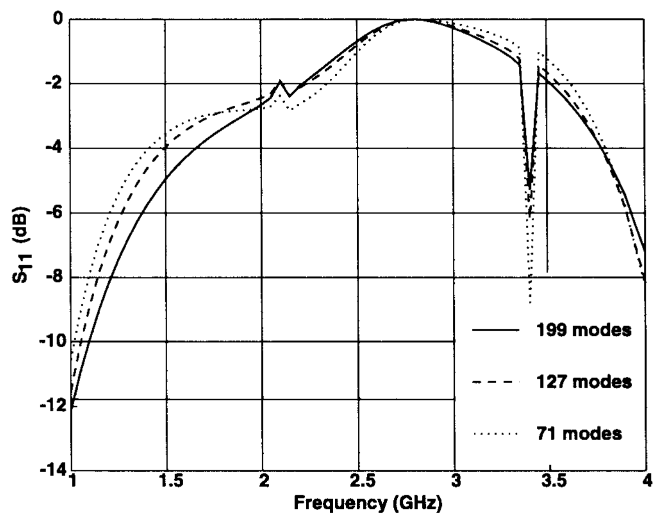

Fig. 8. Various cascading modes showing convergence of S_{11}.

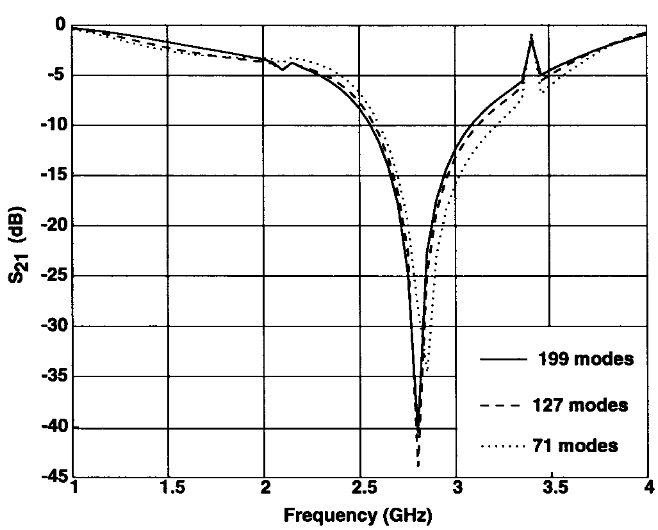

Fig. 9. Various cascading modes showing convergence of S_{21}.

decomposed into three layers, as shown in Fig. 5, with layers 1 and 3 being the top and bottom covers, respectively. The covers are perfect conductors and, hence, their GSM's are diagonal matrices with -1 as diagonal elements. Layer 2 is a metal layer with ports. The excitation ports are modeled by the delta-gap voltage model proposed by Eleftheriades and Mosig [16] (the current basis functions for the excitation ports are shown in the shaded region of Fig. 4). This allows the direct computation of network parameters without the need to extend the line beyond its physical length. The GSM of layer 2 is computed using the method described in this paper. The number of modes considered in the GSM for layers 1 and 3 is 287. Layer 2 has 287 modes and two circuit ports. After cascading the three layers, the modes are augmented. The final scattering matrix has rank two, representing the circuit ports of the filter. The reflection and transmission coefficients S_{11} and S_{21} are calculated in Figs. 6 and 7, respectively, and compare favorably with previously reported results [16]. Convergence curves for the scattering parameters are shown for various numbers of modes in Figs. 8 and 9. As desired, convergence to a result is asymptotically approached as the number of modes considered increases. The need for a large number of modes is in intuitive agreement since dimensions are small compared to the guide wavelength. This example represents an extreme test of the method developed here.

V. Conclusion

A GSM technique has been developed based on a MoM formulation. The method explicitly incorporates device ports and circuit ports in the formulation. Cascading formulas were presented to calculate the composite scattering matrix of a multilayer structure. This matrix is a complete description of the structure. The technique was verified by simulating a shielded microstrip stub filter. The interaction of layers is handled using a GSM method where an evolving composite GSM matrix must be stored to which only the GSM of one layer at a time is evaluated and then cascaded. Thus, computation increases approximately linearly as the number of layers increases. Memory requirements are determined by the number of modes and, thus, is independent of the number of layers. The resulting composite matrix can be reduced in rank to the number of circuit ports to be interfaced to a circuit simulator.

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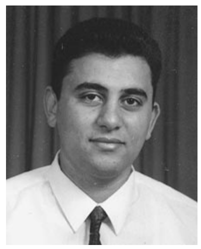

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