Computer-Aided Analysis of Nonlinear Microwave Circuits Using Frequency-Domain Nonlinear Analysis Techniques: The State of the Art

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ABSTRACT

Frequency-domain nonlinear analysis techniques for the simulation of active microwave circuits solve the linear and nonlinear network equations entirely in the frequency-domain. By so doing, they avoid the aliasing problems inherent in piecewise harmonic balance approaches. Consequently, frequency-domain techniques have extremely wide dynamic range and easily accommodate high order multitone excitation. However, this is at the expense of requiring more restrictive nonlinear device models. There are a large number of frequency-domain nonlinear analysis techniques but all are based on functional expansions which enable the frequency components of the output spectrum to be calculated directly from the input spectrum. These techniques have been used to analyze many nonlinear circuits and are the only candidates for the hierarchical simulation of nonlinear microwave circuits. This paper first uses frequency-domain concepts to discuss nonlinear distortion phenomena, then, a review of the frequency-domain nonlinear analysis literature is made with the aim of presenting the major advances in these techniques.

I. INTRODUCTION

The purpose of this and a companion paper [1] is to present the state-of-the-art of techniques for nonlinear microwave circuit analysis when circuit excitation is sinusoidal or a sum of sinewaves. Most state-of-the-art techniques share in common the partitioning of a nodal matrix description of the circuit into its linear and nonlinear parts and use efficient linear frequency-domain analysis to handle the linear elements. This is the feature that distinguishes practical RF and microwave circuit analyses from transient circuit analysis techniques. This paper reviews methods which solve for the steady-state response of a nonlinear circuit by operating entirely in the frequency-domain. We call these methods *frequency-domain nonlinear analysis techniques*. The companion paper briefly considers time-domain techniques and concentrates on reviewing traditional harmonic balance methods which combine a frequency-domain solution of the linear subcircuit with calculations of the instantaneous voltage and currents in the nonlinear subcircuit. These methods are generally referred to as *hybrid nonlinear analysis techniques* or *harmonic balance techniques*. While somewhat arbitrary, this distinction has been made by a number of authors and is followed here.

Frequency-domain nonlinear circuit analysis methods represent logical developments from fre-

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quency-domain linear circuit analysis. Initially, frequency-domain methods were restricted to weakly nonlinear systems but today can be used for strongly nonlinear systems with large signal excitations. They can be used to simulate situations in which a small signal is more than 400 dB below a large signal [1], and can be used with multi-tone excitations (e.g., 28 incommensurable tones [2]). By comparison, traditional harmonic balance techniques [1] have dynamic ranges of 200 dB or less which degrade as tones become close in frequency. Because of computation and memory requirements, they are practically limited to 3 tones. However, active device modeling is more cumbersome with frequency-domain nonlinear analysis techniques than with hybrid techniques.

Frequency-domain nonlinear circuit analysis techniques have been in development for more than half a century, and have applied to circuit analysis and to behavioral modeling of nonlinear analog circuits. The common underlying principle of frequency-domain nonlinear analysis techniques is that the spectrum of the output of a broad class of nonlinear elements, circuits, and systems can be calculated directly given the spectrum input to the nonlinear system. Some techniques determine an output frequency component by summing calculations of individual intermodulation products. For example, the product of two tones is, in the time-domain, the product of two sinusoids. The trigonometric expansion of this yields two intermodulation products which have frequencies which are the sum and difference, respectively, of the frequencies of the tones. Power series techniques use trigonometric identities to expand the power series and calculate each intermodulation product individually. Algorithms sum these by frequency to yield the output spectrum. At the coarse end of the scale are Volterra series-based techniques that evaluate groups of intermodulation products at a single frequency. Some frequency-domain nonlinear analysis techniques are noniterative, although these are restricted to unilateral systems. Others, known as frequency-domain spectral balance techniques, are iterative being the frequency-domain equivalent of the harmonic balance techniques. Intermediate between these extremes are techniques which operate by converting a nonlinear element into a linear element shunted by a number of controlled current sources. This process is iterative and, at each iteration, a residual nonlinear element is left which reduces from one iteration to another.

In this paper we first review a power series expansion analysis technique, the generalized power series analysis (GPSA), which has been previously reported [3–5]. Using terms defined in GPSA some common nonlinear distortion phenomena are discussed. Following these, both noniterative and iterative nonlinear circuit analysis techniques (Volterra series analysis and frequency-domain spectral balance, respectively) and some special techniques are reviewed. The review that follows does not pretend to cover all possible frequency-domain nonlinear analysis techniques. The aim, however, is to survey the 'permutations' referred to above. One of the spectral balance techniques reviewed is the arithmetic operator method. This is compared to two of the traditional harmonic balance methods in the companion paper [1].

II. POWER SERIES EXPANSION ANALYSIS

The roots of frequency-domain nonlinear analysis techniques are contained in Volterra's book: *Theory of Functionals and of Integral and Integro-Differential Equations* [6] published in 1930. Following the chronological order by jumping into Volterra series analysis is too great a step to make. We begin our discussion with power series analysis of nonlinear systems. This is used to introduce the concept of intermodulation products and to classify nonlinear phenomena in the frequency-domain.

Power series expansion analysis of a nonlinear subsystem is straightforward and a convenient way to introduce and classify nonlinear phenomena in sinusoidally excited nonlinear analog circuits. Nonlinear phenomena in the time-domain manifests as saturation or as a nonlinear relation between an input quantity and an output quantity. When a single frequency sinusoidal signal excites a nonlinear circuit the response "usually" includes the original signal and harmonics of the input sinewave. We say usually because if the circuit contains nonlinear reactive elements, subharmonics and autonomous oscillations could also be present. The output may not even be almost periodic[1] if there is chaos. The process is even more complicated when the excitation includes more than one sinusoid, as the circuit response may include

[1] By a signal being almost periodic we mean that a signal can be represented by a finite number of incommensurable tones and their sum and difference components.

all sum and difference frequencies of the original signals.

In the following, we present the fundamental concept behind all frequency-domain nonlinear analysis techniques: the calculation of intermodulation products. The term 'intermodulation' is used to describe the process by which power at one frequency, or group of frequencies, is transferred to power at other frequencies. The term is also used to describe the production of sum and difference frequency components, or intermodulation frequencies, in the output of a system with multiple input sinewaves. This is a macroscopic view of intermodulation as the generation of each intermodulation frequency component derives from many separate intermodulation processes. Here, we develop a treatment of intermodulation at the discrete intermodulation process level.

Consider a unilateral nonlinear system described by a power series

$$y = \sum_{l=1}^{\infty} a_l x^l \tag{1}$$

where y is the system output, and x is the input and is the sum of three sinusoids

$$x = c_1 \cos(\omega_1 t + \phi_1) + c_2 \cos(\omega_2 t + \phi_2) + c_3 \cos(\omega_3 t + \phi_3). \tag{2}$$

To simplify things, let $\alpha_1 = \omega_1 t + \phi_1$, $\alpha_2 = \omega_2 t + \phi_2$, and $\alpha_3 = \omega_3 t + \phi_3$ so that

$$x^l = [c_1 \cos(\alpha_1) + c_2 \cos(\alpha_2) + c_3 \cos(\alpha_3)]^l \tag{3}$$

$$= \sum_{p=0}^{l} \sum_{k=0}^{p} \binom{p}{k}\binom{l}{p} \times c_1^k c_2^{p-k} c_3^{l-p} (\cos \alpha_1)^k \times (\cos \alpha_2)^{p-k} (\cos \alpha_3)^{l-p}. \tag{4}$$

Eq. (4) includes a large number of components, the radian frequencies of which are the sum and differences of ω_1, ω_2, and ω_3.

Frequency-Domain Expansions

The expansion in eq. (4) is in the time-domain, but the frequency-domain forms have been pursued by a number of authors. This approach is quite old and has been investigated as the basis for hand calculations as well as for computer-based simulations. One of the early contributors was Wass [7] who, in 1948, developed a procedure for calculating the individual intermodulation products, but these results were not amenable to efficient computer implementation. An approach more suitable for computer calculation was independently developed by Engel et al. in 1967 [8], by Sea in 1968 [9], and by Mednikov in 1969 [10]. An improved calculation strategy was subsequently developed by Sea and Vacroux in 1969 [11]. In 1973, Heiter proposed using a modified power series having order dependent time delays to represent the nonlinearities [12]. This series is of the form

$$y(t) = a_0 + a_1 x(t - \tau_1) + a_2 x^2(t - \tau_2) + a_3 x^3(t - \tau_3) + \cdots \tag{5}$$

and is useful for modeling phase nonlinearities and distortion due to reactive elements [12–15]. Steer and Khan continued these developments in 1983 with the addition of complex coefficients to the power series resulting in the form [3]

$$y(t) = A \sum_{l=0}^{\infty} \left[a_l \left\{ \sum_{k=1}^{N} b_k x_k(t - \tau_{k,l} \right\}^l \right] \tag{6}$$

(called a generalized power series), where $y(t)$ is the output of the system; l is the order of the power series terms; a_l is a complex coefficient; $\tau_{k,l}$ is a time delay that depends on both power series order l and the index of the input frequency component k; and b_k is a real coefficient. This is certainly more complicated than the conventional power series form but allows a large class of nonlinear systems to be modeled. For simulation at the circuit level the conventional form of the power series is adequate to describe individual nonlinear elements, i.e.,

$$y(t) = \sum_{l=0}^{\infty} \left[a_l \left\{ \sum_{k=1}^{N} x_k(t) \right\}^l \right]. \tag{7}$$

The result of the power series expansion work is an algebraic formula for the output components when the input is a sum of sinusoids. With an N component multifrequency input

$$x(t) = \sum_{k=1}^{N} x_k(t) = \sum_{k=1}^{N} |X_k| \cos(\omega_k t + \phi_k) \tag{8}$$

where X_k is the phasor[2] of x_k and using the multinomial expansion, the power series of eq. (7) can be expanded and terms collected according to frequency. As a result, the phasor component of the output, Y_q, with radian frequency ω_q, can be expressed as a sum of intermodulation products (IPs) [3]

$$Y_q = \sum_{n=0}^{\infty} \underbrace{\sum_{n_1,\ldots,n_N}}_{|n_1|+\cdots+|n_N|=n} U_q(n_1, \ldots, n_N) \quad (9)$$

where $\omega_q = \sum_{k=1}^{N} n_k\omega_k$, a set of n_k's defines an intermodulation product (called an IPD for intermodulation product description), and n is the order of intermodulation. The second summation is over all possible combinations of $n_1, \ldots, n_N$ such that $|n_1| + \cdots + |n_N| = n$. The summations are, therefore, over the infinite number of IPs (the U_q's) yielding the qth output component (Y_q). When a nonlinear circuit is excited by a finite number of sinusoids, a possibly infinite number of frequency components may be present. In order to analyze such a problem numerically, the number of frequency components considered in the analysis must be truncated. Here we consider N frequency components. Then each IP in eq. (9) is given by [3]

$$U_q(n_1, \ldots, n_N) = \epsilon_n \mathrm{Re}\,\{(X_k^{\dagger})^{|n_k|}\}_{\omega_q}\, T(n_1, \ldots, n_N) \quad (10)$$

where

$$X_k^{\dagger} = \begin{cases} X_k & n_k \geq 0 \\ X_k^{*} & n_k < 0 \end{cases}, \quad (11)$$

$$\epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases}, \quad (12)$$

$$T(n_1, \ldots, n_N) = \sum_{\alpha=0}^{\infty} \underbrace{\sum_{S_1,\ldots,S_N}}_{S_1+\cdots+S_N=\alpha} \times \left\{\left(\frac{(n+2\alpha)!}{2^{(n+2\alpha)}}\right) a_{n+2_\alpha}\, \Phi\right\} \quad (13)$$

and

$$\Phi = \prod_{k=1}^{N} \frac{(X_k^{\dagger})^{|n_k|}\,|X_k|^{2S_k}}{S_k!(|n_k| + S_k)!} \quad (14)$$

and $\mathrm{Re}\{\,\}_{\omega_q}$ is defined such that, for $\omega_q \neq 0$, it is ignored and, for $\omega_q = 0$, the real part of the expression in braces is taken. S_k is a summation index for the kth frequency component and the summation in the S_k's is over all non-negative S_k meeting the condition $S_1 + \cdots + S_N = \alpha$. The sole purpose of the $\mathrm{Re}\{\,\}_{\omega_q}$ operator is to reduce the amount of calculation for the DC component of the output by half. The formula expressed by eqs. (9)–(14) essentially turns a time-domain description (the power series) into a frequency-domain description. Power series expansion has the advantage of retaining the time-domain description of the nonlinearities but requiring no explicit time-domain calculations in order to find the frequency-domain representation for the output. The power series expansion leads to considerable intuitive understanding of the intermodulation process and eqs. (10)–(14) can be rewritten to highlight the contributions to an intermodulation product. Thus,

$$U_q(n_1, \ldots, n_N) = K(n_1, \ldots, n_N) \times [1 + T'(n_1, \ldots, n_N)] \quad (15)$$

where

$$K(n_1, \ldots, n_N) = \epsilon_n a_n \frac{n!}{2^n} \times \mathrm{Re}\left\{\prod_{k=1}^{N} \frac{(X_k^{\dagger})^{|n_k|}}{|n_k|!}\right\}_{\omega_q} \quad (16)$$

is an intermodulation term, and

$$T'(n_1, \ldots, n_N) = \sum_{\alpha=1}^{\infty} \underbrace{\sum_{S_1,\ldots,S_N}}_{S_1+\cdots+S_N=\alpha} \left\{\left(\frac{(n+2\alpha)!}{n!2^{2\alpha}}\right) \frac{a_{n+2\alpha}}{a_n}\, \Phi'\right\} \quad (17)$$

where

$$\Phi' = \prod_{k=1}^{N} \frac{|X_k|^{2S_k}|n_k|!}{S_k!(|n_k| + S_k)!} \quad (18)$$

T' is a saturation term and is zero for small signals and, depending on the power series coefficients,

[2] Capitalized variables indicate phasor quantities having both magnitude and phase.

can either become increasingly positive or increasingly negative as the signal levels become larger. This corresponds to enhancement and compression, respectively, of an intermodulation product.

Nonlinear System Response to Multi-Frequency Sinewave Excitation

If the excitation of an analog circuit is sinusoidal, then the specifications of circuit performance are generally in terms of frequency-domain phenomena, e.g., intermod levels, gain, and the 1 dB gain compression point. In the time-domain, the nonlinear behavior is evident as saturation or clipping of a waveform so that a sinusoidal waveform is distorted to a perturbed periodic waveform. However, with multi-frequency excitation by signals that are not harmonically related, the waveforms in the circuit are not periodic. Here we first look at the nonlinear response of a circuit to multi-frequency excitation and then classify the nonlinear phenomena.

Consider the response y of a nonlinear system to the two-tone excitation x shown in Figure 1(a). The frequencies f_1 and f_2 are, in general, non-harmonically related and components at all sum and difference frequencies ($mf_1 + nf_2$, $m, n = -\infty, \ldots, -1, 0, 1, \ldots, \infty$) of f_1 and f_2 will appear at the output of the system. If the nonlinearity is second order, so that the maximum value of l in eqs. (1) and (7) is 2, the spectrum of the output of the system is that of Figure 1(b). With a general nonlinearity, so that k can be large, the spectrum of the output will contain a very large number of components. An approximate output spectrum is given in Figure 1(c). The relationship of the various components to the incommensurable components is given in Table I. Also shown is a truncated spectrum which will be used in the following discussion. Most of the frequency components in the truncated spectrum of Figure 1(c) have been named: dc results from rectification; f_3, f_4, f_5, f_8, f_9, f_{10}, and f_{11} are called intermodulation frequency components; f_4, f_5 are commonly called image frequencies as well; f_1, f_2 are the input frequencies; and f_6, f_7 are harmonics.

All of the frequencies in the output of the nonlinear system result from intermodulation. In other words, each frequency component is the summation of intermodulation products. Intermodulation is the process of frequency mixing. However, it is usual to refer those frequencies additional to the input and output frequencies, and their harmonics, as intermodulation frequencies. It is unfortunate that the term intermodulation is used in two related but slightly different contexts. The terms 'intermodulation frequency'

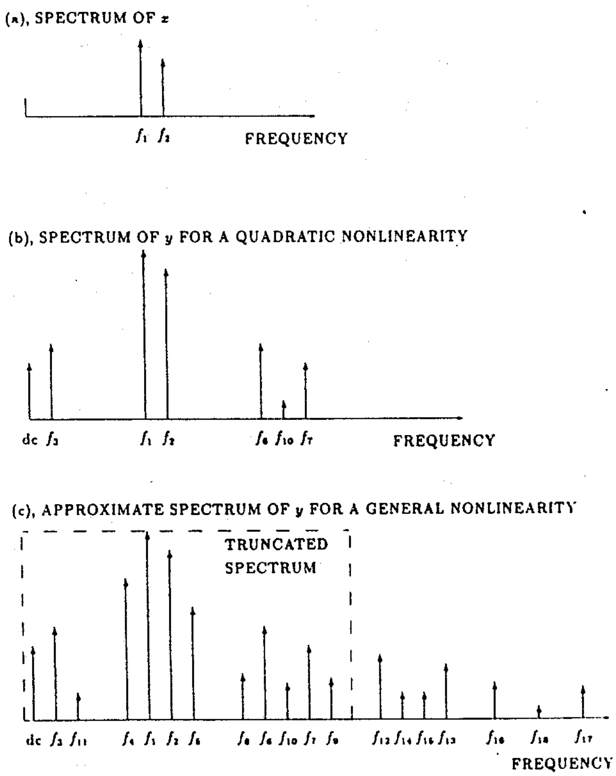

Figure 1. Spectra in a nonlinear system.

TABLE I. Frequency Component Descriptions

f_1	incommensurable tone
f_2	incommensurable tone
f_3	$= f_2 - f_1$
f_4	$= 2f_1 - f_2$
f_5	$= 2f_2 - f_1$
f_6	$= 2f_1$
f_7	$= 2f_2$
f_8	$= 3f_1 - f_2$
f_9	$= 3f_2 - f_1$
f_{10}	$= f_1 + f_2$
f_{11}	$= 2f_2 - 2f_1$
f_{12}	$= 3f_1$
f_{13}	$= 3f_2$
f_{14}	$= 2f_1 + f_2$
f_{15}	$= 2f_2 + f_1$
f_{16}	$= 4f_1$
f_{17}	$= 4f_2$
f_{18}	$= 2f_1 + 2f_2$

or 'intermodulation component' refer to the undesired frequencies generated in mixing process as described here. The term 'intermodulation product' (IP) refers to the entire nonlinear process of sinusoids (or even a single frequency component) mixing to produce components at any frequency.

Gain Compression/Enhancement

Gain compression can be conveniently described in the time-domain or in the frequency-domain. Time-domain descriptions refer to limited power availability or to limitations on voltage or current swings. At low signal levels moderately nonlinear devices such as class A amplifiers behave linearly so that there is one dominant IP with a zero saturation term. As signal levels increase other IPs become important as harmonic levels increase. Depending on the harmonic loading condition, these IPs could be in phase with the original IP contributing to gain enhancement or out of phase contributing to gain compression. As well, the magnitude of the saturation term can be larger. Which effect—saturation or additional IPs—dominates depends on the particular design although the two effects can be used to balance each other and so extend the power at which departures from linearity are significant. In the truncated spectrum, only the fundamental and the second harmonic are present so there are few IPs present in a list of IPs contributing to gain saturation/enhancement (see Table II). The first order IP corresponds to the linearized output. The second order IP is due to the mixing of the fundamental with the second harmonic to produce a component at the fundamental. In class A MESFET amplifiers, gain compression is principally due to growth in the saturation term [e.g., (17)] resulting in the classic saturation characteristic shown as curve (a) in Figure 2.

TABLE II. Intermodulation Products Contributing to Compression/Enhancement

Frequency considered:		f_1
Incommensurable frequencies: Involved in phenomenon		f_1
Intermodulation products:		
order	*output*	*input*
First	f_1	$= f_1$
Second	f_1	$= -f_1 + f_6$
Third	*none*	
Fourth	f_1	$= 3f_1 - f_6$
Fifth	f_1	$= -3f_1 + 2f_6$

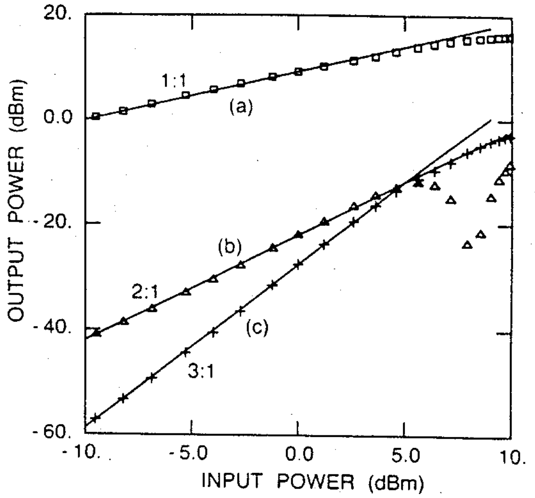

Figure 2. Output power levels of a class A MESFET amplifier versus input power for (a) the fundamental, (b) the second harmonic, and (c) the third harmonic.

Desensitization

Desensitization is the variation of the amplitude of one of the desired components due to the presence of another incommensurable signal. This phenomenon is depicted in Figure 3 where the amplitude of the signal at f_1 is affected by the amplitude of the signal at f_2. Desensitization is the result of growth in the saturation term T' [e.g., (17)] and the generation of IPs is not involved. When signals are small, T' is zero but becomes larger as the signal levels increase. Thus, if there are two incommensurable signals, one small and the other large, at the input of an amplifier, the gain of the small signal can be affected by just the presence of the large signal. It has been pointed out that internal amplifier noise can also affect the gain of a signal [16]. This is also the result of desensitization.

Harmonic Generation

Harmonic generation is the most obvious result of nonlinear distortion. Simply squaring a sinusoidal signal will give rise to a second harmonic

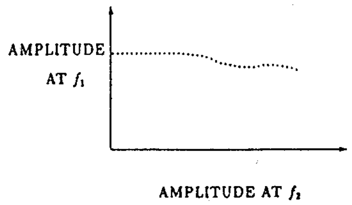

Figure 3. Effect of desensensitization.

component. In the truncated spectrum, only the second harmonic is present so that few IPs are recorded in an IP listing (Table III). The first order IP corresponds to the generation of, for example, current at the second harmonic due to voltage at the second harmonic. This IP, of course, does not give rise to the second harmonic. Second and higher order mixing leads to a large number of IPs. At small signals only the second order IPs are significant leading to the classic slope of 2 characteristic of the second harmonic power as a function of the input fundamental power, curve (b) of Figure 2. As the input power increases higher order IPs become significant. If these are out of phase with the dominant second order IP, then destructive cancellation can occur at higher input powers as can be seen in curve (b) of Figure 2 at an input of approximately 8 dBm.

Intermodulation

Intermodulation is the generation of spurious frequency components at the sum and difference frequencies of the input frequencies. In the truncated spectrum f_3, f_4, f_5, f_8, f_9, f_{10}, and f_{11} are intermodulation frequencies. Focusing on just one of these, f_4, we see from Table IV that there are a large number of low order IPs contributing to this effect. For the reasons given above, the first order response is usually large, but does not describe the origins of this phenomenon. The lowest order mixing that gives rise to intermodulation is a second order process and we see that, for the truncated spectrum, there are four second order IPs. These can destructively and constructively interfere so that the power of an intermodulation frequency component can vary erratically at high input powers.

Cross-Modulation

Cross-modulation is the modulation of one component by another incommensurable component. Here, it would be modulation of f_1 by f_2 or modulation of f_2 by f_1. The intermodulation products contributing to this phenomenon are listed in Table V up to third order. For discrete tone excitation, cross-modulation can not be easily distinguished from desensitization. However, with cross-modulation, information contained in the sidebands of one incommensurable tone can be transferred to the other incommensurable tone. This transfer of information does not occur with desensitization.

Detuning

Detuning is the generation of dc charge or dc current resulting in change of an active device's operating point. The generation of DC current with a large signal is commonly referred to as rectification and its effect can often be reduced by biasing using voltage and current sources. However, DC charge generation in nonlinear reactances is more troublesome as it can neither be detected nor effectively reduced. A list of the detuning IPs is given in Table VI. The zero order IP corre-

TABLE III. Intermodulation Products Contributing to Harmonic Generation

Frequency considered:		f_1
Incommensurable frequencies: Involved in phenomenon		f_1
Intermodulation products:		
order	*output*	*input*
First	f_6	$= f_6$
Second	f_6	$= 2f_1$
Third	f_6	$= -f_1 + f_6$
Fourth	f_6	$= -2f_1 + 2f_6$

TABLE IV. Intermodulation Products Contributing to Intermodulation

Frequency considered:		f_4
Incommensurable frequencies: Involved in phenomenon		f_1, f_2
Intermodulation products:		
order	*output*	*input*
First	f_4	$= f_4$
Second	f_4	$= f_1 - f_3$
Second	f_4	$= -f_1 + f_8$
Second	f_4	$= -f_2 + f_6$
Second	f_4	$= -f_5 + f_{10}$
Third	f_4	$= 2f_1 - f_2$
Third	f_4	$= f_1 + f_2 - f_5$
Third	f_4	$= -f_1 - f_3 + f_6$
Third	f_4	$= f_1 - f_6 + f_8$
Third	f_4	$= f_1 + f_6 - f_{10}$
Third	f_4	$= f_1 + f_7 - f_9$
Third	f_4	$= f_1 - f_7 + f_{10}$
Third	f_4	$= f_2 - 2f_3$
Third	f_4	$= -f_2 + f_3 + f_8$
Third	f_4	$= -f_2 - f_3 + f_{10}$
Third	f_4	$= f_2 + f_6 - f_7$
Third	f_4	$= f_2 + f_8 - f_{10}$
Third	f_4	$= f_2 - f_9 + f_{10}$
Third	f_4	$= -f_3 - f_4 + f_8$
Third	f_4	$= f_3 - f_5 + f_6$
Third	f_4	$= -f_3 - f_5 + f_7$
Third	f_4	$= f_5 + f_6 - f_9$
Third	f_4	$= f_5 - f_7 + f_8$

TABLE V. Intermodulation Products Contributing to Cross-Modulation

Frequency considered: f_1
Incommensurable frequencies: Involved in phenomenon f_1, f_2
Intermodulation products:

order	output	input
Second	f_1	$= f_2 - f_3$
Second	f_1	$= -f_2 + f_{10}$
Second	f_1	$= f_3 + f_4$
Second	f_1	$= -f_4 + f_8$
Second	f_1	$= -f_5 + f_7$
Third	f_1	$= -f_1 + f_2 + f_4$
Third	f_1	$= -f_1 + f_3 + f_8$
Third	f_1	$= -f_1 - f_3 + f_{10}$
Third	f_1	$= 2f_2 - f_5$
Third	f_1	$= -f_2 + f_3 + f_6$
Third	f_1	$= -f_2 - f_3 + f_7$
Third	f_1	$= -f_2 + f_4 + f_5$
Third	f_1	$= f_2 - f_6 + f_8$
Third	f_1	$= f_2 + f_6 - f_{10}$
Third	f_1	$= f_2 + f_7 - f_9$
Third	f_1	$= f_2 - f_7 + f_{10}$
Third	f_1	$= -2f_3 + f_5$
Third	f_1	$= -f_3 - f_4 + f_6$
Third	f_1	$= -f_3 - f_5 + f_9$
Third	f_1	$= f_3 - f_5 + f_{10}$
Third	f_1	$= f_4 + f_6 - f_8$
Third	f_1	$= f_4 - f_6 + f_{10}$
Third	f_1	$= f_4 - f_7 + f_9$
Third	f_1	$= f_4 + f_7 - f_{10}$
Third	f_1	$= f_5 + f_6 - f_7$
Third	f_1	$= f_5 + f_8 - f_{10}$
Third	f_1	$= f_5 - f_9 + f_{10}$

sponds to the conventional view of rectification and, from eqs. (10)–(14), this IP is

$$U_q(0, \ldots, 0) = a_0 + \sum_{\alpha=1}^{\infty} \underbrace{\sum_{S_1,\ldots,S_N}}_{S_1+\cdots+S_N=\alpha} \times \left\{ \left(\frac{(2\alpha)!}{2^{(2\alpha)}} \right) a_{2\alpha} \prod_{k=1}^{N} \left(\frac{|X_k|^{S_k}}{S_k!} \right)^2 \right\}. \tag{19}$$

For very small signals, $U_q(0, \ldots, 0)$ is just the DC offset a_0. As the input signal components, the X_k's become larger and $U_q(0, \ldots, 0)$ will grow as a result of rectification.

AM–PM Conversion

The conversion of amplitude modulation to phase modulation (AM–PM conversion) is a troublesome nonlinear phenomenon in microwave circuits and results from the amplitude of a signal affecting the delay through the system. The most convenient way of modeling this effect at the subsystem level is to introduce order dependent time delays in the power series description of the transfer function as proposed by Heiter [12]. With these, the formula for an intermodulation component of the output contains a phase shifting factor. As a result, the various IPs at a particular frequency can be out of phase. Since the magnitude of each IP is signal level dependent, the frequency component, which is the sum of the IPs, will also have a level-dependent phase shift. No specific IPs can be assigned to this affect but frequency-domain power series expansion analysis leads to expressions describing the AM–PM conversion [12].

Sub-Harmonic Generation and Chaos

In systems with memory effects, i.e., with reactive elements, sub-harmonic generation is possible. The intermodulation products for sub-harmonics can not be expressed in terms of the input incommensurable components. Sub-harmonics are ini-

TABLE VI. Intermodulation Products Contributing to Detuning

Frequency considered: dc
Incommensurable frequencies: Involved in phenomenon f_1, f_2
Intermodulation products:

order	output	input
Zero	dc	$= 0f_1 + 0f_2 + 0f_3 + 0f_4 + 0f_5 + 0f_6 + 0f_7 + 0f_8 + 0f_9 + 0f_{10}$
First	*none*	
Second	*none*	
Third	dc	$= -2f_1 + f_6$
Third	dc	$= f_1 - f_2 + f_3$
Third	dc	$= -f_1 - f_2 + f_{10}$
Third	dc	$= -f_1 + f_3 + f_4$
Third	dc	$= -f_1 - f_4 + f_8$
Third	dc	$= -f_1 - f_5 + f_7$
Third	dc	$= -2f_2 + f_7$
Third	dc	$= -f_2 - f_3 + f_5$
Third	dc	$= -f_2 - f_4 + f_6$
Third	dc	$= -f_2 - f_5 + f_9$
Third	dc	$= f_3 - f_6 + f_8$
Third	dc	$= -f_3 - f_6 + f_{10}$
Third	dc	$= -f_3 - f_7 + f_9$
Third	dc	$= f_3 - f_7 + f_{10}$
Third	dc	$= -f_4 - f_5 + f_{10}$

tiated by noise, possibly a turn-on transient, and so in a simulation it must be explicitly incorporated into the assumed set of steady-state frequency components. The lowest common denominators of the sub-harmonic frequencies then become the basis incommensurable component.

Chaotic behavior can only be simulated in the time-domain. The nonlinear frequency-domain methods as well as the conventional harmonic balance methods simplify a nonlinear problem by imposing an assumed steady-state solution on the nonlinear circuit problem. Chaotic behavior is not periodic and so the simplification is not valid. Together with the ability to simulate transient behavior, the capability to simulate chaotic behavior is the unrivaled realm of time-domain methods.

III. VOLTERRA SERIES ANALYSIS

The oldest and most analytically developed frequency-domain methods are the methods based on Volterra series developed around 1910 by Volterra [6]. Volterra has shown that every continuous functional $G(x)$ can be represented by the expansion

$$G(x) = \sum_{n=1}^{\infty} F_n(x) \tag{20}$$

where $F_n[x(t), b \leq t \leq a]$ is a regular homogeneous functional such that

$$F_n(x) = \int_a^b \cdots \int_a^b g_n(t; \xi_1, \ldots, \xi_n)x(\xi_1)x(\xi_2) \cdots x(\xi_n)d\xi_1 d\xi_2 \cdots d\xi_n \tag{21}$$

and the function g_n is the nth order characteristic or kernel of the functional.

In 1942, Wiener applied this type of functional series to the analysis of nonlinear systems [17]. He suggested that a weak nonlinearity could be represented with just the first few terms of such a series. His ideas have subsequently been developed by many researchers including significant contributions by Bedrosian and Rice [18] and Bussgang, Ehrman, and Graham [19]. This review is based largely on those works as well as the more recent book by Weiner and Spina [20].

Given a time-domain input–output relation $y(t) = f(x(t))$, the output $y(t)$ can be expressed as a Volterra functional series of the input $x(t)$. Thus,

$$y(t) = \sum_{n=1}^{\infty} y_n(t) \tag{22}$$

where

$$y_n(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n)x(t - \tau_1) \cdots x(t - \tau_n)d\tau_1 \cdots d\tau_n \tag{23}$$

and the function $h_n(\tau_1, \ldots, \tau_n)$ is known as the nth order Volterra kernel. It can be used as a time-domain description of many nonlinear systems including nonlinear microwave circuits that do not exhibit hysteresis. In this case, the nth order kernel, h_n, is called the nonlinear impulse response of the circuit of order n. The first order response is just the linearized response of the system and the zero order response is a dc offset. The zero and first order responses describe the system entirely when the input to the nonlinear system is negligibly small. This is a very general procedure that is applicable to an arbitrary input.

Rarely do we need to deal directly with the Volterra series which is in the time-domain. Mostly we are concerned with the frequency-domain derivative form which is expressed in terms of Volterra nonlinear transfer functions. These are obtained by taking the n-fold Fourier transform of h_n

$$H_n(f_1, f_2, \ldots, f_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \ldots, \tau_n) \times \exp[-j2\pi(f_1\tau_1 + \cdots + f_n\tau_n)]d\tau_1 d\tau_2 \cdots d\tau_n \tag{24}$$

where H_n is called the nonlinear transfer function of order n. For the nth order frequency-domain output Y_n, we have

$$Y_n(f_1, \ldots, f_n) = H_n(f_1, \ldots, f_n) \times X(f_1) \cdots X(f_n). \tag{25}$$

Thus, in the frequency domain, the Volterra series expansion has the form

$$Y(f_1, f_2, \ldots) = H_n(f_1)X(f_1) \tag{26}$$

$$+ H_2(f_1, f_2)X(f_1)X(f_2) + H_3(f_1, f_2, f_3)X(f_1) \times X(f_2)X(f_3) + \cdots. \tag{27}$$

Volterra series methods were first applied to transistor circuit simulation in 1967 [21] and since then have been successfully applied to the characterization and analysis of many nonlinear circuits [18–20,22–40] were the Volterra nonlinear transfer functions can be derived algebraically [18, 24,26], experimentally [32,41,42], or numerically [40]. The algebraic determination of the nonlinear transfer functions from the element constitutive relations can be cumbersome, and generally determination of H_n for $n > 3$ is impractical. Thus, only weak nonlinearities are usually considered. Experimental characterization is noise limited so that system characterization is again generally restricted to third order.

Nonlinear subsystems can be described by Volterra nonlinear transfer functions, provided that they are mildly nonlinear, resulting in a behavioral model that is noniterative and efficient to compute [28]. Volterra nonlinear transfer functions can be used with frequency conversion circuits where the large signal local oscillator establishes a time-varying mildly nonlinear system converting energy from the input signal to energy in the output signal [43]. They have also been used as describing functions of nonlinear elements in the simulation of many microwave circuits including oscillators [30,37–39].

Most of the other Volterra techniques applied to microwave circuits apply to subsystem modeling and use feedback to describe nonunilateral subcircuits. The approach seems to be particularly amenable to the simulation of microwave oscillators [37–39], as feedback is explicitly introduced in the circuit to facilitate oscillation. These are fairly specific circuits so that the effort required to partition the circuit into subcircuits and to develop the Volterra nonlinear transfer functions is worthwhile. However, this approach is not amenable to general-purpose circuit simulation.

The application of the method of nonlinear currents reported by Crosmun and Maas [40] is a promising technique for the simulation of nonlinear microwave circuits with applicability to large signal circuits. This technique is based on Volterra theory and enables the direct calculation of the response of a circuit with nonlinear elements that are described by a power series. In the method of nonlinear currents, a circuit is first solved for its linearized response described by zero and first order Volterra nonlinear transfer functions. Considering only the linearized response allows standard linear circuit nodal admittance matrix techniques to be used. The second order response, described by the second order Volterra nonlinear transfer functions can then be represented by controlled current sources. Thus, the second order sources are used as excitations again enabling linear nodal admittance techniques to be used. The process is repeated for the third- and higher-order node voltages and is easily automated in a general purpose microwave simulator. The process is terminated at some specific order of the Volterra nonlinear transfer functions. This is a noniterative technique but relies on rapid convergence of the Volterra series, restricting its use to moderatly strong nonlinear circuits. In particular, diodes are strong nonlinearities and require up to a *40*th order power series [44] for an adequate description with a large voltage waveform across the diode. Consequently, because of the equivalence between the order of the power series description and the maximum order of the Volterra nonlinear transfer functions [45], 40th order Volterra nonlinear transfer functions are required. In a similar approach a Volterra series technique has been applied to circuit simulation by Dmitriev and Silyutin [46]. The example they present uses fifth order Volterra nonlinear transfer functions but their report requires manual algebraic derivations.

In many microwave situations, such as mixers, a small signal interacts with a large signal. Derivation of the large signal waveform permits a weakly nonlinear time-varying Volterra series description of the circuit [24,34]. This is an extension of the linear conversion matrix concept [47] to include nonlinear dependencies on the level of the small signal but special conditions on the LO waveform, such as being sinusoidal current, are typically required.

The major advantage of Volterra-series based analysis is that nonlinear phenomena in large weakly nonlinear systems can be accurately described using a well-defined and methodical process. Development of this capability was the major focus of the work done in the late 1960s and early 1970s. For smaller circuits, closed form expressions for the nonlinear effects can be developed and used to optimize designs. To date, Volterra nonlinear transfer function analysis represents the only mechanism for predicting the nonlinear effects of high level noise in a weakly nonlinear system [17,48,49].

The most advanced Volterra approach, as far as microwave circuit analysis is concerned, is that of Van den Eijnde and Schoukens [50] who couple the Volterra treatment introduced by Chua and Ng [51] with the spectral balance technique. In this work, strongly nonlinear systems can be considered as there is no limitation on the order of the Volterra nonlinear transfer functions and they claim that it is possible to consider orders of 100 or more.

Very similar to the Volterra nonlinear transfer function techniques are the describing function methods [52]. By restricting the generality of the Volterra nonlinear transfer functions many circuits are more amenable to analysis. Gustafson et al. use describing function methods in the study of many microwave circuits including free-running oscillators, phased locked loops, and amplifiers.

Volterra series analysis of nonlinear microwave circuits is invariably restricted to elements with univariate dependencies. However, there are many instances where this is not a good approximation. For example, the transconductance of a GaAs MESFET can not be adequately represented by univariate elements which would require that the transconductance be solely a function of the gate-source voltage. Before Volterra-based techniques can be used in a general purpose microwave circuit simulator, the restriction to univariate nonlinear elements must be overcome. This situation requires bivariate nonlinear transfer functions [24 (pp. 131–137),53], but the algebraic and experimental complexity of determining the high order bivariate transfer functions has practically eliminated their use.

IV. ALGEBRAIC FUNCTIONAL EXPANSION

In contrast to the approach used in Volterra series analysis, the algebraic functional expansion methods use an algebraic approach to nonlinear functional expansion based on the noncommutative power series developed by Lamnabhi et al. [54–57]. This method is related to Volterra series, but the analysis procedure is quite different and relies on symbolic computation.

To analyze nonlinear circuits using this approach, the nonlinear differential equations describing the circuit are written using a specialized symbolic notation. The solution can then be found using a digital computer and a symbolic mathematics package.

The advantage of this technique is that it can be applied to the same sort of problems Volterra series techniques can analyze, but a higher order approximation can be used since all the algebraic manipulations are performed on the computer. It has the disadvantage of requiring software to manipulate symbolic equations. This results in an approach that is difficult to integrate with existing computer-aided design tools. However, being noniterative, these techniques are attractive for the behavioral modeling of nonlinear microwave subsystems.

V. FREQUENCY-DOMAIN SPECTRAL BALANCE

Frequency-domain spectral balance techniques are similar to the harmonic balance methods discussed in the companion paper [1]. The term spectral balance is used to distinguish the frequency-domain techniques from the harmonic balance techniques as the latter term has come to be solely applied to mixed time- and frequency-domain methods. The circuit to be analyzed is arranged as shown in Figure 4 and is divided into linear and nonlinear subcircuits. This explicit separation, however, is not required [58] but considerably simplifies the present discussion. The linear subcircuit has Q nodes; P of which are common to the nonlinear subcircuit, and M of which are connected to independent voltage sources. The nonlinear subcircuit has P nodes and is composed of nonlinear elements. Here we consider N frequency components, so that at the pth node, the instantaneous current into the linear subcircuit is

$$\hat{\imath}_p(t) = \sum_{q=1}^{N} \mathrm{Re}[\hat{I}_p(\omega_q)e^{j\omega_q t}] \tag{28}$$

where $\hat{I}(\omega_q)$ is a phasor current at radian frequency ω_q. Similarly, the current into the nonlinear subcircuit at the pth node is

$$i_p(t) = \sum_{q=1}^{N} \mathrm{Re}[I_p(\omega_q)e^{j\omega_q t}] \tag{29}$$

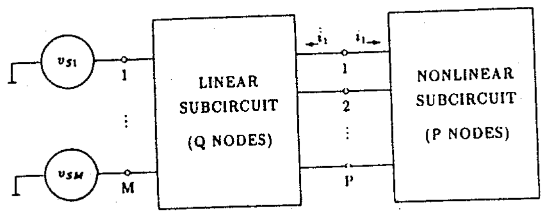

Figure 4. Circuit partitioned into linear and nonlinear subcircuits.

and the voltage at the pth node is

$$v_p(t) = \sum_{q=1}^{N} \mathrm{Re}[V_p(\omega_q)e^{j\omega_q t}] \tag{30}$$

To satisfy Kirchoff's current law,

$$i_p(t) + \hat{i}_p(t) = 0 \tag{31}$$

for all p from 1 to P, i.e., at all nodes of the nonlinear subcircuit. We can rewrite this in the frequency-domain as

$$\mathbf{F}(\mathbf{V}) = \mathbf{I} + \hat{\mathbf{I}} = \mathbf{0} \tag{32}$$

where $\hat{\mathbf{I}}$ and $\mathbf{I}$ are real valued vectors of length $2NP$ consisting of the real and imaginary components of the phasor currents into the linear and nonlinear subcircuits, respectively, at all frequencies. Similarly, $\mathbf{V}$ is the vector of node voltage phasors. The steady-state solution is found by solving the system of nonlinear equations represented by eq. (32), called the 'determining equation' [59]. The preferred solution approach is to use Newton's method to construct the iterative procedure

$$\mathbf{V}^{i+1} = \mathbf{V}^{i} - \mathbf{J}^{-1}(\mathbf{V}^{i})\mathbf{F}(\mathbf{V}^{i}) \tag{33}$$

where the superscripts are iteration numbers. The matrix $\mathbf{J}$ is the Jacobian of $\mathbf{F}$ and requires partial derivatives of the real and imaginary components of the phasor currents with respect to the real and imaginary components of the node voltage phasors for all nodes (1 to P) and all frequency components (1 to N). For the linear subcircuit, this information is contained in the nodal admittance matrix and is readily available for the nonlinear subcircuit if the elements are described by power series [5]. With power series descriptions, obtaining the derivatives of the nonlinear current phasors with respect to each voltage phasor amounts to taking derivatives of (10) with respect to the X_k's. Power series expansion methods are applicable to strongly nonlinear elements but the analysis is practically limited to univariate elements as the analogous bivariate generalized power series expansions require excessive computation [60].

VI. FREQUENCY-DOMAIN SPECTRAL BALANCE USING THE ARITHMETIC OPERATOR METHOD

Recently, an alternative technique called the arithmetic operator method (AOM) has been introduced; first for efficient power series expansion analysis [58,61], and then for any analytical function [62]. This technique utilizes the frequency convolution concept rather than the multinomial expansion technique behind the development of the generalized power series output formula, eq. (10). AOM calculates the total contribution at an output frequency rather than summing up individual intermodulation products.

The most advanced method is that presented by Chang and Steer [62] and will be considered here. The arithmetic operator method manipulates spectra in the frequency-domain evaluation of basic arithmetic operations. Chang and Steer [58,62] introduced the frequency-domain spectrum transform matrix and derived the frequency-domain forms of basic operators—multiplication and division—and their derivatives (as required in Newton's method).

The frequency-domain description of a signal is a set of complex numbers which, for computational necessity, must be truncated. Retaining K frequencies, the time-domain signal x is represented in the frequency-domain by a spectral vector $\mathbf{x}$ defined as

$$\mathbf{x} = [X_{0r}\, X_{1r}\, X_{1i} \ldots X_{kr}\, X_{ki} \ldots X_{Kr}\, X_{Ki}]^T \tag{34}$$

where X_{kr} represents the real part of the frequency component of x at the radian frequency ω_k and X_{ki} is the imaginary part of this component so that the phasor of the kth spectrum component $X_k = X_{kr} + jX_{ki}$. The spectral vectors $\mathbf{y}$ and $\mathbf{z}$ of the time-domain signals y and z are similarly defined. The basic operations $y = x \pm z$ are simply implemented in the frequency-domain as the corresponding elements of $\mathbf{x}$ and $\mathbf{z}$ are added or subtracted. That is

$$\mathbf{y} = \mathbf{x} \pm \mathbf{z}. \tag{35}$$

The derivative forms of these operators are equally straightforward. If y, x, and z are all signal u dependent, then the derivative of $\mathbf{x}$ with respect to the kth component of $\mathbf{u}$ [defined as is $\mathbf{x}$ in eq. (34)] is

$$\dot{\mathbf{x}}_{k,q} = [\partial X_{0r}/\partial U_{k,q}\ \partial X_{1r}/\partial U_{k,q}\ \partial X_{1i}/\partial U_{k,q} \cdots \partial X_{Kr}/\partial U_{k,q}\ \partial X_{Ki}/\partial U_{k,q}]^T, \tag{36}$$

where $q = r$ or i indicates the real or imaginary part. With the spectral vectors $\dot{\mathbf{y}}_{k,q}$ and $\dot{\mathbf{z}}_{k,q}$ similarly defined, the derivative forms of the addition and subtraction arithmetic operators are

$$\dot{\mathbf{y}}_{k,q} = \dot{\mathbf{x}}_{k,q} \pm \dot{\mathbf{z}}_{k,q}. \quad (37)$$

The spectral addition and subtraction, and their derivatives, are straightforward as they involve addition or subtraction of the corresponding elements of the spectra operated on. However, more complicated operators, such as multiplication and division, do not only involve corresponding elements and so the *spectrum mapping function* and *spectrum transform matrix* were introduced. In general, a spectrum will contain DC, fundamental, harmonics, and intermodulation components and so there is a simple arithmetic relationship of the frequencies of the commensurable spectrum components.

Multiplication and Division

In the frequency-domain the multiplication operation $y = xz$ is

$$\mathbf{y} = \mathbf{T}_x\mathbf{z} = \mathbf{T}_z\mathbf{x} \quad (38)$$

where $\mathbf{T}_x$ is called the spectrum transform matrix of $\mathbf{x}$ and is a matrix formulation of the spectrum mapping function—a frequency convolution operation [62]. The spectrum mapping function relates the components of the output spectrum to the spectra of two inputs where the output is the product of the two inputs. Table VII is an example of the spectrum mapping function where $y = xz$ is the system output and x and z are inputs. Three frequency components f_0 ($=DC$), f_1, and f_2

TABLE VII. Example of Spectrum Mapping Function

y	x		z	
k_y	k_x	s_x	k_z	s_z
0	0	+1	0	+1
	1	+1	1	−1
	2	+1	2	−1
1	0	+1	1	+1
	1	+1	0	+1
	1	−1	2	+1
	2	+1	1	−1
2	0	+1	2	+1
	1	+1	1	+1
	2	+1	0	+1

($=2f_1$) are considered here, k_y, k_x, and k_z are frequency indices for the y, x, and z components, and s_x and s_z indicate signs so that $f_{ky} = |s_x f_{kx} + s_z f_{kz}|$.

In the frequency-domain, the multiplication operation has the derivative form

$$\dot{\mathbf{y}}_{k,q} = \mathbf{T}_x\dot{\mathbf{z}}_{k,q} + \mathbf{T}_z\dot{\mathbf{x}}_{k,q}. \quad (39)$$

After interchanging y and z in (38) and (39), the frequency-domain form of the division operation $y = z/x$ is seen to be

$$\mathbf{y} = \mathbf{T}_x^{-1}\mathbf{z} \quad (40)$$

and its derivative form is

$$\dot{\mathbf{y}}_{k,q} = \mathbf{T}_x^{-1}(\dot{\mathbf{z}}_{k,q} - \mathbf{T}_y\dot{\mathbf{x}}_{k,q}) \quad (41)$$

Equations (35), (38), and (40) represent four basic frequency-domain operators: spectral addition, subtraction, multiplication, and division. Theoretically, any analytic function can be evaluated using these. Arithmetic operator methods in conjunction with the spectral balance technique can be used with strongly nonlinear circuits and active device models are not restricted to power series expressions. However, the element expressions must be analytic and algebraic. As spectral operations are considerably more complicated than the corresponding algebraic operations, modeling is restricted to using reasonably straightforward expressions. For example, the Curtice [63] and Materka-Kacprzak [64] models can be used, as the modeling expressions are reasonably simple. However, the physically based model of Khatibzadeh and Trew [65] is not amenable to AOM because of the extensive arithmetic and table look-up involved as well as its not being analytic.

In the companion paper [1], the arithmetic operator method is compared with two conventional harmonic balance techniques. For single-tone excitation, it is comparable to the harmonic balance techniques in terms of memory usage and computation time. However, for two-tone excitation, it is quicker and has much greater dynamic range. The extension of dynamic range is principally because there are no Fourier conversions between the time- and frequency-domains.

Example

For an example of the application of AOM in the spectral balance analysis of microwave circuits, the GaAs MESFET amplifier, which was previously reported by Chang et al. is considered [58]. In the equivalent circuit used to model the MESFET, shown in Figure 5. C_{gs}, C_{gd}, and C_{ds} were taken to be one-dimensional nonlinear elements, and were modeled by power-series descriptions as given in ref. [58]. Values used for linear elements were also given in ref. [58]. However, unlike ref. [58], the drain-source current I_{ds} which is a function of both intrinsic voltages v_{gs} and v_{ds} is modeled by a modified Curtice model from eq. [63] using a hyperbolic function and some v_{ds}-dependent terms:

$$I_{ds} = V_{ps} \tanh(\gamma v_{ds}) + A_6 v_{ds} \qquad (42)$$

where

$$V_{ps} = A + v_{gs}(t - \tau) \times (A_0 + A_1 V_1 + A_2 V_1^2 + A_3 V_1^3) \qquad (43)$$

and

$$V_1 = v_{gs}(t - \tau)[1 + \beta(V_{ds}^0 - v_{ds})]. \qquad (44)$$

All parameter values in eqs. (42), (43), and (44) except τ, which is assumed to be a constant, were determined by fitting the calculated g_m and g_{ds} curves to measured RF data. These determined parameter values are listed in Table VIII.

A. Hyperbolic Function. Eqs. (35), (38), and (40) were used to calculate I_{ds}, g_m, and g_{ds}. For efficient calculations, the hyperbolic tangent function $\tanh(x)$ was expressed as

$$\tanh(x) = (1 - y)/(1 + y) \qquad (45)$$

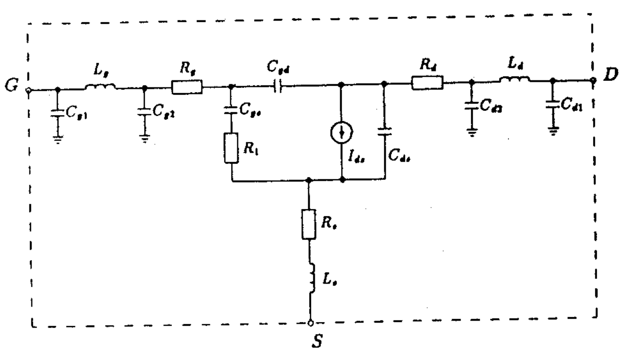

Figure 5. MESFET equivalent circuit.

TABLE VIII. MESFET Parameter Values

Parameter	Value
A_0	0.08494
A_1	0.0558
A_2	−0.01124
A_3	−0.01488
γ	2.4836
β	0.1722
V_{ds}^0	3.0 V
A_6	0.0032
A	0.062

where $y = e^{-2x}$, and the Jacobian matrix of the function $\tanh(x)$ as

$$\mathbf{J}_{\tanh} = \mathbf{T}_z, \quad z = 4y/(1 + y)^2 \qquad (46)$$

where $\mathbf{z}$ is the frequency-domain spectral vector form of z. The exponential e^{-2x} was approximated as

$$e^{-2x} = \left[1 + \frac{u}{2^n} + \frac{1}{2}\left(\frac{u}{2^n}\right)^2 + \cdots + \frac{1}{n!}\left(\frac{u}{2^n}\right)^n\right]^{2^n}, \quad n \le 9 \qquad (47)$$

where

$$u = -2x. \qquad (48)$$

B. Results. A two-tone input excitation using two equal-amplitude signals at 2.35 GHz and 2.4 GHz were used to verify the arithmetic operator method of frequency-domain spectral balance. Figure 6 shows the simulated (curves) and measured (points) values for the power output at 2.35 GHz and the third order intermodulation product at 2.3 GHz as a function of the input power. Good agreement is obtained. With −3 dBm input power and the third order intermodulation (12 ac, 1 dc) considered, the total simulation time using AOM was 2.2 s on a DEC DS3100 (RISC) workstation.

VII. SPECIALIZED TECHNIQUES

The techniques described in the previous sections can be used to analyze arbitrary circuits. In the 1960s, before most of the general techniques were available, specialized techniques were developed for the simulation of diode circuits. Most of these

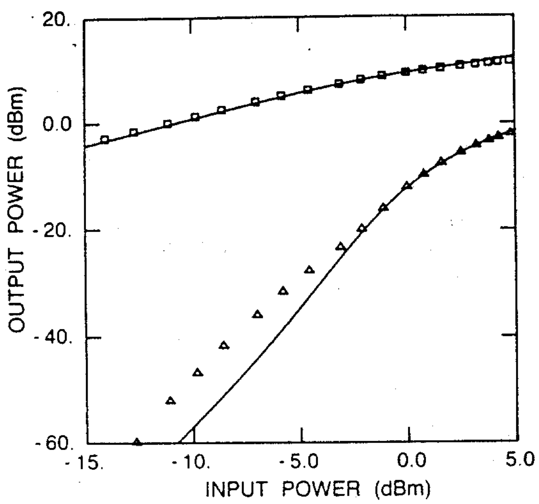

Figure 6. Output power versus input power for two-tone excitation of an amplifier. The tones are of equal power and the input power is the power of one of the tones. The frequencies of the tones are 2.35 GHz and 2.4 GHz, the top curve is the power output at 2.35 GHz and the bottom curve is the power output at 2.3 GHz.

handled a nonlinear resistor only and relied on special expansions of the Shockley diode equation or the reverse bias capacitance equation of a diode. Several workers have developed closed form (noniterative) frequency-domain analyses for simple mixer circuits such as the frequency independent mixer circuit of Figure 7. If the nonlinear capacitance of the diode junction is ignored, and the diode has the ideal current-voltage characteristic given by the Shockley diode equation

$$i = I_S\{\exp(\alpha v) - 1\} \quad (49)$$

where i is the current through the diode and v is the voltage across the junction, the mixer circuit is described by

$$i = I_S\{\exp \alpha[v_S - i(R + R_S)] - 1\}. \quad (50)$$

Rutz-Phillip [66] and Beane [67] used the special recursive polynomials of Mills [68] to expand this in a power series. Subsequently, a Fourier series was obtained. These analyses yielded the current-

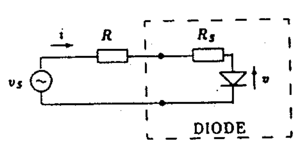

Figure 7. Frequency independent mixer circuit.

voltage solution at the junction terminal without requiring iterations.

Orloff [69] and Gretsch [70] have shown that the intermodulation products can be expressed in terms of products of Bessel functions. This has been based on the Sonine expansion of an exponential [71] and is the conventional method used for the large-input-signal analysis of resistive mixers. Nevertheless it is restricted to diodes with current-voltage characteristics described by the Shockley diode equation. Here, we review the approach of Gretsch which is representative of this type of analysis. If the voltage across the nonlinear resistance is a sum of voltage sinusoids then the Shockley diode equation can be expanded in terms of Bessel functions, to obtain expressions for individual intermodulation products. The method is based on the Sonine expansion [71]:

$$\exp(z \cos \theta) = I_0(z) + \sum_{m=1}^{\infty} I_m(z)\cos(m\theta) \quad (51)$$

where $I_m(z)$ is the modified Bessel function of the first kind of order m. Thus, the exponential of a sum of N sinusoids can be equated to the product of N infinite summations of Bessel functions.

Using eq. (51), Gretsch derived an expression for an intermodulation component of the Shockley diode equation [eq. (49)] in terms of products of Bessel functions of the voltage phasors at the junction. Thus, an intermodulation component of i, corresponding to U_q in eq. (9), is given by

$$U_q(n_1, \ldots, n_N) = 2I_s \exp(\alpha V_{DC}) \prod_{k=1}^{N} I_{n_k}(\alpha V_k) \quad (52)$$

where, as before, the n_k's define the intermodulation product. The total current at a particular frequency is just the sum of the intermodulation products at this frequency.

Grayzel [72], in investigating varactor frequency converters, used the normalized voltage-charge relationship of the reverse biased capacitance of a pn junction diode

$$\hat{v} = (\hat{q})^{1/(1-\gamma)} \quad (53)$$

where $\hat{v}$ and $\hat{q}$ are the voltage and charge normalized to the breakdown voltage V_B and charge Q_B such that $\hat{v} = (v + \phi)/(V_B + \phi)$ and $\hat{q} = (q + Q_\phi)/(Q_B + Q_\phi)$, where Q_ϕ is the charge on the junction at the contact potential ϕ. Grayzel used time-domain techniques to solve the now simplified problem. His work was continued by Conning [73] for large voltages across the diode so that he could neglect the contact potential which enabled him to expand eq. (53) using the Jacobi–Anger formula to derive an expression for the intermodulation components in terms of Bessel functions.

Nakamura et al. [74] considered an up-converter and assumed that conversion was due solely to the reverse-bias capacitance of a pn junction diode. They used the Sonine expansion to obtain the junction charge in terms of Bessel functions of the voltage components. The individual intermodulation products are then products of Bessel functions and are summed to yield the total response of the mixer. In developing the formula, Nakamura et al. made many implicit assumptions including retaining only the first term of the Bessel function expansion of the charge-voltage relationship and then only using the first term of the power series expansions of the Bessel functions. The assumptions restrict their method to low level pumping and relatively small input signals.

For an abrupt junction diode $\gamma = \frac{1}{2}$ and the voltage-charge relationship, eq. (53), reduces to a quadratic equation. For such a low order description of the nonlinearity it is a simple matter to derive expressions for the individual intermodulation products. This approach was used by Goldstein and Frank [75] and Abdullah and Clayton [76] in analyzing parametric amplifiers.

Many other nonlinear characteristics can be directly expanded to yield simple formulas for the output frequency components. Recently, derivations for the output frequency components of transistor circuits have been completed by Abuelma'atti and Gardiner [77–79] and lend valuable insight into the performance of these systems. In 1989, Withington and Kollberg [80] investigated superconducting quasiparticle mixers using a specialized expansion of the quasiparticle junction tunneling characteristic and a relaxation-based spectral balance analysis. The quasiparticle tunnel junction response function is particularly sharp and, under local oscillator excitation, produces a rich set of harmonics. With conventional harmonic balance simulation, a much larger number of harmonics must be included in a simulation in order to avoid aliasing. Withington and Kollberg expanded the response function using the Jacobi–Anger equality to obtain a formula for each frequency component of the response in terms of summations of Bessel functions. Using these formulas as the describing function of the nonlinear device, they developed a computationally efficient relaxation-based spectral balance simulator.

Specialized derivations involve considerable manual algebraic manipulations and apply to very particular forms of nonlinear element characteristics. They can not be considered as viable candidates for a general purpose microwave circuit simulation. But in the specialized situations, simulation speed and accuracy are unrivaled.

VIII. OTHER TECHNIQUES

Recently Ushida and Chua [81] reported a robust frequency-domain relaxation method for the analysis of nonlinear circuits with multifrequency excitation. Relaxation techniques avoid the necessity of formulating or inverting a Jacobian as required when Newton's method is used to solve the harmonic balance equations. Instead, they use a simple update algorithm to modify the estimates of the node voltages so that the voltage phasor at a particular node and at a particular frequency are updated in proportion to the harmonic balance error at that node and frequency. While being fast, relaxation techniques may not converge when the circuit is stiff, i.e., when the nonlinearities are too strong. Ushida and Chua's technique introduces a compensation element to weaken the nonlinearity so that the convergence properties of the relaxation technique are considerably improved. For each nonlinear element they introduce relaxation models composed of associated linear time-invariant elements and/or linear controlled sources and independent sources. At each iteration of the relaxation method each nonlinear element is converted to an associated relaxation model so that the resulting circuit can be solved using conventional nodal analysis. Nevertheless, relaxation-based simulation is not as robust as Newton-based harmonic balance analysis and so the range of circuits that can simulated will always be less.

IX. CONCLUSION

Frequency-domain nonlinear analysis has its roots in Volterra series analysis where the phasor of a frequency component of the output of a nonlinear system is the summation of intermodulation products. These intermodulation products are given as the product of a level-independent transfer function (known as a Volterra nonlinear transfer function) and an intermodulation term which is the product of integer powers of the phasors of the input to the system. The Volterra nonlinear transfer functions permit block diagram representations of weakly nonlinear systems [18]. This modeling approach and related power series expansions have permitted simple descriptions of distortion phenomena in nonlinear systems—as classified in the second section of this paper. Considering the effect of nonlinear distortion in the frequency-domain has enabled simple closed form descriptions of nonlinear phenomena to be developed and consequent minimization of undesired effects [82–85].

Many different approaches have been taken to extend the Volterra nonlinear transfer function technique to more strongly nonlinear systems and to more general microwave circuits. At the circuit level the method of nonlinear currents using power series descriptions of the nonlinearities has been used to model elements in the nonlinear circuit by introducing controlled and independent current sources. This is achieved noniteratively but relies on rapid convergence of the power series [28]. In one variation of this technique only the first order reduction is used so that linearized admittances replace voltage proportional current sources [81]. In many cases a strongly nonlinear circuit becomes moderately nonlinear and relaxation techniques can be used to solve the spectral balance equations.

General purpose frequency-domain nonlinear analysis techniques iteratively solve the spectral balance equations equating the phasors of the currents flowing into the nonlinear subcircuit to the corresponding current phasors flowing out of the linear subcircuit. The nonlinear current phasors are summations of intermodulation products calculated using power series expansions [4,58], analytic function expansions [62], or specialized expansions of the nonlinear device characteristics [70,80]. These are robust techniques and can be used with strongly nonlinear systems and multi-tone excitation. In the companion paper [1] one of the frequency-domain spectral balance techniques is quantitatively compared to two of the promising conventional harmonic balance techniques. Frequency-domain spectral balance techniques can be used to simulate situations in which a small signal is more than 400 dB below a large signal and can be used with multi-tone excitation. By comparison, traditional harmonic balance techniques have dynamic ranges of less than 200 dB which degrades as tones become close in frequency. Because of computation and memory requirements, they are practically limited to three tones. However, active device modeling is more cumbersome with frequency-domain nonlinear analysis techniques than with hybrid techniques.

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REFERENCES

1. R. J. Gilmore and M. B. Steer, "Nonlinear circuit analysis using the method of harmonic balance—a review of the art," *Int. J. of Microwave and Millimeter Wave Computer-Aided Engineering,* Vol. 1, April 1991, pp. 159–180.
2. J. Karaoguz, "Integrated link level and circuit level simulation of mobile communication systems," Master of Science Thesis, Department of Electrical and Computer Engineering, North Carolina State University, 1989.
3. M. B. Steer and P. J. Khan, "An algebraic formula for the complex output of a system with multi-frequency excitation," *Proc. of the IEEE,* January 1983, pp. 177–179.
4. G. W. Rhyne and M. B. Steer, "Generalized power series analysis of intermodulation distortion in a MESFET amplifier: Simulation and experiment," *IEEE Trans. Microwave Theory Tech.*, MTT-35, December 1987, pp. 1248–1255.
5. G. W. Rhyne, M. B. Steer, and B. D. Bates, "Frequency-domain nonlinear circuit analysis using generalized power series," *IEEE Trans. Microwave Theory Tech.*, MTT-36, February 1988, pp. 379–387.
6. V. Volterra, *Theory of Functionals and of Integral and Integro-Differential Equations,* Blackie & Sons, London, 1930; Dover, New York, 1959.
7. C. A. A. Wass, "A table of intermodulation products," *J. IEE,* Vol. 95, pt. 3, 1948, pp. 31–39.
8. G. M. Engel, D. A. Conner, and W. J. Steen, "Determination of intermodulation product amplitudes," *1967 Proc. National Electronics Conf.*, pp. 719–723.
9. R. G. Sea, "An algebraic formula for amplitudes of intermodulation products involving an arbitrary number of frequencies," *Proc. IEEE,* Vol. 56, August 1968, pp. 1388–1389.
10. O. I. Mednikov, "The theory of two-resonant-cir-

cuit parametric oscillators," *Radio Eng. and Electron. Phys.*, Vol. 14, June 1969, pp. 973–976.
11. R. G. Sea and A. G. Vacroux, "On the computation of intermodulation products for a power series nonlinearity," *Proc. IEEE,* March 1969, pp. 337–338.
12. G. L. Heiter, "Characterization of nonlinearities in microwave devices and systems," *IEEE Trans. Microwave Theory Tech.*, MTT-21, December 1973, pp. 797–805.
13. R. S. Tucker and C. Rauscher, "Modelling the 3rd-order intermodulation-distortion properties of a GaAs FET," *J Electron. Lett.*, Vol. 13, August 1977, pp. 508–509.
14. R. S. Tucker, "Third-order intermodulation distortion and gain compression in GaAs FET's," *IEEE Trans. Microwave Theory Tech.*, MTT-27, May 1979, pp. 400–408.
15. B. S. Troitskiy, "Analysis and design of nonlinear circuits by the method of power series inversion," *Elektosvyaz,* No. 8, 1987, pp. 50–52.
16. V. I. Kirillov, "Interference protection of regenerative parametric amplifiers," *Telecom. and Radio Engrg.*, March 1972, pp. 57–60.
17. N. Wiener, "Response of a nonlinear device to noise," *M.I.T. Radiation Lab. Report V-16S,* April 1942.
18. E. Bedrosian and S. O. Rice, "The output properties of Volterra systems (nonlinear systems with memory) driven by harmonic and Gaussian inputs," *Proc. IEEE,* Vol. 59, December 1971, pp. 1688–1707.
19. J. J. Bussgang, L. Ehrman, and J. W. Graham, "Analysis of nonlinear systems with multiple inputs," *Proc. IEEE,* Vol. 62, August 1974, pp. 1088–1119.
20. D. D. Weiner and J. F. Spina, *Sinusoidal Analysis and Modeling of Weakly Nonlinear Circuits,* New York, 1980, Van Nostrand Reinhold Co.
21. S. Narayanan, "Transistor distortion analysis using Volterra series representation," *Bell System Tech. J.*, Vol. 46, May 1967, pp. 991–1024.
22. I. W. Sandberg, "Expansions for nonlinear systems," *Bell System Tech. J.*, Vol. 61, February 1982, pp. 159–199.
23. G. M. Lambrianous and C. S. Aitchison, "Optimization of third-order intermodulation product and out power from an X-band MESFET amplifier using Volterra series analysis," *IEEE Trans. Microwave Theory Tech.*, MTT-33, December 1985, pp. 1395–1403.
24. J. W. Graham and L. Ehrman, "Nonlinear system modeling and analysis with applications to communications receivers," Rome Air Development Center, Rome, NY, June 1973.
25. M. Schetzen, "Nonlinear system modeling based on the Wiener theory," *Proc. IEEE,* Vol. 69, December 1981, pp. 1557–1573.
26. R. A. Minasian, "Intermodulation distortion analysis of MESFET amplifiers using Volterra series representation," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-28, January 1980, pp. 1–8.
27. C. L. Law and C. S. Aitchison, "Prediction of wideband power performance of MESFET distributed amplifiers using the Volterra series representation," *IEEE Trans. Microwave Theory Tech.*, MTT-34, December 1986, pp. 1308–1317.
28. S. A. Maas, *Nonlinear Microwave Circuits,* Norwood, MA: Artech House, 1988.
29. S. Narayanan, "Application of Volterra series to intermodulation distortion analysis of transistor feedback amplifier," *IEEE Trans. Circuit Theory,* CT-17, November 1970, pp. 518–527.
30. T. Endo and L. O. Chua, "Quasi-periodic oscillation via Volterra series," *IEEE Int. Symposium on Circuits and Systems,* May 1986, pp. 57–60.
31. A. M. Khadr and R. H. Johnston, "Distortion in high-frequency FET amplifiers," *IEEE J. Solid-State Circuits,* SC-9, August 1974, pp. 180–189.
32. V. Krozer, K. Fricke, and H. L. Hartnagel, "A novel analytical approach for the nonlinear microwave circuits and experimental characterization of the nonlinear behaviour of a new MESFET device structure," *IEEE MTT-S Int. Microwave Symposium Digest,* June 1989, pp. 351–354.
33. A. Javed, P. A. Goud, and B. A. Syrett, "Analysis of a microwave feedforward amplifier using Volterra series representation," *IEEE Trans. Commun.*, COM-25, March 1977, pp. 355–360.
34. R. B. Swerdlow, "Analysis of intermodulation noise in frequency converters by Volterra series," *IEEE Trans. Microwave Theory Tech.*, MTT-26, April 1978, pp. 305–313.
35. T. K. Sarkar and D. D. Weiner, "Scattering analysis of nonlinearly loaded antennas," *IEEE Trans. Antennas Propagat.*, AP-24, March 1976, pp. 125–131.
36. T. K. Sarkar, D. D. Weiner, and R. F. Harrington, "Analysis of nonlinearly loaded multiport antenna structures over an imperfect ground plane using the Volterra-series method," *IEEE Trans. Electromagn. Compat.*, EMC-20, May 1978, pp. 278–287.
37. Y. Hu, J. Obregon, and J. C. Mollier, "Nonlinear analysis of microwave FET oscillators using Volterra series," *IEEE Trans. Microwave Theory Tech.*, MTT-37, November 1989, pp. 1689–1693.
38. L. O. Chua and Y. S. Tang, "Nonlinear oscillation via Volterra series," *IEEE Trans. Circuits and Systems,* CAS-29, March 1982, pp. 150–168.
39. J. Xu and S. Li, "A feedback model of millimeter wave harmonic oscillators and its application," *Int. J. of Infrared and Millimeter Waves,* Vol. 10, No. 9, 1989, pp. 1093–1101.
40. A. M. Crosmun, S. A. Maas, "Minimization of intermodulation distortion in GaAs MESFET small-signal amplifiers," *IEEE Trans. Microwave Theory Tech.*, MTT-37, September 1989, pp. 1411–1417.

41. S. Boyd, Y. S. Tang, and L. O. Chua, "Measuring Volterra kernals," *IEEE Trans. on Circuits and Systems*, CAS-30, August 1987, pp. 571–577.
42. L. O. Chua and Y. Liao, "Measuring Volterra kernels (II)," *Int. J. of Circuits Theory and Applications*, Vol. 17, 1989, pp. 151–190.
43. I. W. Saudberg, "On Volterra expansions for time varying nonlinear systems," *IEEE Trans. Circuits Syst.*, CAS-30, February 1983, pp. 61–67.
44. M. B. Steer, "Multifrequency analysis of nonlinear circuits," in *IEEE Southeastcon Convention Digest*, 1985, pp. 116–120.
45. M. B. Steer, P. J. Khan, and R. S. Tucker, "Relationship between Volterra series and generalized power series," *Proc. of the IEEE*, December 1983, pp. 1453–1454.
46. V. D. Dmitriev and A. I. Silyutin, "A method of designing broadband nonlinear circuits based on a modification of the nonlinear current method," *Radioelectron Commun. Syst.*, Vol. 29, No. 11, 1986, pp. 50–55.
47. D. N. Held and A. R. Kerr, "Conversion loss and noise of microwave and millimeter-wave mixers: Part 1—Theory and Part 2—Experiment," *IEEE Trans. Microwave Theory Tech.*, MTT-26, February 1978, pp. 49–61.
48. R. E. Maurer and S. Narayanan, "Noise loading analysis of a third-order nonlinear system with memory," *IEEE Trans. Commun. Technol.*, COM-16, October 1968, pp. 701–712.
49. M. Rudko and D. D. Weiner, "Volterra systems with random inputs: A formalized approach," *IEEE Trans. Commun.*, COM-26, February 1978, pp. 217–227.
50. E. Van den Eijnde and J. Schoukens, "Steady-state analysis of a periodically excited nonlinear system," *IEEE Trans. Circuits and Systems*, CAS-37, February 1990, pp. 232–242.
51. L. O. Chua and C. Y. Ng, "Frequency-domain analysis of nonlinear systems: general theory," *Electronic Circuits Systems*, Vol. 3, July 1979, pp. 165–185.
52. L. Gustafsson, G. H. B. Hanson and K. I. Lundström, "On the use of describing functions in the study of nonlinear active microwave circuits," *IEEE Trans. Microwave Theory and Tech.*, MTT-20, June 1972, pp. 402–409.
53. S. O. Rice, "Volterra systems with more than one input port—Distortion in a frequency converter," *Bell Syst. Tech. J.*, Vol. 52, October 1973, pp. 1255–1270.
54. M. Lamnabhi, "A new symbolic calculus for the response of nonlinear systems," *Systems and Control Lett.*, Vol. 2, October 1982, pp. 154–162.
55. M. Fliess and F. Lamnabhi-Lagarrigue, "Application of a new functional expansion to the cubic anharmonic oscillator," *J. Math. Phys.*, Vol. 23, April 1982, pp. 495–502.
56. M. Fliess, M. Lamnabhi, and F. Lamnabhi-Lagarrigue, "An algebraic approach to nonlinear functional expansions," *IEEE Trans. Circuits Systems*, CAS-30, August 1983, pp. 554–570.
57. M. Lamnabhi, "Functional analysis of nonlinear circuits: a generating power series approach," *IEE Proc. H*, Vol. 133, October 1986, pp. 375–384.
58. C. R. Chang, M. B. Steer, and G. W. Rhyne, "Frequency-domain spectral balance using the arithmetic operator method," *IEEE Trans. Microwave Theory Tech.*, MTT-37, November 1989, pp. 1681–1688.
59. A. Ushida and L. O. Chua, "Frequency-domain analysis of nonlinear circuits driven by multi-tone signals," *IEEE Trans. Circuits and Systems*, CAS-31, September 1984, pp. 766–779.
60. P. J. Lunsford, G. W. Rhyne, and M. B. Steer, "Frequency domain bivariate generalized power series analysis of nonlinear analog circuits," *IEEE Trans. Microwave Theory Tech.*, in press.
61. J. H. Haywood and Y. L. Chow, "Intermodulation distortion analysis using a frequency domain harmonic balance technique," *IEEE Trans. Microwave Theory Tech.*, MTT-36, August 1988, pp. 1251–1257.
62. C. R. Chang and M. B. Steer, "Frequency-domain nonlinear microwave circuit simulation using the arithmetic operator method," *IEEE Trans. Microwave Theory Tech.*, MTT-38, August 1990, pp. 1139–1143.
63. W. R. Curtice, "GaAs MESFET modeling and nonlinear CAD," *IEEE Trans. Microwave Theory Tech.*, MTT-36, February 1988, pp. 220–230.
64. T. Kacprzak and A. Materka, "Compact dc model of FaAs FET's for large-signal computer calculation," *IEEE J. of Solid-State Circuits*, SC-18, April 1983, pp. 211–213.
65. M. A. Khatibzadeh and R. J. Trew, "A large-signal, analytic model for the GaAs MESFET," *IEEE Trans. on Microwave Theory and Tech.*, Vol. 36, February 1988, pp. 231–238.
66. E. M. Rutz-Philipp, "Power conversion in nonlinear resistive elements related to interference phenomena," *IBM Journal*, Vol. 11, September 1967, pp. 544–552.
67. E. Beane, "Prediction of mixer intermodulation levels as function of local oscillator power," *IEEE Trans. Electromagnetic Compatability*, Vol. EMC-13, May 1971, pp. 56–63.
68. H. D. Mills, "On the equation, $i = i_0 . \exp\alpha(V - Ri) - 1$," *IBM J.*, Vol. 11, 1967, pp. 553–554.
69. L. M. Orloff, "Intermodulation analysis of crystal mixer," *Proc. IEEE*, Vol. 52, February 1964, pp. 173–179.
70. W. R. Gretsch, "The spectrum of intermodulation generated in a semiconductor diode junction," *Proc. IEEE*, Vol. 54, November 1966, pp. 1528–1535.
71. M. Abramowitz and I. A. Stegun, "*Handbook of*

Mathematical Functions," Dover Publications, New York, 1965.
72. A. I. Grayzel, "The overdriven varactor upper sideband upconverter," *IEEE Trans. Microwave Theory Tech.*, MTT-15, October 1967, pp. 561–565.
73. S. W. Conning, "An analysis of the high power varactor upconverter," *Proc IREE Australia,* July 1966, pp. 163–173.
74. S. Nakamura, H. Yamamoto, and A. Furukawa, "Output power and intermodulation of high power parametric up-converter," *Review of the Electrical Communication Laboratory*, Vol. 15, September–October 1967, pp. 571–595.
75. M. Goldstein and H. Frank, "Nonlinear analysis of parametric amplifiers," *IEEE Trans. Circuit Theory,* November 1969, pp. 532–535.
76. F. Abdullah and F. M. Clayton, "A large-signal theory for broad-band frequency converters using abrupt junction varactor diodes," *IEEE Trans. Microwave Theory Tech.*, MTT-25, February 1977, pp. 127–136.
77. M. T. Abuelma'atti and J. G. Gardiner, "Approximate method for hand computation of amplifier intermodulation product levels," *IEE Proceedings–G,* Vol. 128, February 1981, pp. 32–34.
78. M. T. Abuelma'atti, "Harmonic and intermodulation generation due to hot electrons in semiconductors," *IEE Proc.*, Vol. 136, Pt. G, December 1989, pp. 323–326.
79. M. T. Abuelma'atti, "Harmonic and intermodulation performance of logarithmic-law devices excited by sinusoidal signals," *Int. J. of Electronics,* Vol. 66, No. 4, 1989, pp. 535–541.
80. S. Withington and E. L. Kollberg, "Spectral-domain analysis of harmonic effects in superconducting quasiparticle mixers," *IEEE Trans. Microwave Theory Tech.*, MTT-37, January 1989, pp. 231–238.
81. A. Ushida and L. O. Chua, "Steady-state response of non-linear circuits: A frequency domain relaxation method," *Int. J. Circuit Theory and Applications,* Vol. 17, 1989, pp. 249–269.
82. R. A. Pucel, "Profile design for distortion reduction in microwave field-effect transistors," *J Electron. Lett.*, Vol. 14, March 1978, pp. 204–206.
83. J. A. Higgins, "Intermodulation distortion in GaAs FETs," *1978 IEEE MTT-S Int. Microwave Symp. Dig.*, June 1978, pp. 138–141.
84. J. A. Higgins and R. L. Kuvas, "Analysis and improvement of intermodulation distortion in GaAs power FET's," *IEEE Trans. Microwave Theory Tech.*, MTT-28, January 1980, pp. 9–17.
85. L. C. Thomas, "Eliminating broadband distortion in transistor amplifiers," *Bell System Tech. J.*, March 1968, pp. 315–342.

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