# A Full-Wave System Simulation of a Folded-Slot Spatial Power Combining Amplifier Array

Mostafa N. Abdulla, Usman A. Mughal, Huan-Shang Tsai†, Michael B. Steer, and Robert A. York†
Department of Electrical and Computer Engineering
North Carolina State University, Raleigh, NC 27695-7914.
†Department of Electrical and Computer Engineering
University of California Santa Barbara, Santa Barbara, CA 93106.

*Abstract*— **An integrated electromagnetic and circuit modeling environment is developed for a CPW-based spatial power combining array with folded slot antennas. Simulated and measurements effective isotropic power gain (EIPG) of an individual cell and of a 4×4 array is presented.**

## I. Introduction

The quest for high power from solid state devices at millimeter-wave frequencies and above has led to intense interest in combining power from spatially distributed sources [1]. A defining characteristic of these systems is that the electromagnetic environment has a significant effect on system performance. Through electromagnetic coupling the active devices interact and system performance can only be adequately simulated by incorporating electromagnetic modeling in circuit simulation. The challenge is to develop an electromagnetic analysis that supports circuit ports so that it can be used with an appropriate circuit simulator. The major contributions of the work described here are

1. Development of a Method of Moment (MoM) formulation of arbitrary shape slot structure that
 (a) provides a multiport linear model of the structure with circuit ports defined in terms of port voltages and port currents and so is compatible with a circuit simulator; and
 (b) supports calculation of the near and far field distributions and calculation of the EIPG of the overall system.
2. Experimental characterization of a CPW-based folded slot active array.

The development uses the mixed potential integral equations (MPIE) and the method of moments (MoM) techniques to achieve full-wave analysis for the passive linear part and a linear circuit simulator for the active part (amplifiers).

## II. Theory

The MPIE approach has been previously applied to the study of coplanar waveguide discontinuities and has shown very good accuracy, efficiency and versatility in terms of the geometries it can solve. Using the equivalence principle [3], the aperture can be closed and then replaced by an equivalent magnetic surface current $\mathbf{M}_s$ below the ground plane (superscript $b$) and $-\mathbf{M}_s$ above the ground plane (superscript $a$). Therefore, the original problem is decompose into two isolated problems. The total electric and magnetic fields in region "$a$" and region "$b$" are expressed in terms of electric scalar and vector potentials:

$$\mathbf{E}^a(\mathbf{M}_s) = \frac{1}{\epsilon}\nabla\times\mathbf{F}^a(\mathbf{M}_s). \quad (1)$$

$$\mathbf{E}^b(-\mathbf{M}_s) = \frac{1}{\epsilon_0}\nabla\times\mathbf{F}^b(-\mathbf{M}_s). \quad (2)$$

$$\mathbf{H}^a(\mathbf{M}_s) = \mathbf{H}^a{}_{inc} + j\omega\mathbf{F}^a(\mathbf{M}_s) - \nabla\Psi^a(\mathbf{M}_s) \quad (3)$$

$$\mathbf{H}^b(-\mathbf{M}_s) = \mathbf{H}^b{}_{inc} + j\omega\mathbf{F}^b(-\mathbf{M}_s) - \nabla\Psi^b(-\mathbf{M}_s). \quad (4)$$

These potentials are expressed in terms of Green's functions of a multilayer inhomogeneous region

$$\mathbf{F}^a(\mathbf{M}_s) = \int\int_S \overline{\overline{\mathbf{G}}}{}_F^a(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}_s(\mathbf{r}')\,dS'. \quad (5)$$

$$\Psi^a(\mathbf{M}_s) = \int\int_S \overline{\overline{\mathbf{G}}}{}_\Psi^a(\mathbf{r},\mathbf{r}')\cdot\rho_{\mathbf{m}s}(\mathbf{r}')\,dS'. \quad (6)$$

$$\mathbf{F}^b(\mathbf{M}_s) = \int\int_S \overline{\overline{\mathbf{G}}}{}_F^b(\mathbf{r},\mathbf{r}')\cdot[-\mathbf{M}_s(\mathbf{r}')]dS'. \quad (7)$$



0-7803-5135-5/99/$10.00 © 1999 IEEE

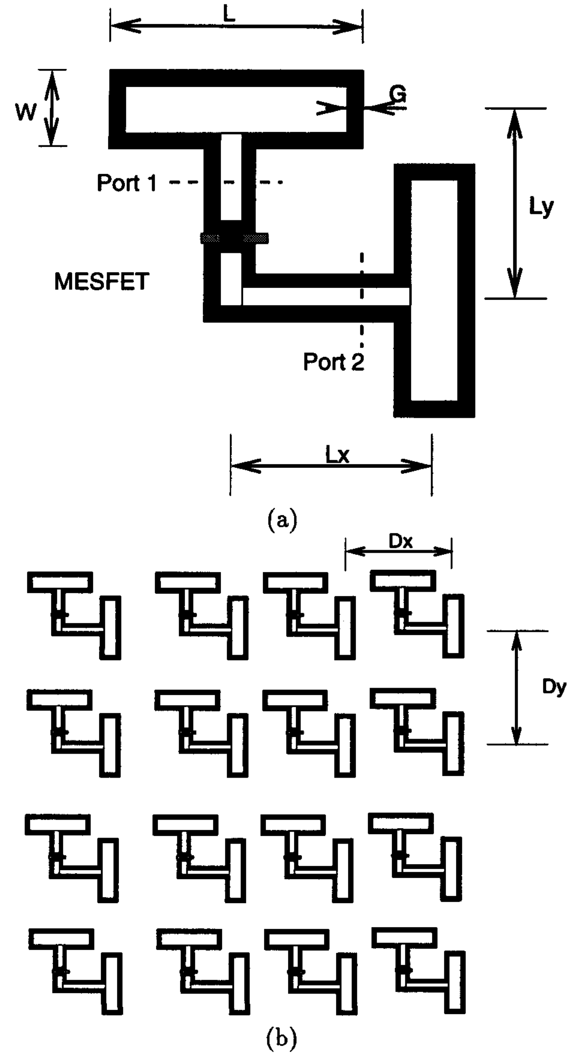


Fig. 1. (a) The amplifier unit cell (b) 4× 4 array [2].

$$\Psi^b(\mathbf{M}_s) = \int\int_S \overline{\overline{\mathbf{G}}}{}_\Psi^b(\mathbf{r},\mathbf{r}') \cdot [-\rho_{\mathbf{m}_s}(\mathbf{r}')]dS'. \quad (8)$$

where $\overline{\overline{\mathbf{G}}}{}_F^a(\mathbf{r},\mathbf{r}')$, $\overline{\overline{\mathbf{G}}}{}_\Psi^a(\mathbf{r},\mathbf{r}')$, $\overline{\overline{\mathbf{G}}}{}_F^b(\mathbf{r},\mathbf{r}')$, and $\overline{\overline{\mathbf{G}}}{}_\Psi^b(\mathbf{r},\mathbf{r}')$ are the spatial-domain dyadic Green's functions of vector and scalar potentials from the magnetic sources $\mathbf{M}_s(\mathbf{r}')$ and $\rho_{\mathbf{m}_s}(\mathbf{r}')$ in the regions $a$ and $b$ respectively. These spatial-domain dyadic Green's functions are calculated using a numerical evaluation of the Sommerfeld integration [5],[4],and [6]. The surface magnetic current radiates an electromagnetic field in the two regions above and below the slots, so that the continuity of the tangential electric field on the surface of the slot is satisfied. The remaining boundary condition to be applied is the continuity of the tangential magnetic field on the surface of the slot aperture which leads to the following MPIE equation that is solved using the MoM. Thus

$$\mathbf{H}^a(\mathbf{r}) = \mathbf{H}^b(\mathbf{r})\,|_{\text{aperture}} \quad (9)$$

The method of moments (MoM) formulation is developed by expanding and testing the MPIE using Galerkin's method to form a linear system of equations which is the MoM matrix set of equations [7]. The magnetic current is expanded as

$$\mathbf{M_s} = \sum_{n=1}^{N} V^x{}_n \mathbf{T}_n^x(\mathbf{r}) + \sum_{m=1}^{M} V^y{}_m \mathbf{T}_m^y(\mathbf{r}). \quad (10)$$

where $\mathbf{T}_n^x$ and $\mathbf{T}_m^y$ are the rooftop basis functions. Upon introducing these distribution functions into the MPIE's and testing them with $\mathbf{T}_k^x$, $k = 1$ to $N$, and $\mathbf{T}_l^y$, $l = 1$ to $M$, the following system of integral equation is obtained:

$$\begin{bmatrix} \mathbf{Y}_{xx} & \mathbf{Y}_{xy} \\ \mathbf{Y}_{yx} & \mathbf{Y}_{yy} \end{bmatrix} \begin{bmatrix} \mathbf{V}_x \\ \mathbf{V}_y \end{bmatrix} = \begin{bmatrix} \langle \Delta\mathbf{H}_x{}^a, \mathbf{T}_k^x \rangle \\ \langle \Delta\mathbf{H}_y{}^b, \mathbf{T}_l^y \rangle \end{bmatrix} \quad (11)$$

Here $\langle , \rangle$ specifies the inner product operation and the subscript $t$ refers to the tangential components in the x-y plane. $\mathbf{Y}_{xx}$, $\mathbf{Y}_{xy}$, $\mathbf{Y}_{yx}$, and $\mathbf{Y}_{yy}$ are the MoM admittance submatrices. The vectors, $\mathbf{V}_x$, and $\mathbf{V}_y$ are the unknown coefficients of the magnetic current amplitudes. $\langle \Delta\mathbf{H}_{x_{inc}}, \mathbf{T}_k^x \rangle$,and $\langle \Delta\mathbf{H}_{y_{inc}}, \mathbf{T}_l^y \rangle$ are the excitation vectors from the incident fields which are identically zero everywhere except at the positions of the sources (at the ports in the case of transmitting mode).

## III. Port

The MoM admittance matrix of the slot system, with electrical ports at the location of the active devices inserted in the slot, includes the interactions associated with the non-port magnetic currents. we are interested in the port impedance matrix and these ports are differential ports located on the slot at the interface of longitudinally adjacent MoM cells. After rewriting the excitation vectors in terms of currents and separate between the port cells and non port cells and then (11) can be expressed as

$$\begin{bmatrix} \mathbf{Y}^{cc} & \mathbf{Y}^{ct} \\ \mathbf{Y}^{tc} & \mathbf{Y}^{tt} \end{bmatrix} \begin{bmatrix} \mathbf{V}^c \\ \mathbf{V}^t \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}^t \end{bmatrix} \quad (12)$$

Here superscript $t$ denotes terminal port quantities and superscript $c$ denotes quantities pertinent to magnetic currents induced on the slot surface. The excitation vector $[\mathbf{I}^t]$ is due to the delta-gap current generators at each port and is given a value of 1 Amp. Equation 12 defines the complete moment matrix, which includes the interactions associated with the non-port magnetic currents. However, we are interested in the port impedance matrix, from which one can infer the coupling coefficients between unit cells on the slot. We will now find these network matrices for the ports. We obtain the voltages from (12) as

$$\begin{bmatrix} \mathbf{V}^c \\ \mathbf{V}^t \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^{cc} & \mathbf{Y}^{ct} \\ \mathbf{Y}^{tc} & \mathbf{Y}^{tt} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}^t \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} \mathbf{Z}^{cc} & \mathbf{Z}^{ct} \\ \mathbf{Z}^{tc} & \mathbf{Z}^{tt} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}^t \end{bmatrix}. \quad (14)$$

From (14), we identify the *port impedance matrix* $[\mathbf{Z}^t] = [\mathbf{Z}^{tt}]$, since $[\mathbf{V}^t] = [\mathbf{Z}^t][\mathbf{I}^t]$ from circuit theory.

## IV. Amplifier and Antenna parameters

This paper focuses on developing an electromagnetic model, compatible with a general purpose microwave circuit simulator, for a CPW-based folded slot active array the transverse view of which is shown in Fig 1(b). Each dimension of this system is around two wavelengths. Each amplifying unit cell is an amplifier connected to the receiving and transmitting folded-slot antennas by CPW lines as shown in Fig 1(a). The dimensions of the folded slot are L=18 mm, W=7 mm, and the gap width is 1 mm. The substrate has a dielectric constant of 10.8 and thickness of 0.635 mm. The array consists of 4×4 unit cells configured as shown in Fig 1(b). The x-spacing is 38.5 mm and the y-spacing is 27.5 mm. The amplifiers are GaAs MESFET's (NE32184A). From the manufacturer's data sheet, the transconductance of this MESFET, $g_m$, is typically 33 mS. Using the same parameters as in [2], leads to $Z_{in} = Z_{out} = Z_{out} = 125\ \Omega$. The overall gain of the array can be expressed in term of the effective isotopic power gain (EIPG) which is the only directly measurable quantity, defined as $G_{FSd} G_{FSa} G_A$, where $G_A$ is the gain of the amplifier and $G_{FSd}$ and $G_{FSa}$ are the directional gains of the folded-slot antenna on the air and dielectric sides, respectively. The EIPG was simulated using farfield calculation of the radiated field result with plane wave excitation of the active array.

## V. Simulation Results

The input impedance for a folded slot antenna with L=78 mm, W=6 mm, gap width of 2 mm, dielectric constant 2.2, and thickness of 0.831mm was calculated using the MoM method. This is compared with the results obtained previously using FDTD simulation [2] and to measurements in Fig. 2. The EIPG of a single amplifier cell and the array are shown and compared with measurements in Fig. 3(a) and (b) respectively. The simulated values has the same frequency dependence and the same maximum value as the measured values. The EIPG is around 11 dB for the unit amplifier and 32 dB for the array. Plots show a difference in the maximum gain frequency from simulated and measurement results. The simulated maximum EIPG occurred at 3.9 GHz which is less than measurement by 0.4 GHz. We believe that this shift in frequency is due to differences between the fabricated and simulated dimensions [2] and also to the limitations in the use of manufactures typical data in constructing the model of the MMIC. Nonlinear behavioral modeling of the MMIC is in progress so that saturation can be studied.

## VI. Acknowledgment

This work was supported by the U.S. Army Research Office through Clemson University as a Multidisciplinary Research Initiative on Quasi-Optics,agreement number DAAG55-97-K-0132.

## References

[1] R. A. York, and Z. B. Popović, "Active and Quasi-Optical Arrays for Solid-State Power Combining," *John Wiley & Sons, Inc.*, 1997.

[2] H. S. Tsai, M. J. W. Rodwell,and R. A. York, "Planar Amplifier Array with Improved bandwidth Using Folded-slots," *IEEE Microwave and Guided wave letters*, Vol.4,No.4 April 1994, pp. 112-114.

[3] R. F. Harrington,"Time-Harmonic Electromagnetic Fields," *New York: McGraw-Hill*, 1961.

[4] M. Naguib, H. Ghali, and H. El-Hennawy, "Mixed Analytical Numerical calculation of Microstrip Antenna Greens Function Using Personal Computer C-language,",*Proc. of the 1995 IEEE AP-S Int. Symposium*.

[5] M. N. Abdulla, "Analysis and Design of Microstrip Antennas," *M.Sc. thesis , Ain Shams University ,Cairo , Egypt*, 1996.

[6] T. Itoh, "Numerical Techniques for Microwave and Millimeter-Wave Passive Structures," *John Wiley & Sons, Inc.*, 1989.

[7] Johnson J. H. Wang, "Generalized Moment Methods in Electromagnetics," *John Wiley & Sons, Inc.*, 1991.

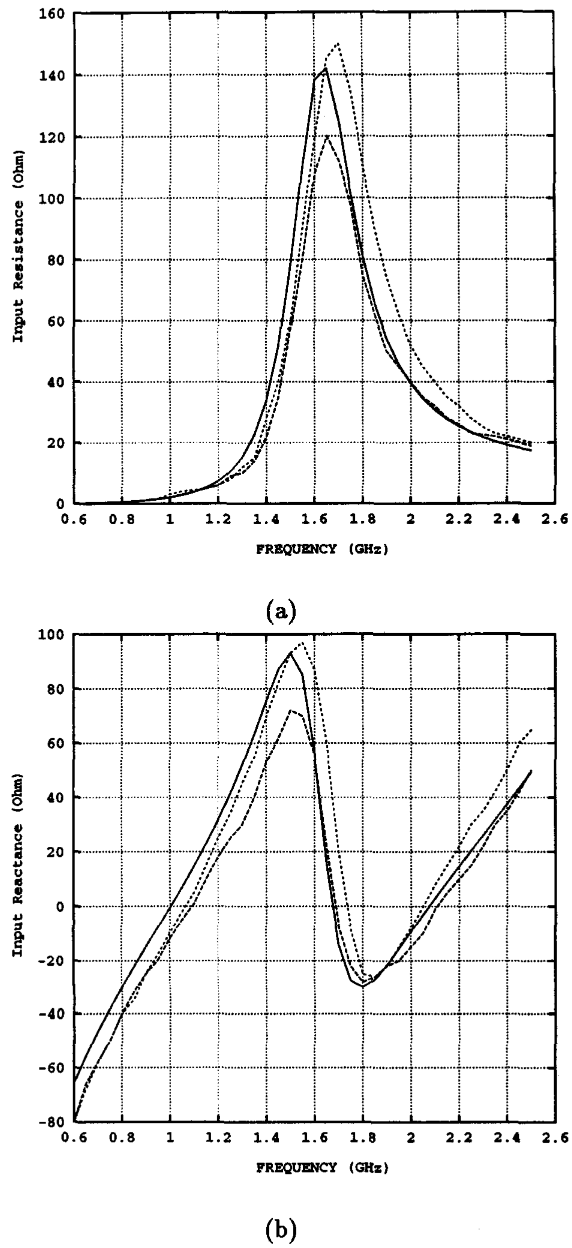


Fig. 2. The calculated and measured input impedance of the folded slot. MoM simulation (solid line), FDTD (short dashed line), and measurement (long dashed line): (a) real; and (b) imaginary.

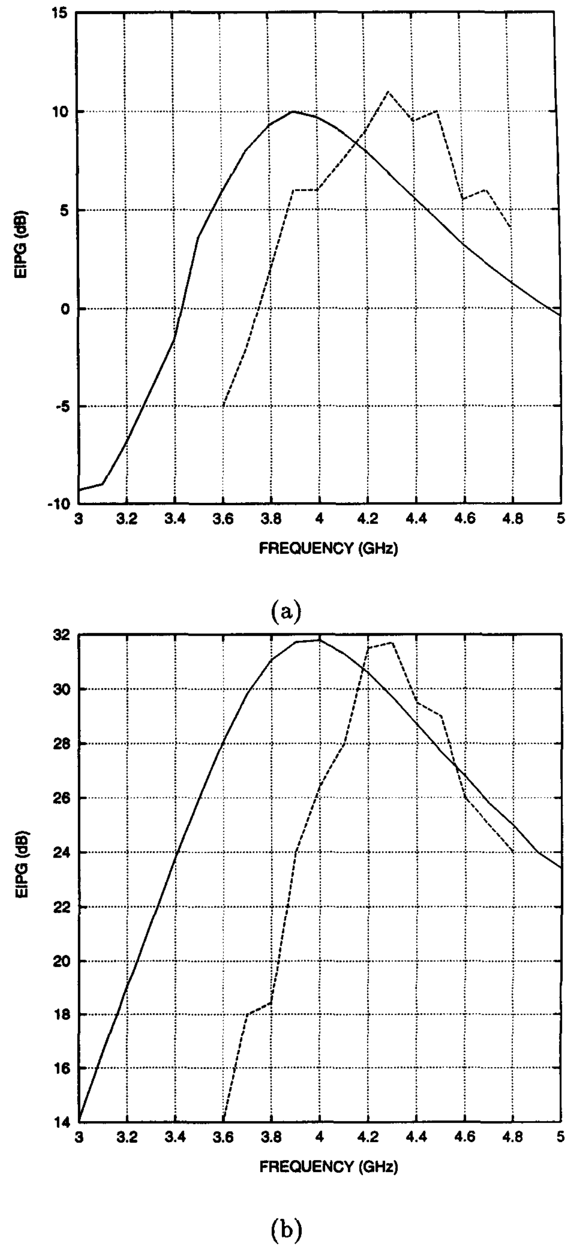


Fig. 3. The calculated and measured EIPG. MoM simulation(solid line), and measurement (dashed line):(a) unit cell; and (b) array.