

## Electromagnetic Modeling of Finite Grid Structures in Quasi-Optical Systems

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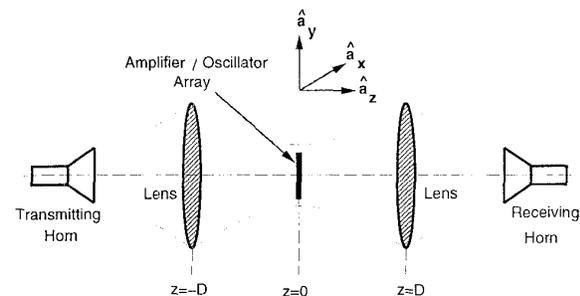
**Abstract.** A full-wave moment method technique developed for the analysis of quasi-optical systems is used to model finite grid structures. This technique incorporates an electric field dyadic Green's function for a grid centered between two lenses in free space which is derived by separately considering paraxial and non-paraxial fields. Results for the driving point reflection coefficient of a  $3 \times 3$  grid are computed and compared with measurements.

### 1. Introduction

Quasi-optical techniques provide a means for combining power from numerous solid-state millimeter-wave sources without lossy metallic interconnections. Power from the sources in an array is combined over a distance of many wavelengths to channel power predominately into a single paraxial mode, see Fig. 1. Progress toward large, high-power, efficient arrays is hampered by the relatively crude state of design technology including the lack of suitable computer aided engineering tools. In particular, the many unit active circuits in a large array cannot be individually optimized for efficiency and stability. This is because no simulation process has been developed to model impedances and stability criteria for a finite array where most of the array elements see different circuit conditions.

The essential component of quasi-optical modeling is development of circuit-level models of quasi-optical structures. The conventional modeling approach using moment method techniques requires the derivation of a mixed, scalar and vector, potential Green's function. In this paper a full-wave moment method implementation is developed for the analysis of finite grid structures in the quasi-optical lens system shown in Fig. 1. A series of developments [1-3] culminated in a straight forward methodology for developing a novel Green's function of a quasi-optical system. The elec-

tric field dyadic Green's function of a quasi-optical system is derived in two parts: one part describing the effect of the quasi-optical paraxial fields and the other part describing the remaining fields. This form of the dyadic Green's function is particularly convenient for quasi-optical systems because of its relative ease of development. It did, however necessitate the development of an advanced method of moments approach combining spatial domain and spectral domain techniques to model a quasi-optical open cavity resonator [4]. With this formulation the field solver can be conveniently used in the development of circuit-level models of quasi-optical systems.



**Figure 1:** Quasi-optical lens system configuration for amplifier/oscillator arrays.

### 2. Quasi-Optical Lens System Dyadic Green's Function

The approach used in [2, 3] to develop a dyadic Green's function for a quasi-optical open cavity resonator is used here to develop a Green's function for the system shown in Fig. 1. In this configuration the array is centered between two lenses in free space where both lenses are assumed to be identical and equally spaced from the array. The dyadic Green's function is

derived in two parts

$$\bar{\bar{\mathbf{G}}}_E = \bar{\bar{\mathbf{G}}}_{En} + \bar{\bar{\mathbf{G}}}_{Em} \quad (1)$$

where  $\bar{\bar{\mathbf{G}}}_{En}$  and  $\bar{\bar{\mathbf{G}}}_{Em}$  describe the non-modal and modal fields, respectively. The non-modal fields are found by removing the paraxial components  $\bar{\bar{\mathbf{G}}}_{Ep}$  from the free space dyadic Green's function,  $\bar{\bar{\mathbf{G}}}_{E0}$ , to give

$$\bar{\bar{\mathbf{G}}}_E = \bar{\bar{\mathbf{G}}}_{E0} - \bar{\bar{\mathbf{G}}}_{Ep} + \bar{\bar{\mathbf{G}}}_{Em} \quad (2)$$

The final dyadic Green's function is evaluated in two parts

$$\bar{\bar{\mathbf{G}}}_E = \bar{\bar{\mathbf{G}}}_{El} + \bar{\bar{\mathbf{G}}}_{E0} \quad (3)$$

where  $\bar{\bar{\mathbf{G}}}_{El} = \bar{\bar{\mathbf{G}}}_{Em} - \bar{\bar{\mathbf{G}}}_{Ep}$  represents the contribution of the lenses (quasi-optical modes) and  $\bar{\bar{\mathbf{G}}}_{E0}$  represents the free space (direct radiation) contribution.

Following a similar derivation of that in [2,3] results in the following:

$$\bar{\bar{\mathbf{G}}}_{El} = - \sum_{mn} \frac{R_{mn} \psi_{mn}}{(1 - R_{mn} \psi_{mn})} \dot{E}_{mn} E_{mn} \bar{\bar{\mathbf{I}}}_T \quad (4)$$

with the free space dyadic Green's function defined as

$$\bar{\bar{\mathbf{G}}}_{E0}(\mathbf{r}|\mathbf{r}') = \left( \bar{\bar{\mathbf{I}}} + \frac{1}{k_0} \nabla \nabla' \right) \frac{\exp(jk_0 |\mathbf{r} - \mathbf{r}'|)}{4\pi |\mathbf{r} - \mathbf{r}'|} \quad (5)$$

where  $\bar{\bar{\mathbf{I}}}$  and  $\bar{\bar{\mathbf{I}}}_T$  are the unit dyad and transverse unit dyad, respectively. The scalar electric modal field  $E_{mn}$  is given by the Hermite-Gaussian traveling wave-beam defined in [1] and  $R_{mn}$  and  $\psi_{mn}$  represent the reflection coefficient and phase, respectively, of the traveling wave-beam modes [2,3].

### 3. Method of Moments

The moment method technique developed in [4] which incorporates a dyadic Green's function for the quasi-optical open cavity resonator and combines both spatial and spectral domains is used here to incorporate the dyadic Green's function of the quasi-optical lens system. The boundary value problem for the current distribution on the conductor grid surface, located at  $z = 0$ , is formulated as an electric field integral equation. The grid surface is segmented into rectangular cells and a Galerkin method is used employing sinusoidal basis functions to give the following linear system

$$[\mathbf{Z}] [\mathbf{I}] = [\mathbf{V}] \quad (6)$$

to be solved for the unknown currents  $I_n$ . With the dyadic Green's function in (3) being comprised of two components, the moment matrix elements in  $[\mathbf{Z}]$  may also be divided into two parts

$$Z_{ji} = Z_{l,ji} + Z_{0,ji} \quad (7)$$

where

$$Z_{l,ji} = - \int_y \int_x \int_{y'} \int_{x'} \mathbf{J}_j(x, y) \cdot \bar{\bar{\mathbf{G}}}_{El}(x, y; x', y') \cdot \mathbf{J}_i(x', y') dx' dy' dx dy \quad (8)$$

and

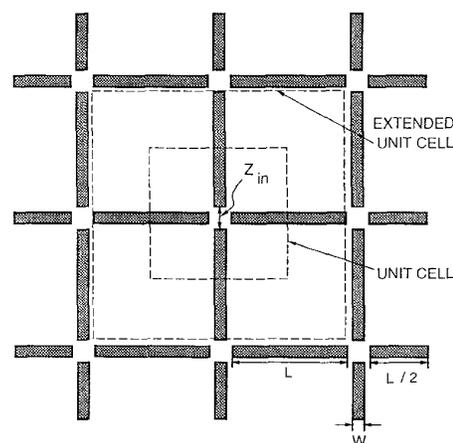
$$Z_{0,ji} = - \int_y \int_x \int_{y'} \int_{x'} \mathbf{J}_j(x, y) \cdot \bar{\bar{\mathbf{G}}}_{E0}(x|x'; y|y') \cdot \mathbf{J}_i(x', y') dx' dy' dx dy. \quad (9)$$

The free space moment matrix elements in (9) are also formulated in the spectral domain [4,5] for computing the self-terms to avoid singularities that occur in (5).

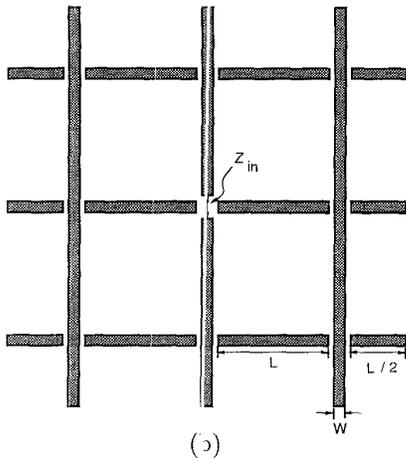
For the excitation vector  $\mathbf{V}$  in (6) a delta-gap voltage generator is used and the driving point impedance is computed as the ratio of the current flowing through the gap to the voltage at the gap.

### 4. Comparison of Computed and Experimental Results

Measurements and simulations were performed in free space for the  $3 \times 3$  grid shown in Fig. 2. The grid consists of 9 unit cells where each unit cell is of dimension  $51.8 \text{ mm} \times 51.8 \text{ mm}$  with the metallic grid lines having a length of  $L = 42 \text{ mm}$  and a width of  $W = 6.35 \text{ mm}$ . The gap spacing where the active device would be was  $9.8 \text{ mm}$ .

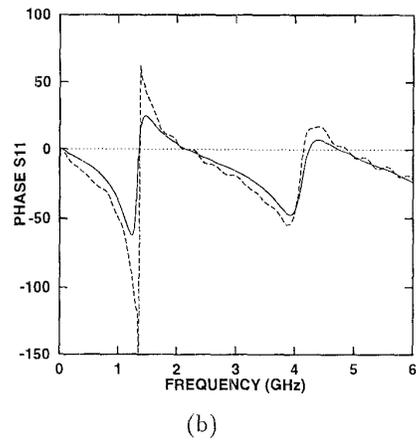
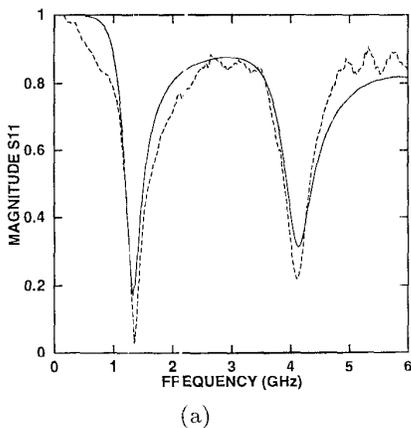


(a)

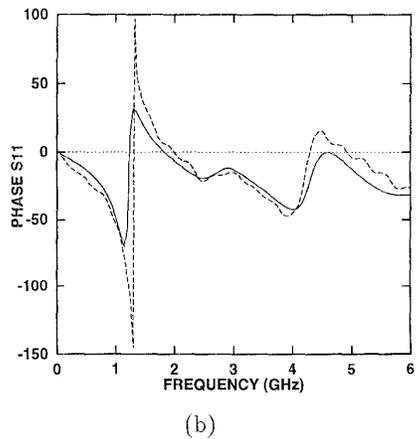
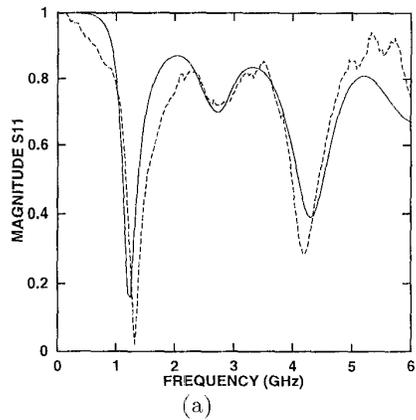


**Figure 2:** A  $3 \times 3$  grid with the driving point impedance being measured in the middle gap: (a) other gaps opened; (b) other gaps shorted.

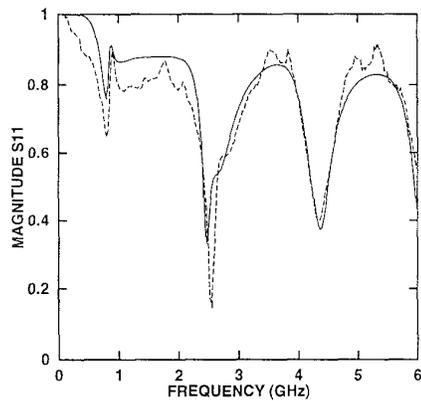
Fig. 3 shows the driving point reflection coefficient for an extended unit cell ( $93.8 \text{ mm} \times 93.8 \text{ mm}$ ) with the same grid line width and gap spacing. Next the entire  $3 \times 3$  grid structure was considered where the driving point reflection coefficient was measured in the middle gap. Figs. 4 and 5 shows the driving point reflection coefficient with the other gaps opened and shorted, respectively. From these results we can observe that there is significant mutual coupling between the grid elements. Measurements and simulations were also performed for the other gaps in the grid. The results indicate that the input impedance of edge and corner gaps differ from that of the middle gap due to the finite extent of the grid. This variation as well as the coupling between cells is ignored in other attempts at modeling quasi-optical systems [6]. The real value of this modeling technique will come from the simulation of much larger arrays.



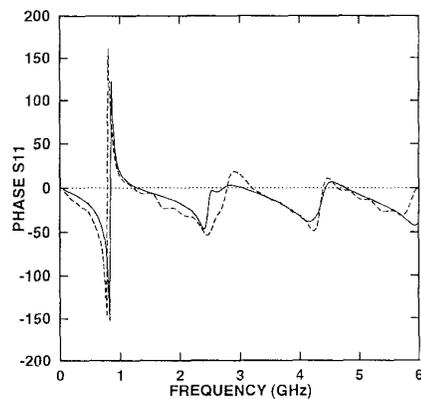
**Figure 3:** Driving point reflection coefficient: (a) magnitude; (b) phase; of the extended unit cell: solid line, simulation; dashed line, measurement.



**Figure 4:** Driving point reflection coefficient: (a) magnitude; (b) phase; of the  $3 \times 3$  opened grid: solid line, simulation; dashed line, measurement.



(a)



(b)

**Figure 5:** Driving point reflection coefficient: (a) magnitude; (b) phase; of the  $3 \times 3$  shorted grid: solid line, simulation; dashed line, measurement.

## 5. Conclusions

A full-wave moment method implementation has been developed for the analysis of finite grid structures in a quasi-optical lens system. This implementation includes the derivation of a dyadic Green's function for this quasi-optical system and a moment method scheme utilizing both spatial and spectral domains for efficient computation of the moment matrix elements. As a verification of the moment method, simulated results have been shown to compare favorably with measurements. The significance of the modeling work are: (1) finite sized grids are considered (there is no need to make the simplifying assumption of an infinite grid of identical unit-cells as is required in all other quasi-optical system modeling approaches); and (2) it is broadband (from DC to any frequency) as required in CAD. From a system development point of view the design of each element in the array can be individu-

ally optimized to achieve an optimum global solution in terms of stability, output power and efficiency.

## Acknowledgment

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