# A Statistical Relationship for Power Spectral Regrowth in Digital Cellular Radio

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## ABSTRACT

A method is presented for calculating the spectral regrowth generated by a digitally modulated carrier passed through a nonlinear microwave circuit. Spectral regrowth is calculated from the probability density function (pdf) of the modulation amplitude and a complex power series nonlinear model based on measure AM-AM and AM-PM data of a nonlinear device. Calculation of spectral regrowth allows efficient evaluation of the impact of modulation format on spectral regrowth.

## INTRODUCTION

The high level of frequency reuse in cellular wireless communication systems is largely because they are designed to tolerate relatively high levels of interference. One source of interference is interference produced by transmissions in other adjacent channels generating distortion tones outside the band. The performance specifications that are required to control adjacent channel distortion profoundly affect RF active component design. Hence the current interest in developing schemes to accurately predict the adjacent channel power ratio (ACPR) of an RF amplifier in a timely manner.

Recently, characterization of spectral regrowth from nonlinear microwave circuits has been approached using AM-AM and AM-PM transfer characteristics[1,2]. Spectral regrowth is simulated by passing a modulated signal through the transfer characteristics and applying a FFT at the output and measuring the resulting power spectrum. In this work, spectral regrowth is analyzed by performing a nonlinear transformation of the input signal amplitude statistics. Signal statistics are represented by a probability density function of the envelope amplitude variation. Linear and intermodulation power are calculated directly from a nonlinear transformation of the input statistics using a complex power series representation of the AM-AM AM-PM characteristics and statistical moment theory.

The probability density function, pdf, is a compact representation of the statistical envelope amplitude characteristics of a digitally modulated signal. The pdf is a function of the envelope amplitude and the area under the pdf yields the probability of occurrence for amplitudes on a given interval. The pdf of a given modulation format can be estimated ahead of time from a long sequence of input symbols. The pdf is estimated from a normalized histogram of the amplitude variation of a waveform given by [3]

$$\hat{f}_x(x) = \frac{N_i}{N\Delta x} \qquad (1)$$

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where $N_i$ is the number of counts in the $i$th bin, N is the total number of bins, and $\Delta x$ is the bin width. The pdf of a CDMA OQPSK signal was estimated from (1) by taking 400 equal width bins between the minimum and maximum value of the modulation envelope and counting the number of occurrence over $2^{14}$ input symbols. The estimated pdf is shown in figure 1.

The mean of a random signal is the first moment of the signal and is defined by the expectation operator [4]

$$\mu_x = E[X] = \int_{-\infty}^{\infty} x \hat{f}_x(x)\, dx \qquad (2)$$

The first moment or mean of a function of a random variable, Y=g(x), can be calculated by substituting g(x) into (2)

$$\mu_y = E[Y] = E[g(x)] = \int_{-\infty}^{\infty} g(x) \hat{f}_x(x)\, dx \qquad (3)$$

where g(x) is a nonlinear function of the amplitude random variable. A complex power series is used to describe the nonlinear function g(x)

$$\tilde{g}(x) = \tilde{a}_1 x + \tilde{a}_3 x^3 + \tilde{a}_5 x^5 + \tilde{a}_7 x^7 + \ldots \qquad (4)$$

where the $a_n$ are complex power series coefficients[5,6]. Only the odd order terms can be determined from single tone complex compression characteristics of a nonlinear device [8]. Fortuitously, the odd order terms are of the most concern because they produce intermodulation distortion in band and adjacent to the desired signal. Odd order coefficients of (4) can be determined from curve fits to measured AM-AM and AM-PM data.

Consider a memory less nonlinearity described by (4) truncated to fifth order. The mean of the signal plus distortion is calculated by substituting (4) for g(x) into (3)

$$\begin{aligned} &= \int_{-\infty}^{\infty} \tilde{g}(x) \hat{f}_x(x)\, dx \\ &= \int_{-\infty}^{\infty} (\tilde{a}_1 x + \tilde{a}_3 x^3 + \tilde{a}_5 x^5) \hat{f}_x(x)\, dx \\ &= \tilde{a}_1 \int_{-\infty}^{\infty} x \hat{f}_x(x)\, dx + \tilde{a}_3 \int_{-\infty}^{\infty} x^3 \hat{f}_x(x)\, dx + \tilde{a}_5 \int_{-\infty}^{\infty} x^5 \hat{f}_x \\ &= \tilde{a}_1 \mu_x + \tilde{a}_3 \mu_{x^3} + \tilde{a}_5 \mu_{x^5} \end{aligned} \qquad (5)$$

The mean of the distortion is, by inspection of (5),

$$\mu_D = \tilde{a}_3 \mu_{x^3} + \tilde{a}_5 \mu_{x^5} \qquad (6)$$

The intermodulation distortion ratio is calculated by taking the ratio of the sum of mean squares of the signal minus in-band distortion divided by the distortion

$$IMR = 10\log_{10} \frac{|\tilde{a}_1 \mu_x + \tilde{a}_3 \mu_{x^3} + \tilde{a}_5 \mu_{x^5}|^2}{|\tilde{a}_3|^2 \mu_{x^3}^2 + |\tilde{a}_5|^2 \mu_{x^5}^2} \qquad (7)$$

Equation (7) is a statistical relationship for spectral regrowth based on statistical properties of the modulation format and the complex power series representation of a nonlinear device.

## EXAMPLE

The AM-AM and AM-PM curves for a 23dB gain 835MHz driver amplifier were obtained from a single frequency input power sweep using a vector network analyzer. A complex power series to order 13 was fitted to the AM-AM and AM-PM data by using least squares optimization. The complex power series model was verified using a time-variant harmonic balance simulator to compare two-tone intermodulation and ACPR against measured data[4]. The measure versus modeled results for intermodulation in figure 2. The complex power series model is in close agreement measured data for both two-tone intermodulation over a wide range of input power.

The first, third, and fifth order power series coefficients and the estimated pdf data from figure 1 where used with (7) to calculate ACPR. All of the mean integrations for (7) reduced to summations because the pdf was estimated on equal width intervals. The resulting ACPR calculation results are shown with measured ACPR in figure 3. Calculated ACPR agrees well with measured data for signal levels below heavy compression. The lower limit of ACPR measurement is limited by the baseband filter rejection of the CDMA OQPSK signal generator.

## CONCLUSION

A statistical relationship for ACPR based on modulation amplitude statistics and a complex power series nonlinear model has been presented. ACPR is related to a nonlinear transformation of the probability density function for the envelope amplitude distribution of a digital modulated signal. The complex power series model provides an accurate representation of the AM-AM AM-PM characteristics of a bandpass nonlinear microwave device. Calculation of ACPR from the statistical properties of the modulation permits efficient evaluation of the impact of modulation format on spectral regrowth.

## REFERENCES

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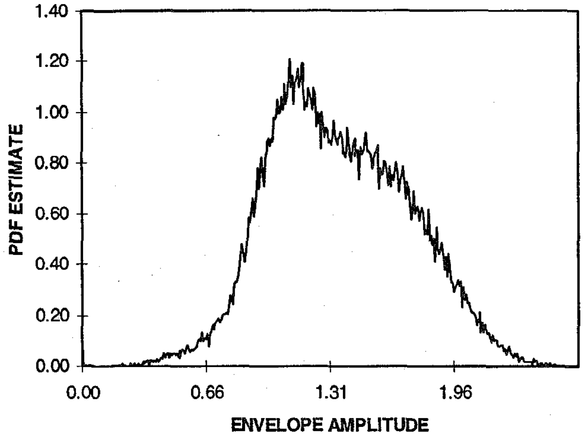

**Figure 1**: Estimated amplitude probability density function.

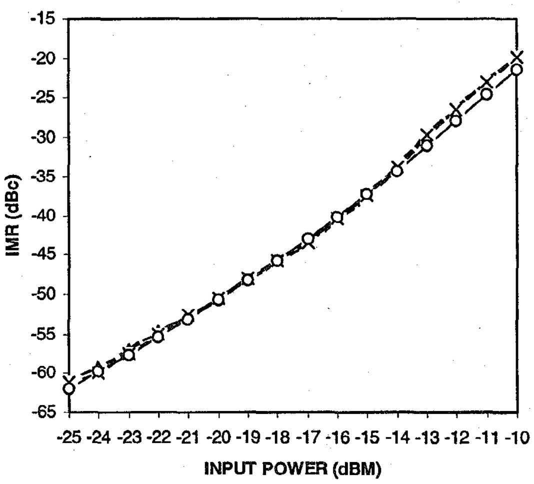

**Figure 2:** Measured and molded two-tone IMR, x-measured and o-modeled.

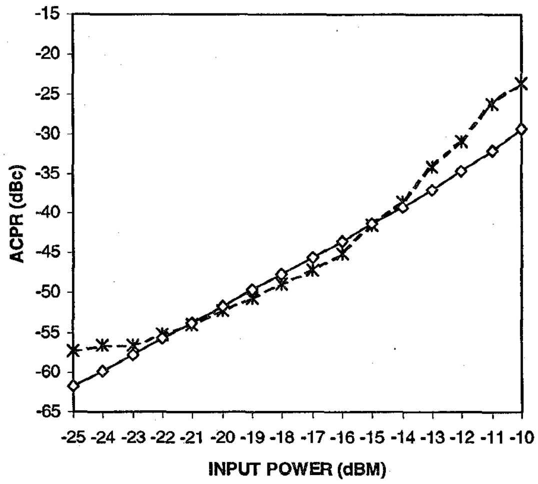

**Figure 3:** Measured and calculated ACPR, *-measured, dimonds-calculated.