The Relationship between Bivariate Volterra Analysis and Power Series Analysis with Application to the Behavioral Modeling of Microwave Circuits

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ABSTRACT

Volterra nonlinear transfer functions can be used in the behavioral modeling of many nonlinear microwave circuits. They can be developed experimentally, numerically, and, to a limited extent, analytically. This article presents an enhanced analytic method for developing bivariate Volterra nonlinear transfer functions based on their relationship to power series. The technique is applied to the Volterra-series-based behavioral modeling of a MESFET amplifier using experimental characterization of the MESFET.

I. INTRODUCTION

The computer-aided simulation and synthesis of large active microwave systems require nonlinear frequency-domain behavioral modeling of component circuits and subsystems. As system designs become larger and more complex, the designer has an increasing need to simulate the entire system, or a very large section of the system, that is too large for a circuit simulator. If smaller subsections of the system can be characterized by behavioral models, then a system level analysis can be done using these models. Volterra theory provides a general purpose way to characterize an arbitrary nonautonomous analog subsystem. All of the information needed to predict the behavior of a nonlinear system is contained in either the time-domain Volterra kernels, or the frequency-domain Volterra nonlinear transfer functions. Microwave behavioral modeling is in its infancy, and currently most subsystems are described linearly using circuit parameter (e.g., scattering parameter) descriptions, linear transfer functions (possibly with gain), and small-signal conversion matrices. Mildly nonlinear subsystems can be described by low-order Volterra nonlinear transfer functions resulting in a behavioral model that is noniterative and efficient to compute [1]. Volterra nonlinear transfer functions can also be used with frequency conversion circuits where a large signal establishes a time-varying mildly nonlinear system converting energy from the input signal to energy in the output signal [2]. They have also been used as describing functions of nonlinear elements in the harmonic balance simulation of microwave circuits [3–6], but this work is not directly applicable to behavioral modeling.

This article addresses the problem of developing the Volterra nonlinear transfer functions of strongly nonlinear microwave circuits using power

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series-based experimental characterization of nonlinear elements. Previously, we have shown how the univariate (single input) Volterra nonlinear transfer functions can be obtained for a system having a univariate power series description [7]. Here, we establish this relationship for the more complicated case of a system with two inputs. The power series–Volterra nonlinear transfer function relationship enables the convenient development of the behavioral model of experimentally characterized nonlinearities. Furthermore, it enables the development of large signal Volterra nonlinear transfer functions. These large signal transfer functions are valid for strong nonlinear departures from an RF operating point. These contributions are presented using a MESFET amplifier example. In this example, we develop the Volterra nonlinear transfer functions of a MESFET amplifier using an experimentally derived bivariate power series description of its nonlinearity. The work reported here is also applicable to extending Volterra-based harmonic balance simulation of microwave circuits [4] to include bivariate nonlinearities such as the current characteristics of a transistor.

II. BACKGROUND

In circuit and system modeling using Volterra series techniques, rarely does one need to deal directly with Volterra series themselves. Volterra series are a generalized form of power series, and are used as time-domain descriptions of nonlinear systems. Mostly, one is concerned with the frequency-domain derivative form which is expressed in terms of Volterra nonlinear transfer functions [8,9]. If a circuit has a univariate (i.e., a single input-to-single output) Volterra series representation, the frequency-domain output $Y(f)$ of the system can be written as the summation of different order responses

$$Y(f) = \sum_{n=0}^{\infty} Y_n(f) \tag{1}$$

as illustrated in Figure 1. Here $Y_n(f)$ is the nth order response, and corresponds to the response of the nth order term in the power series description of the nonlinearity [7]. If the system is mildly nonlinear only the few terms in the summation are required. The first-order response is just the linearized response of the system and the zeroeth

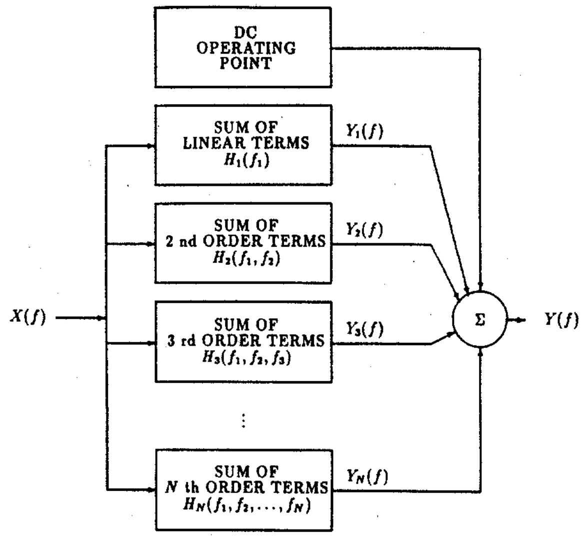

Figure 1. Illustration of nonlinear analysis using Volterra series.

order response is a DC offset. The zero- and first-order responses describe the system entirely when the system is linear or when the input to the nonlinear system is sufficiently small. As the input to a nonlinear system increases, second and higher order responses must be included to adequately describe the system. With multifrequency excitation

$$\begin{aligned} x(t) &= \sum_{q=1}^{Q} 2|E_q|\cos(2\pi f_q t + \theta_q) \\ &= \sum_{q=-Q}^{Q} E_q \exp(j2\pi f_q t) \end{aligned} \tag{2}$$

the nth order response can be written

$$\begin{aligned} Y_n(f) = \sum_{q_1=-Q}^{Q} \cdots \sum_{q_n=-Q}^{Q} \\ \times H_n(f_{q_1}, \ldots, f_{q_n}) E_{q_1} \cdots E_{q_n} \\ \times \exp[j2\pi(f_1 + f_2 \cdots f_n)] \end{aligned} \tag{3}$$

Where the term $H_n(f_{q_1}, \ldots, f_{q_n})E_{q_1} \cdots E_{q_n}$ is an nth-order intermodulation product of frequency $(f_{q_1} + \cdots + f_{q_n})$. Each intermodulation product is the product of a Volterra nonlinear transfer function H_n, and an intermodulation term $E_{q_1} \ldots E_{q_n}$ which describes the interaction of the input signals. The power of the Volterra series method is that a nonlinear system is characterized by a number of functions, the H_n's, which do not depend on the form or level of the input. Determination of the H_n's is reasonably straightforward for weakly nonlinear systems since only the H_n's for $n \leq 3$ are usually required.

The Volterra nonlinear transfer functions can be derived algebraically [1,10,11], experimentally [12–15], or numerically [4]. The algebraic determination of the H_n's from the element constitutive relations can be cumbersome, and generally determination of H_n for $n > 3$ is impractical. Experimental characterization is noise limited so that system characterization is again generally restricted to third order or less. The problem of determining the nonlinear transfer functions is even more difficult when considering systems or subsystems with two input ports. This situation requires bivariate nonlinear transfer functions [10, pp. 131–137; 16], but the algebraic and experimental complexity of determining the bivariate transfer functions has practically eliminated their use.

III. DEVELOPMENT

The aim of this section is to express the Volterra nonlinear transfer functions in terms of power series coefficients. Our strategy is to compare the expression of an intermodulation product developed using a bivariate power series description and the expression developed using bivariate Volterra nonlinear transfer functions.

A. Generalized Power Series

The bivariate Generalized Power Series [17] can be expressed in the form

$$y(t) = A \sum_{i=1}^{I} A_i \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{m,n,i} \times \left[\sum_{k=-K}^{K} b_{k,i} x_k(t - \tau_{k,n,i}) \right]^n \times \left[\sum_{l=-L}^{L} d_{l,i} z_l(t - \lambda_{l,m,i}) \right]^m \tag{4}$$

where $x(t)$ is given by

$$x(t) = \sum_{k=0}^{K} \epsilon_k |X_k| \cos(\omega_k t + \phi_k) = \sum_{k=-K}^{K} X_k e^{j\omega_k t} \tag{5}$$

where ϵ_k is the Neumann factor ($\epsilon_k = 1$, $n = 0$; $\epsilon_k = 2$, $n \neq 0$). For the right side of eq. (5) to remain real, we require $X_{-k} = X_k^*$ and $\omega_{-k} = -\omega_k$, where * denotes the complex conjugate. The output, $z(t)$ is, likewise, described. The coefficients (A, A_i, $a_{m,n,i}$, $b_{k,i}$, $d_{l,i}$) may be complex, in which case the frequency components of $y(t)$ are interpreted as being phase shifted. We can also express the output as

$$y(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} y_{m;n}(t) \tag{6}$$

where

$$y_{m;n}(t) = \sum_{i=1}^{I} \tilde{a}_{m,n,i} \left[\sum_{k=-K}^{K} \tilde{b}_{k,i,n} x_k(t) \right]^n \times \left[\sum_{l=-L}^{L} \tilde{d}_{l,i,n} z_l(t) \right]^m \tag{7}$$

$$\tilde{a}_{m,n,i} = AA_i a_{m,n,i} \tag{8}$$

$$\tilde{b}_{k,i,n} = b_{k,i} e^{-j\omega_k \tau_{k,n,i}} \tag{9}$$

$$\tilde{d}_{l,i,n} = d_{l,i} e^{-j\omega_l \lambda_{l,n,i}} \tag{10}$$

The exponentation in eq. (7) can be factored to yield the following equation for the m; nth order intermodulation product

$$y_{m;n}(t) = \sum_{i=1}^{I} \tilde{a}_{m,n,i} \times \left[\sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \prod_{\zeta=1}^{n} \tilde{b}_{k_\zeta,i,n} x_{k_\zeta}(t) \right] \times \left[\sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_n=-L}^{L} \prod_{\nu=1}^{m} \tilde{d}_{l_\nu,i,n} z_{l_\nu}(t) \right] \tag{11}$$

or

$$y_{m;n}(t) = \sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_n=-L}^{L} \left[\prod_{\zeta=1}^{n} x_{k_\zeta}(t) \right] \left[\prod_{\nu=1}^{m} z_{l_\nu}(t) \right] \sum_{i=1}^{I} \tilde{a}_{m,n,i} \times \left[\prod_{\theta=1}^{n} \tilde{b}_{k_\theta,i,n} \right] \left[\prod_{\mu=1}^{m} \tilde{d}_{l_\mu,i,n} \right] \tag{12}$$

B. Volterra Series

An intermodulation product of the output of a two input system can also be expressed using Volterra series representation. The total output of a system described by a bivariate Volterra series is given by

$$y(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} y_{m;n}(t) \tag{13}$$

where

$$\begin{aligned} y_{m;n}(t) = &\int_{\hat\tau_m=-\infty}^{+\infty} \int_{\hat\tau_{m-1}=-\infty}^{+\infty} \cdots \int_{\hat\tau_1=-\infty}^{+\infty} \int_{\bar\tau_n=-\infty}^{+\infty} \int_{\bar\tau_{n-1}=-\infty}^{+\infty} \cdots \int_{\bar\tau_1=-\infty}^{+\infty} \\ &h_{m;n}(\hat\tau_1, \hat\tau_2, \ldots \hat\tau_m; \bar\tau_1, \bar\tau_2, \ldots \bar\tau_n) \\ &\times \left[\prod_{h=1}^{m} z(t-\hat\tau_h)\right]\left[\prod_{i=1}^{n} x(t-\bar\tau_i)\right] \\ &d\bar\tau_1\, d\bar\tau_2 \ldots d\bar\tau_n\, d\hat\tau_1\, d\hat\tau_2 \ldots d\hat\tau_m \end{aligned} \tag{14}$$

in the sense

$$y_{0;0}(t) = h_{0;0} \tag{15}$$

where ˆ and ˉ refer to the first and second input sets, respectively. The $y_{0;0}$ term is often omitted by other authors but is kept here for generality and is necessary to describe DC offsets. Here $h_{m;n}$ is called the Volterra kernel. For the frequency-domain description of the Volterra kernels, we need to use the *m;n*th Fourier transform of $h_{m;n}$ so that the *m;n*th-order bivariate Volterra nonlinear transfer function is given by[1]

$$\begin{aligned} &H_{m;n}(\hat f_1, \hat f_2, \ldots \hat f_m; \bar f_1, \bar f_2, \ldots \bar f_n) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_{m;n}(\hat\tau_1, \hat\tau_2, \ldots \hat\tau_m, \bar\tau_1, \bar\tau_2, \ldots \bar\tau_n) \\ &e^{-j2\pi(\hat f_1\hat\tau_1+\hat f_2\hat\tau_2+\ldots\hat f_m\hat\tau_m+\bar f_1\bar\tau_1+\bar f_2\bar\tau_2+\ldots\bar f_n\bar\tau_n)} \\ &d\hat\tau_1\, d\hat\tau_2 \ldots d\hat\tau_m\, d\bar\tau_1\, d\bar\tau_2 \ldots d\bar\tau_n \end{aligned} \tag{16}$$

$$H_{0;0} = h_{0;0} \tag{17}$$

and, using inverse Fourier transformation,

$$\begin{aligned} &h_{m;n}(\hat\tau_1, \hat\tau_2, \ldots \hat\tau_m, \bar\tau_1, \bar\tau_2, \ldots \bar\tau_n) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_{m;n}(\hat f_1, \hat f_2, \ldots \hat f_m; \bar f_1, \bar f_2, \ldots \bar f_n) \\ &e^{+j2\pi(\hat f_1\hat\tau_1+\hat f_2\hat\tau_2+\ldots\hat f_m\hat\tau_m+\bar f_1\bar\tau_1+\bar f_2\bar\tau_2+\ldots\bar f_n\bar\tau_n)} \\ &d\hat f_1\, d\hat f_2 \ldots d\hat f_m\, d\bar f_1\, d\bar f_2 \ldots d\bar f_n \end{aligned} \tag{18}$$

The Fourier transformation of the *m;n*th order time-domain response $y_{m;n}(t)$ yields the frequency-domain response of the nonlinear system [10, p. 132]

$$\begin{aligned} Y_{m;n}(f) = &\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_{m;n}(\hat f_1, \hat f_2, \ldots \hat f_m; \bar f_1, \bar f_2, \ldots \bar f_n) \\ &\delta(f - \hat f_1 - \hat f_2 - \ldots \hat f_m - \bar f_1 - \bar f_2 - \ldots \bar f_n) \\ &\left[\prod_{h=1}^{m} Z(\hat f_h)\right]\left[\prod_{i=1}^{n} X(\bar f_i)\right] \\ &d\hat f_1\, d\hat f_2 \ldots d\hat f_m\, d\bar f_1\, d\bar f_2 \ldots d\bar f_n \end{aligned} \tag{19}$$

Assuming $x(t)$ and $z(t)$ are of the form given by eq. (5), the frequency-domain representations of the inputs are given by

$$X(f) = \sum_{k=-K}^{K} X_k\, \delta\left(f - \frac{\omega_k}{2\pi}\right) \tag{20}$$

$$Z(f) = \sum_{l=-L}^{L} Z_l\, \delta\left(f - \frac{\omega_l}{2\pi}\right) \tag{21}$$

Thus the phasor of the *m;n*th order intermodulation product is

$$\begin{aligned} Y_{m;n}(f) = &\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_{m;n}(\hat f_1, \hat f_2, \ldots \hat f_m; \bar f_1, \bar f_2, \ldots \bar f_n) \\ &\delta(f - \hat f_1 - \hat f_2 - \ldots \hat f_m - \bar f_1 - \bar f_2 - \ldots \bar f_n) \\ &\left[\prod_{h=1}^{m} \sum_{l=-L}^{L} Z_l\, \delta\left(\hat f_h - \frac{\omega_l}{2\pi}\right)\right] \\ &\times \left[\prod_{i=1}^{n} \sum_{k=-K}^{K} X_k\, \delta\left(\bar f_i - \frac{\omega_k}{2\pi}\right)\right] \\ &d\hat f_1\, d\hat f_2 \ldots d\hat f_m\, d\bar f_1\, d\bar f_2 \ldots d\bar f_n \end{aligned} \tag{22}$$

[1]We use the standard notation of capital letters for the frequency-domain description of a variable.

Expanding the multiplication by introducing the summation variables $l_1, l_2, \ldots l_m$ and $k_1, k_2 \ldots k_n$ yields

$$Y_{m;n}(f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_{m;n}(\hat{f}_1, \hat{f}_2, \ldots \hat{f}_m; \bar{f}_1, \bar{f}_2, \ldots \bar{f}_n) \delta(f - \hat{f}_1 - \hat{f}_2 - \ldots \hat{f}_m - \bar{f}_1 - \bar{f}_2 - \ldots \bar{f}_n) \left[\sum_{l_1=-L}^{L} Z_{l_1} \delta\left(\hat{f}_1 - \frac{\omega_{l_1}}{2\pi}\right)\right] \times \left[\sum_{l_2=-L}^{L} Z_{l_2} \delta\left(\hat{f}_2 - \frac{\omega_{l_2}}{2\pi}\right)\right] \cdots \left[\sum_{l_m=-L}^{L} Z_{l_m} \delta\left(\hat{f}_m - \frac{\omega_{l_m}}{2\pi}\right)\right] \left[\sum_{k_1=-K}^{K} X_{k_1} \delta\left(\bar{f}_1 - \frac{\omega_{k_1}}{2\pi}\right)\right] \times \left[\sum_{k_2=-K}^{K} X_{k_2} \delta\left(\bar{f}_2 - \frac{\omega_{k_2}}{2\pi}\right)\right] \cdots \left[\sum_{k_n=-K}^{K} X_{k_n} \delta\left(\bar{f}_n - \frac{\omega_{k_n}}{2\pi}\right)\right] d\hat{f}_1\, d\hat{f}_2 \ldots d\hat{f}_m\, d\bar{f}_1\, d\bar{f}_2 \ldots d\bar{f}_n \quad (23)$$

and changing the order of summation and integration

$$Y_{m;n}(f) = \sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_m=-L}^{L} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_{m;n}(\hat{f}_1, \hat{f}_2, \ldots \hat{f}_m; \bar{f}_1, \bar{f}_2, \ldots \bar{f}_n) \delta(f - \hat{f}_1 - \hat{f}_2 - \ldots \hat{f}_m - \bar{f}_1 - \bar{f}_2 - \ldots \bar{f}_n) Z_{l_1} \delta\left(\hat{f}_1 - \frac{\omega_{l_1}}{2\pi}\right) Z_{l_2} \delta\left(\hat{f}_2 - \frac{\omega_{l_2}}{2\pi}\right) \cdots Z_{l_m} \delta\left(\hat{f}_m - \frac{\omega_{l_m}}{2\pi}\right) X_{k_1} \delta\left(\bar{f}_1 - \frac{\omega_{k_1}}{2\pi}\right) X_{k_2} \delta\left(\bar{f}_2 - \frac{\omega_{k_2}}{2\pi}\right) \cdots X_{k_n} \delta\left(\bar{f}_n - \frac{\omega_{k_n}}{2\pi}\right) d\hat{f}_1\, d\hat{f}_2 \ldots d\hat{f}_m\, d\bar{f}_1\, d\bar{f}_2 \ldots d\bar{f}_n \quad (24)$$

Carrying out the integration is relatively easy since the integrand is nonzero only when $\bar{f}_i = \omega_{k_i}/2\pi$ and $\hat{f}_i = \omega_{l_i}/2\pi$, thus

$$Y_{m;n}(f) = \sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_m=-L}^{L} H_{m;n}\left(\frac{\omega_{l_1}}{2\pi}, \frac{\omega_{l_2}}{2\pi}, \cdots \frac{\omega_{l_m}}{2\pi}; \frac{\omega_{k_1}}{2\pi}, \frac{\omega_{k_2}}{2\pi}, \cdots \frac{\omega_{k_n}}{2\pi}\right) \delta\left(f - \frac{\omega_{l_1}}{2\pi} - \frac{\omega_{l_2}}{2\pi} - \cdots \frac{\omega_{l_m}}{2\pi} - \frac{\omega_{k_1}}{2\pi} - \frac{\omega_{k_2}}{2\pi} - \cdots \frac{\omega_{k_n}}{2\pi}\right) Z_{l_1} Z_{l_2} \ldots Z_{l_m} X_{k_1} X_{k_2} \ldots X_{k_n} \quad (25)$$

Taking the inverse Fourier transform yields the *m;n*th order time-domain response.

$$y_{m;n}(t) = \sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_m=-L}^{L} H_{m;n}\left(\frac{\omega_{l_1}}{2\pi}, \frac{\omega_{l_2}}{2\pi}, \cdots \frac{\omega_{l_m}}{2\pi}; \frac{\omega_{k_1}}{2\pi}, \frac{\omega_{k_2}}{2\pi}, \cdots \frac{\omega_{k_n}}{2\pi}\right) e^{+j(\omega_{l_1} + \omega_{l_2} + \ldots \omega_{l_m} + \omega_{k_1} + \omega_{k_2} + \ldots \omega_{k_n} +)t} Z_{l_1} Z_{l_2} \ldots Z_{l_m} X_{k_1} X_{k_2} \ldots X_{k_n} \quad (26)$$

and using eqs. (20) and (21) yields the time-domain form of the *m;n*th intermodulation product.

$$Y_{m;n}(t) = \sum_{k_1=-K}^{K} \sum_{k_2=-K}^{K} \cdots \sum_{k_n=-K}^{K} \sum_{l_1=-L}^{L} \sum_{l_2=-L}^{L} \cdots \sum_{l_m=-L}^{L} \left[\prod_{\zeta=1}^{n} x_{k_\zeta}(t)\right]\left[\prod_{\nu=1}^{m} z_{l_\nu}(t)\right] H_{m;n} \left(\frac{\omega_{l_1}}{2\pi}, \frac{\omega_{l_2}}{2\pi}, \cdots \frac{\omega_{l_m}}{2\pi}; \frac{\omega_{k_1}}{2\pi}, \frac{\omega_{k_2}}{2\pi}, \cdots \frac{\omega_{k_n}}{2\pi}\right) \quad (27)$$

C. Comparison

Equations (27) and (12) both represent *m;n*th-order intermodulation products. By comparing eqs. (27) and (12) we can observe that a system represented by a bivariate power series is equivalent to a system represented by a Volterra series if

$$H_{m;n}\left(\frac{\omega_{l_1}}{2\pi}, \frac{\omega_{l_2}}{2\pi}, \ldots \frac{\omega_{l_m}}{2\pi}; \frac{\omega_{k_1}}{2\pi}, \frac{\omega_{k_2}}{2\pi}, \ldots \frac{\omega_{k_n}}{2\pi}\right) = \sum_{i=1}^{I} \tilde{a}_{m,n,i}\left[\prod_{\theta=1}^{n} \tilde{b}_{k_\theta,i,n}\right]\left[\prod_{\mu=1}^{m} \tilde{d}_{l_\mu,i,n}\right] \quad (28)$$

That is, if

$$H_{m;n}\left(\frac{\omega_{l_1}}{2\pi}, \frac{\omega_{l_2}}{2\pi}, \ldots \frac{\omega_{l_m}}{2\pi}; \frac{\omega_{k_1}}{2\pi}, \frac{\omega_{k_2}}{2\pi}, \ldots \frac{\omega_{k_n}}{2\pi}\right) = A\sum_{i=1}^{I} A_i a_{m,n,i} \times \left[\prod_{\theta=1}^{n} \tilde{b}_{k_\theta,i,n} e^{-j\omega_{k_\theta}\tau_{k_\theta,i,n}}\right] \times \left[\prod_{\mu=1}^{m} \tilde{d}_{l_\mu,i,n} e^{-j\omega_{l_\mu}\lambda_{l_\mu,i,m}}\right] \quad (29)$$

$$H_{0;0} = A\sum_{i=1}^{I} A_i a_{0,0,i} \quad (30)$$

Thus given the coefficients of a bivariate power series description of a circuit or system, the Volterra nonlinear transfer functions are immediately available.

IV. DISCUSSION

Further insight to the system representation of these two kinds of analyses can be seen using graphical representations of the system model. For simplification, let us begin by looking at the univariate form of the generalized power series [18].

$$y(t) = A\sum_{i=1}^{I} A_i \sum_{n=0}^{\infty} a_{n,i} \left[\sum_{k=-K}^{K} b_{k,i} x_k(t - \tau_{k,n,i})\right]^n \quad (31)$$

For simplification we will restrict $A = A_i = I = 1$ in eqs. (31) and (4). The discussion can be extended to arbitrary A, A_i, and I, but the comparisons between Volterra and power series are more easily seen for the simpler case.

First, consider the effects of the parameters b_k and $\tau_{k,n}$ in eq. (31). The parameter b_k is the amplification of the signal at the input frequency ω_k, and must be symmetric with respect to k, i.e., $b_k = b_{-k}$. The parameter $\tau_{k,n}$ is the time delay, equivalent to a phase shift at the input frequency ω_k. Therefore, for a given n, the effect of b_k and $\tau_{k,n}$ is equivalent to passing the input through a linear system characterized by amplitude $b(\omega)$ and delay $\tau_n(\omega)$. We will define $G_{b,\tau,n}$ to be the linear system characterized by b_k and $\tau_{k,n}$.

Earlier work with generalized power series [18] allowed the coefficients a_n to be complex. This can be somewhat confusing at first glance because it appears that the right side of eq. (31) could be complex. However, when calculating the results of steady-state simulation of a system, only the positive frequency components of the output need to be calculated. Thus a complex a_n is implicitly a function of the output frequency. To be mathematically complete, we must state that a_n^* is used to calculate the negative frequency components of the output, but since the negative output frequency components are not directly calculated, the complex coefficients are, in practice, constants. We will define G_{a_n} to be the linear system characterized by a_n. We could, if we liked, allow a_n to be explicitly a function of the output frequency, thus G_{a_n} could be any arbitrary linear system, not just a linear system that has a constant transfer function for positive frequencies. This generalization, however, would be even more confusing with the present notation.

On the other hand, to simplify the calculation, we can restrict τ to be independent of n, or conversely, we could let b be dependent on n, thus generalizing $G_{b,\tau,n}$ to be any arbitrary linear system. From examination of eq. (31), we can see that the system representation for univariate generalized power series can be given as shown in Figure 2. Note that a_0 is just the DC output of the system when no inputs are applied. Figure 1 gives the standard system representation for Volterra series analysis. By comparing Figures 1 and 2, we can easily see that power series analysis is equivalent to Volterra analysis if we restrict the Volterra nonlinear transfer functions, H_n to be representable as shown in Figure 3. The same relationship can be applied to the bivariate case. Figure 4 shows the equivalent bivariate Volterra nonlinear transfer function that is represented by bivariate generalized power series.

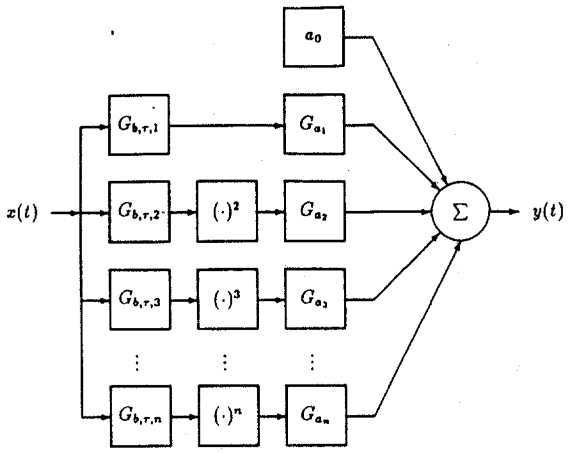

Figure 2. Univariate power series system representation.

V. VOLTERRA NONLINEAR TRANSFER FUNCTION MODEL OF A *MESFET*

One form of Volterra series analysis of nonlinear circuits casts the nonlinear circuit into block diagram form, each block being described by linear transfer functions for linear subcircuits and by Volterra nonlinear transfer functions for nonlinear subcircuits [19]. Here we apply this technique to the Volterra series-based analysis of a MESFET amplifier and compare the simulated results to experimental results.

The MESFET amplifier used here was previously analyzed by Chang et al. [20]. The model schematic is shown in Figure 5, and the measured parameter values are given in Table I. Using the substitution theorem [21, section 2.2.1], the equivalent signal flow graph of the circuit is shown in Figure 6. The input is the value of the source voltage V_{IN}. This signal passes through the linear systems $G_{\mathrm{IN\text{-}GS}}$ and $G_{\mathrm{IN\text{-}DS}}$. Each of these systems is characterized by all of the linear components of the circuit. V_{GS} and V_{DS} are the inputs to the two-input nonlinear block B characterized by τ and Table II. Using (28), we can directly calculate the bivariate Volterra nonlinear transfer function and thus can calculate the value I_{DS}. The value of the output voltage V_L is the output of the linear system

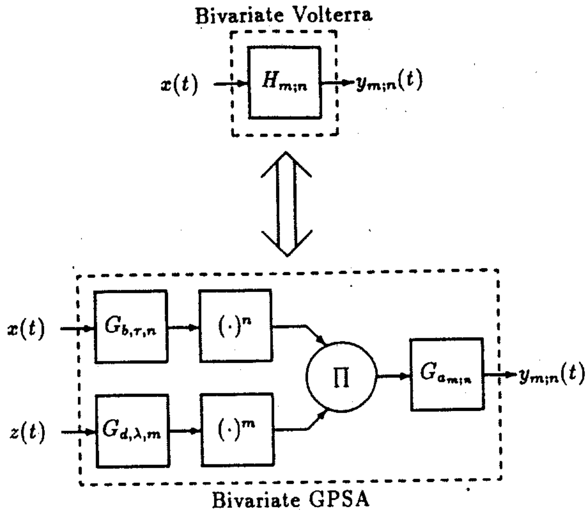

Figure 4. Equivalency of bivariate Volterra analysis and generalized power series analysis (GPSA).

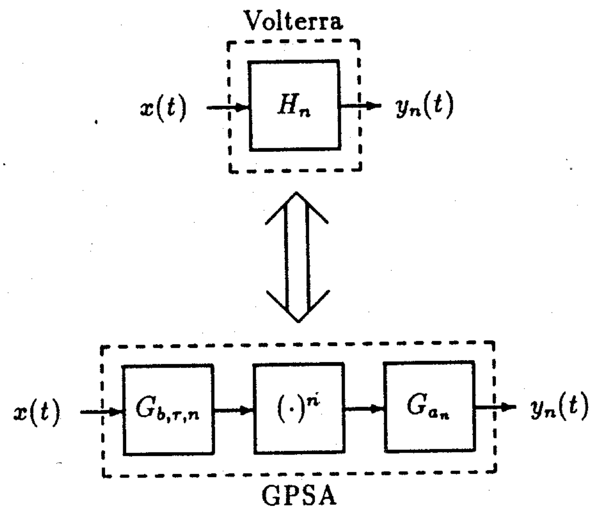

Figure 3. Equivalency of univariate Volterra analysis and generalized power series analysis (GPSA).

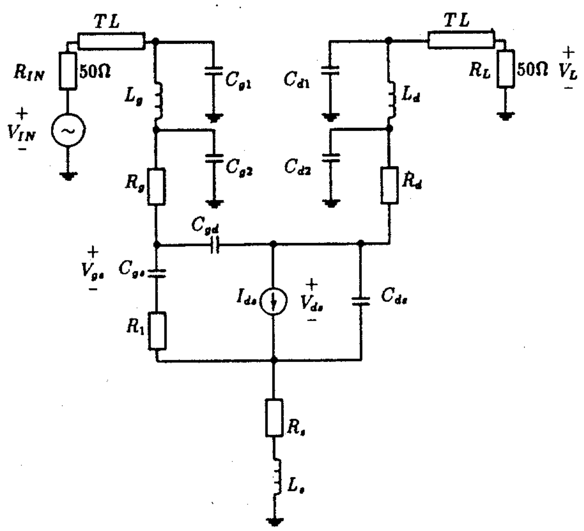

Figure 5. Full circuit for measurement of MESFET amplifier characteristics.

TABLE I. Element Values for MESFET Circuit

Element	Value
C_{g1}	0.1386 pF
L_g	0.69414 nH
C_{g2}	0.30707 pF
R_g	2.9 Ω
R_s	2.4 Ω
L_s	0.00323 nH
R_d	5.3 Ω
L_d	0.41143 nH
C_{d2}	0.09012 pF
C_{d1}	0.00341 pF
R_1	10 Ω
τ	6.56 ps
C_{ds}	0.25050 pF
C_{gs}	0.50150 pF
C_{gd}	0.08637 pF

G_{OUT} which has as its input I_{DS}. The two linear systems $G_{FB\text{-}DS}$ and $G_{FB\text{-}GS}$ provide the feedback to the two inputs of the bivariate block.

Our aim now is to solve for the steady-state output of the system given a steady-state input signal V_{IN}. The two feedback paths do not allow a straightforward closed-form solution. However, the steady-state output of the system can be solved relatively easily by iteration for low-input powers. For the first iteration, we assume that the feedback contributions are negligible and calculate the system response. Then, using the new calculated value of I_{DS}, we calculate a new V_{DS} and V_{GS} and use the new values for the next iteration. This process is continued until there is no appreciable change of any of the values from one iteration to the next.

Figure 7 gives the results of a two-tone test for the circuit using two equal-amplitude signals at input frequencies of 2.35 GHz and 2.4 GHz. The

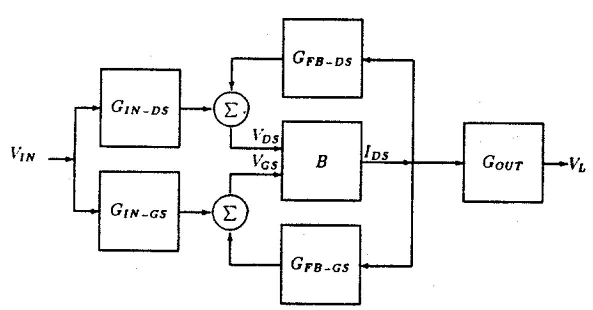

Figure 6. Nonlinear signal flow graph for MESFET amplifier. $G_{IN\text{-}GS}$, $G_{IN\text{-}DS}$, $G_{FB\text{-}GS}$, $G_{FG\text{-}DS}$, and G_{OUT} are linear systems. B is a bivariate nonlinear system.

TABLE II. Coefficients for Bivariate Power Series of I_{ds} About DC Operating Point

Order of V_{gs} Term	Order of V_{ds} Term: 0	1	2	3	4	5	6	7
0	0.0211092	0.00467136	0.00004985	0.00026358	0.00006759	−0.000138752	−0.000008659	0.0000154853
1	0.0689287	0.00307239	−0.000616581	0.00244648	−0.000132799	−0.000942032	0.0000306236	0.000105969
2	0.0552606	−0.0158083	0.00336844	0.00931433	−0.00214949	−0.00326047	0.000315678	0.000336661
3	−0.0241834	−0.0132546	0.00541917	0.0127004	−0.00146277	−0.00362775	0.000207059	0.000290322
4	−0.0328935	0.0236952	−0.00941775	−0.0136877	0.00546728	0.00528516	−0.000791546	−0.000575213
5	0.00865757	0.0340615	−0.0131895	−0.0366089	0.00563955	0.0110368	−0.000849452	−0.000954538
6	0.00689521	−0.00467676	0.0034243	−0.00190339	−0.00231212	−0.000264063	0.000324688	0.00010378
7	−0.00616748	−0.0241809	0.00856464	0.0253214	−0.00426292	−0.00775008	0.000655445	0.000683864
8	−0.0029458	−0.00895554	0.00252446	0.0105268	−0.001237	−0.00305257	0.00019882	0.000254013

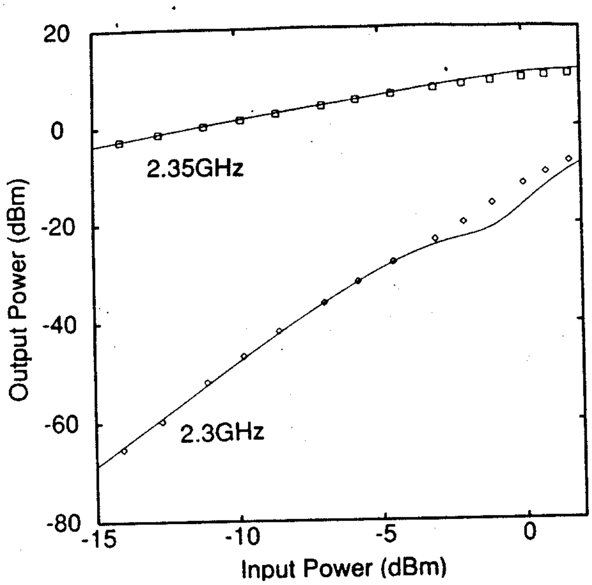

Figure 7. The results of a two-tone test for the circuit using two equal-amplitude signals at input frequencies of 2.35 GHz and 2.4 GHz. The horizontal axis is the input power for one of the tones, and the vertical axis is the output power at the fundamental frequency of 2.35 GHz and the image frequency of 2.3 GHz. The solid lines are the simulated values and the points are experimentally measured values.

horizontal axis is the input power for one of the tones, and the vertical axis is the output power at the fundamental frequency of 2.35 GHz and the image frequency of 2.3 GHz. The points are experimentally measured values and the solid lines are the simulated values and show good agreement. These simulated values also agree closely with those given by Chang et al. [20]. Note that a linear simulation would predict no power at 2.3 GHz.

Another technique for solving nonlinear circuits, that is based on the work of Volterra, is the method of nonlinear currents [19,21]. This technique has the advantage of a closed form solution to a nonlinear system which consists of (a) linear subsystems and (b) simple nonlinear systems described by univariate power series. This method has been used to analyze MESFET amplifiers [4], but since the nonlinear blocks are characterized by univariate power series, the V_{DS}-V_{GS} cross-product terms of the bivariate power series are ignored. Thus, the coefficients in Table II that are neither in the first row nor the first column are assumed to be negligible. The work presented here can be used to extend the method of nonlinear currents to include more general nonlinearities.

VI. CONCLUSION

The relationship between Volterra series analysis and generalized power series analysis has been established for both the single-input (univariate) and two-input (bivariate) system representations. A general power series is equivalent to a Volterra system if the form of the Volterra nonlinear transfer function is restricted to a linear system followed by an ideal integer exponentiation $(\cdot)^n$ followed by a second linear system. A similar restriction applies to the bivariate case.

The experimentally derived bivariate power series characteristics of the drain current of a MESFET amplifier can be used to calculate the bivariate Volterra nonlinear transfer function, and thus the output of a MESFET circuit can be represented by a nonlinear signal flow graph. If feedback is present in the flow graph, iterative techniques can be used to solve for the steady-state response entirely in the frequency domain.

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For biography and photo of **Michael Steer,** see page 37 of issue Number 1.