ANALYSIS OF NONLINEAR CIRCUITS DRIVEN BY MULTI-TONE SIGNALS USING GENERALIZED POWER SERIES

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ABSTRACT

This paper presents a frequency-domain nonlinear circuit analysis technique known as generalized power series analysis. This method is based on harmonic balance and treats nonlinear elements as well as linear elements in the frequency domain. It is suited to the analysis of analog circuits with an indefinite number of nonlinear elements driven by multiple signals that are not harmonically related. A unique feature is the availability of derivative information, allowing efficient minimization of the harmonic balance error.

INTRODUCTION

There are several methods currently available for analyzing nonlinear circuits driven by multi-tone signals. Included are the conventional time domain [1], [2] and Volterra series methods [3], [4] as well as the more recent techniques based on harmonic balance [5]-[8]. In general, the harmonic balance methods are more efficient and are more appropriate for the analysis of communication and high frequency circuits.

All of the harmonic balance techniques treat the linear elements in the frequency domain, but differ in their treatment of the nonlinear elements. In [5], [8] the nonlinear elements are analyzed in the time domain and Fourier Transforms used to interface with the linear elements. In [6] a least squares approximation is used to analyze the nonlinear elements. In [9], [10] a method is presented for expanding the nonlinearities in generalized power series (a power series having complex coefficients and time delays). This method has recently been coupled with harmonic balance to yield a powerful analysis technique [7], [11]. In this paper, we review the basic formulas associated with generalized power series and present for the first time the formulas used in a harmonic balance analysis based on generalized power series.

REVIEW OF GENERALIZED POWER SERIES

In this section, we review the basic formulas used in generalized power series analysis. From the method of Steer and Khan, the output y(t) of a system having an N component multifrequency input

$$x(t) = \sum_{k=1}^{N} x_k(t) = \sum_{k=1}^{N} |X_k| \cos(\omega_k t + \phi_k) \tag{1}$$

is expressed as the generalized power series [10]

$$y(t) = \sum_{l=0}^{\infty} \left[a_l \left\{ \sum_{k=1}^{N} b_k x_k(t - \tau_{k,l}) \right\}^l \right] \tag{2}$$

where $y(t)$ is the output of the system; l is the order of the power series terms; a_l is a complex coefficient; $\tau_{k,l}$ is a time delay that depends on both power series order and the index of the input frequency component; and b_k is a real coefficient. The phasor of the q th component of y of radian frequency ω_q is given by [10]

$$Y_q = \sum_{n=0}^{\infty} \sum_{\substack{n_1,\ldots,n_N \\ |n_1|+\ldots+|n_N|=n}} Y_q' \tag{3}$$

where $\omega_q = \sum_{k=1}^{N} n_k \omega_k$, a set of n_k's defines an intermodulation product, and n is the order of intermodulation. The summations are therefore over the infinite number of intermodulation products (the Y_q''s) yielding the q th output component. Generally when a nonlinear circuit is excited by a finite number of sinusoids, an infinite number of frequency components will be present. In order to analyze such a problem numerically, the number of frequency components considered in the analysis must be truncated. Here we consider N frequency components. Each intermodulation product in (3) is found from

$$Y_q' = \mathrm{Re}\left\{ \epsilon_n \left(\prod_{k=1}^{N} (X_k')^{|n_k|} \right) T \right\}_{\omega_q} \tag{4}$$

where

$$T = \sum_{\sigma=0}^{\infty} \left\{ \left(\frac{(n+2\sigma)!}{2^{(n+2\sigma)}} \right) a_{n+2\sigma} R_{n+2\sigma} Z \right\} \tag{5}$$

and

$$Z = \sum_{\substack{s_1,\ldots,s_N \\ s_1+\cdots+s_N=\sigma}} \left\{ \left(\prod_{k=1}^{N} \frac{|X_k|^{2s_k}}{s_k!(|n_k|+s_k)!} \right) \left(\prod_{k=1}^{N} b_k^{(|n_k|+2s_k)} \right) \right\} . \tag{6}$$

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In these expressions X_k is the phasor of x_k ,

$$X'_k = \begin{cases} X_k & n_k \geq 0 \\ X_k^* & n_k < 0 \end{cases} \quad (7)$$

$$R_{n+2\sigma} = \exp(-j \sum_{k=1}^{N} n_k \omega_k \tau_{k,n+2\sigma}) \quad (8)$$

$$\epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases} \quad (9)$$

and $\mathrm{Re}\{\ \}_{\omega_q}$ is defined such that for $\omega_q \neq 0$ it is ignored and for $\omega_q = 0$ the real part of the expression in braces is taken. The output of a nonlinear element described by generalized power series can thus be calculated using an algebraic formula if the input is a sum of sinusoids.

GENERALIZED POWER SERIES AND HARMONIC BALANCE

The analysis method presented in this paper is based on minimization of an objective function derived from the principle of harmonic balance. The nonlinear elements are described using generalized power series while the linear elements are handled using standard frequency domain nodal techniques. This results in an efficient analysis procedure which is described below. We show how the objective function is calculated and then present an efficient method for minimizing the objective function as well as an algorithm for implementing the analysis technique on a digital computer.

The analysis of a nonlinear circuit proceeds by dividing the circuit into linear and nonlinear subcircuits as shown in Fig. 1. One subcircuit is composed of the linear components along with any voltage or current sources, while the other is composed of nonlinear elements each of which is characterized by a generalized power series. The nonlinear subcircuit has M nodes and at the p th node the instantaneous current into the linear subcircuit is the sum of N frequency components so that

$$i_p = \sum_{q=1}^{N} \mathrm{Re}[I_{p,q} e^{j\omega_q t}] \ . \quad (10)$$

Similarly, the current into the nonlinear subcircuit at the p th node is

$$i'_p = \sum_{q=1}^{N} \mathrm{Re}[I'_{p,q} e^{j\omega_q t}] \quad (11)$$

where $I_{p,q}$ and $I'_{p,q}$ are the phasors of the q th frequency components of current flowing into the linear and nonlinear subcircuits respectively. We refer to these phasors as node current phasors. The voltage at the p th node is

$$v_p = \sum_{q=1}^{N} \mathrm{Re}[V_{p,q} e^{j\omega_q t}] \quad (12)$$

where $V_{p,q}$ is the phasor of the q th frequency component of voltage at the p th node (referred to as a node voltage phasor). Under steady state conditions, Kirchoff's current law must be satisfied, so $i_p + i'_p = 0$ for all p from 1 to M. Thus, the steady state solution of the circuit can be found by minimizing the harmonic balance error

$$E = \sum_{p=1}^{M} \sum_{q=1}^{N} |I_{p,q} + I'_{p,q}|^2 \ . \quad (13)$$

For efficient computation, the error function E is written as

$$E = \sum_{k=1}^{2} \sum_{p=1}^{M} \sum_{q=1}^{N} F_{k,p,q}^2(V) = \sum_{i=1}^{2MN} G_i^2(V) \quad (14)$$

where

$$F_{1,p,q}(V) = \mathrm{Re}(I_{p,q} + I'_{p,q}) \quad (15)$$

and

$$F_{2,p,q}(V) = \mathrm{Im}(I_{p,q} + I'_{p,q}) \ . \quad (16)$$

The elements $G_i(V)$ are equal to the elements $F_{k,p,q}$ where the subscript i represents a unique choice for k,p, and q. In these expressions, V is the vector of the node voltage phasors. Evaluation of the error function (14) as a function of the node voltage phasors requires calculation of the node current phasors given the node voltage phasors. For the linear subcircuit the node current phasors are easily calculated using standard frequency domain nodal techniques whereas the currents in the nonlinear subcircuit can be calculated using the algebraic formula (3)-(9) since the nonlinear elements are described by generalized power series.

Minimization of the objective function can be achieved using a variety of iteration schemes. One suitable technique to minimize such a sum of squares is the Newton-Raphson method. This method seeks to find the minimum of E with respect to V using the iterative procedure

$$^{i+1}\begin{bmatrix} \mathrm{Re}(V_{1,1}) \\ \mathrm{Im}(V_{1,1}) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Re}(V_{p,q}) \\ \mathrm{Im}(V_{p,q}) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Re}(V_{M,N}) \\ \mathrm{Im}(V_{M,N}) \end{bmatrix} = {}^{i}\begin{bmatrix} \mathrm{Re}(V_{1,1}) \\ \mathrm{Im}(V_{1,1}) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Re}(V_{p,q}) \\ \mathrm{Im}(V_{p,q}) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Re}(V_{M,N}) \\ \mathrm{Im}(V_{M,N}) \end{bmatrix} - J^{-1}({}^iV)G({}^iV) \quad (17)$$

where the leading superscripts are iteration numbers. The matrix J is the Jacobian where the element in the (2j-1) th row and k th column at the i th iteration is

$$\left[J({}^iV)\right]_{2j-1,k} = \frac{\partial G_{2j-1}({}^iV)}{\partial \mathrm{Re}({}^iV_k)} \tag{18}$$

and at the (2j) th row and k th column

$$\left[J({}^iV)\right]_{2j,k} = \frac{\partial G_{2j}({}^iV)}{\partial \mathrm{Im}({}^iV_k)} . \tag{19}$$

Calculation of the Jacobian requires partial derivatives of the node current phasors with respect to the real and imaginary parts of the node voltage phasors for all nodes of the nonlinear subcircuit and all frequency components. Partial derivatives of a nonlinear node current phasor with respect to the real and imaginary parts of a node voltage phasor are obtained by differentiating the algebraic formula (3)-(9) and using the chain rule. Using the notation in (1)-(9), the derivative of the phasor of the q th component of the output of the nonlinearity with respect to the magnitude of the phasor of the m th input component, $X_m = |X_m|e^{j\phi_m}$, is found from (3) by differentiating

$$\frac{\partial Y_q}{\partial |X_m|} = \sum_{n=0}^{\infty} \sum_{\substack{n_1, \ldots, n_N \\ |n_1| + \cdots + |n_N| = n}} \frac{\partial Y_q'}{\partial |X_m|} \tag{20}$$

where

$$\frac{\partial Y_q'}{\partial |X_m|} = \frac{|n_m|}{|X_m|} Y_q' + \epsilon_n \left(\prod_{k=1}^{N} X_k'^{|nk|} \right) \left(\sum_{\sigma=0}^{\infty} a_{n+2\sigma} \frac{(n+2\sigma)!}{2^{(n+2\sigma)}} \frac{\partial Z}{\partial |X_m|} \right) \tag{21}$$

and

$$\frac{\partial Z}{\partial |X_m|} = \sum_{\substack{s_1, \ldots, s_N \\ s_1 + \cdots + s_N = \sigma}} \left\{ \prod_{\substack{k=1 \\ k \neq m}}^{N} \left(\frac{|X_k|^{2s_k}}{s_k!(|n_k| + s_k)!} \right) \left(\frac{2s_m |X_m|^{2s_m - 1}}{s_m!(|n_m| + s_m)!} \right) \left(a_{n+2\sigma} R_{n+2\sigma} \prod_{k=1}^{N} b_k^{(|n_k| + 2s_k)} \right) \right\} \tag{22}$$

Similarly, the derivative of the phasor of the q th component of the output of the nonlinearity with respect to the angle of the phasor of the m th component of the input is found to be

$$\frac{\partial Y_q}{\partial \phi_m} = \sum_{n=0}^{\infty} \sum_{\substack{n_1, \ldots, n_N \\ |n_1| + \cdots + |n_N| = n}} \frac{\partial Y_q'}{\partial \phi_m} \tag{23}$$

where

$$\frac{\partial Y_q'}{\partial \phi_m} = j n_m Y_q' . \tag{24}$$

These derivatives calculated for the nonlinear elements can be converted to derivatives with respect to the real and imaginary components of the node voltage phasors by using the chain rule.

The derivative of the current through a linear admittance (Y) with respect to the real component of the phasor voltage across it is

$$\frac{\partial I_q}{\partial \mathrm{Re}(V_m)} = \begin{cases} Y & m=q \\ 0 & m \neq q \end{cases} \tag{25}$$

while the derivative with respect to the imaginary component is

$$\frac{\partial I_q}{\partial \mathrm{Im}(V_m)} = \begin{cases} jY & m=q \\ 0 & m \neq q \end{cases} . \tag{26}$$

Thus, the derivatives for the linear subcircuit are easily found from its y-parameters.

An algorithm using the above procedure for nonlinear circuit analysis is outlined in Fig. 2. A program has been written implementing this algorithm and has been used to successfully analyze nonlinear microwave circuits containing a MESFET with large signal excitation (for example, see [7], [11]).

CONCLUSION

We have presented a novel technique, termed generalized power series nonlinear analysis, for analyzing nonlinear analog circuits under large signal conditions. The technique operates entirely in the frequency domain which enables circuits having multifrequency excitation to be simulated. Because the nonlinear elements are represented by generalized power series, derivative information is available. This allows an efficient optimization algorithm to be used. The technique is applicable to large signals and to strong nonlinearities.

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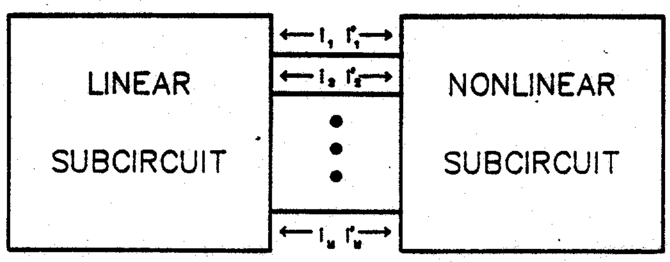

Figure 1
A nonlinear circuit separated into linear and nonlinear subcircuits. The instantaneous current into the linear subcircuit at the p th node is i_p, while i'_p flows into the nonlinear subcircuit.

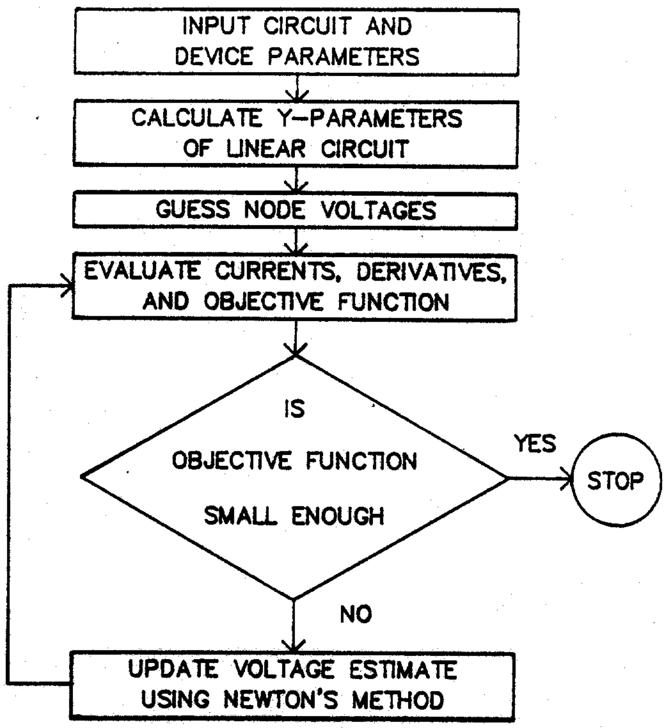

Figure 2
Algorithm for the generalized power series nonlinear circuit analysis using a minimization procedure.