## Parasitic Effects Upon the High-Frequency Performance of Three-Terminal Devices

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## ABSTRACT

Estimates of transistor performance at millimeter-wave frequencies are generally based on extrapolation of low microwave frequency gain measurements assuming a 6 dB per frequency octave gain roll-off. This approach leads to optimistically high predictions of millimeter-wave performance. In this paper we show that a complex conjugate pole-pair, due to domain capacitance, leads to a 12 dB/octave gain roll-off in the millimeter-wave region. As a result the actual f<sub>max</sub> of millimeter-wave transistors can be considerably less than that determined using extrapolation of microwave frequency gain measurements. It is shown that current MESFET, HEMT and PBT designs are limited to frequencies below 150 GHz.

#### INTRODUCTION

Predictions of the performance of microwave transistors at millimeter-wave frequencies have typically been based on extrapolation of microwave gain measurements at a gain roll-off of 6 dB per frequency octave. These suggest that reasonable gain can be obtained at frequencies well above 100 GHz [1,2,3,4,5,6,7,8]. Here we demonstrate that the charge dipole domain that is known to form in the channel of FET-type devices dominates high frequency performance and is a fundamental limitation to obtaining high frequency operation.

In this paper we clarify the various definitions used for transistor gain and prove the equivalence of maximum available gain and unilateral gain definitions of  $f_{max}$  We also examine the high frequency performance of state-of-the-art MESFET, HEMT (High Electron Mobility Transistors) and PBT (Permeable Base Transistors) transistors using physical models developed from microwave measurements.

#### GAIN DEFINITIONS

Three gain definitions, maximum stable gain -  $G_{MS}$ , maximum available gain -  $G_{MA}$ , and Mason's gain (also referred to as unilateral gain) - U are used to quantify the gain performance of microwave transistors. Here we clarify these definitions and prove that  $f_{max}$  is the frequency at which all three gains are unity. This is an important result as  $f_{max}$  is traditionally defined as the frequency at which U is unity whereas extrapolations of  $G_{MS}$ , the gain most conveniently measured, is used to estimate the  $f_{max}$  of a transistor.

G<sub>MA</sub> is the maximum available gain of an active device with input and output matching networks but with no feedback. It is defined as [9, 10]

$$G_{MA} = \frac{|S_{21}|}{|S_{22}|} \left( k \pm \sqrt{(k^2 - 1)} \right)$$
 (1)

where  $S_{11}$ ,  $S_{12}$ ,  $S_{21}$  and  $S_{22}$  are the S-parameters of the active device, the stability factor of the active device alone is  $k = [1 - |S_{11}|^2 - |S_{22}|^2 + |D|^2]/[2|S_{11}|^2|S_{21}|^2]$  and  $D = S_{11}S_{22} - S_{12}S_{21}$ . Only when k > 1 is the active device unconditionally stable so that  $G_{MA}$  is only defined for k > 1. This usually occurs only at high frequencies near  $f_{max}$ . At low frequencies where the active device is potentially unstable an alternative definition of maximum available gain, the maximum stable available gain,  $G_{MS}$ , is used.  $G_{MS}$  is defined in the same way as  $G_{MA}$  but with the added condition that unconditionally stable gain is ensured by resistively loading the active device so that the stability factor of the new circuit (the active device plus the loading resistors) is 1. The mathematical definition of  $G_{MS}$  is given by Vendelin [9] and Ha [10] with the obvious requirement that  $G_{MS}$  can never be larger than  $G_{MA}$  so that

$$G_{MS} = \begin{cases} \frac{|S_{21}|}{|S_{12}|} & \text{for } |k| \le 1\\ \frac{|S_{21}|}{|S_{12}|} \left(k - \sqrt{(k^2 - 1)}\right) & \text{for } |k| > 1 \end{cases}$$
 (2a)

In the simple theories [9, 11], at frequencies at which k < = 1 G<sub>MS</sub> rolls off at 3 dB per frequency decade where as for k > 1 G<sub>MS</sub> and G<sub>MA</sub> roll off at 6 dB per frequency octave.

U is the highest possible gain that an amplifier using the active device and reactive tuning could ever achieve and is given by [12]

$$U = \frac{|S_{21}/S_{12} - 1|^2}{2k|S_{21}/S_{12} - 2Re(S_{21}/S_{12})|}$$
(3)

U is the maximum available power gain when the two-port has been simultaneously conjugately matched and a feedback network across the active device adjusted so that the transistor amplifier is unilateral.

# $f_{\text{max}}$

The frequency where U becomes unity is the maximum frequency of oscillation,  $f_{max}$ , and so the highest frequency at which useful gain can be obtained is somewhat less than this. U and  $f_{max}$  are the figures of merit by which the gain and frequency performance of different transistors are compared. U however, is a difficult quantity to measure so measurements of  $G_{MS}$  are normally made and the gain at the highest measured frequency used to extrapolate, at 6 dB per octave, to the gain performance at millimeter-wave frequencies. Thus  $f_{max}$  can be estimated. Although it is commonly believed that U is unity at a higher frequency than is  $G_{MS}$  (or  $G_{MA}$ ) [9, 11], it is a simple matter to show that the U,

 $G_{MA}$  and  $G_{MS}$  definitions of  $f_{max}$  are equivalent. The proof is as follows. For U=1 we have, from (3)

$$2k \left| \frac{S_{21}}{S_{12}} \right| - 2Re \left( \frac{S_{21}}{S_{12}} \right) = \left[ Re \left( \frac{S_{21}}{S_{12}} - 1 \right) \right]^2 + \left[ Im \left( \frac{S_{21}}{S_{12}} \right) \right]^2$$

This simplifies to

$$\left| \frac{S_{21}}{S_{12}} \right| - 2k \left| \frac{S_{21}}{S_{12}} \right| + 1 = 0$$

This has solutions  $|S_{21}/S_{12}| = k \pm \sqrt{k^2 - 1}$ . This implies that k > 1 so that  $G_{MA}$  is defined when U = 1. Only one of the solutions for  $|S_{21}/S_{12}|$  is correct as the solution that occurs at the lowest frequency corresponds to  $f_{max}$ . Since k is continuous and increases with frequency through k = 1 for practical devices<sup>1</sup>,  $|S_{21}/S_{12}| = k + \sqrt{k^2 - 1}$ . Using this solution in (1) we see that  $G_{MA} = 1 = G_{MS}$  at  $F_{MAX}$ . Thus the unilateral gain, maximum stable available gain, and maximum available gain definitions of  $F_{MAX}$  are equivalent<sup>2</sup>.

The above result has a significant implications. Since  $G_{MS}$  (= $G_{MA}$ ) and  $G_{U}$  are both unity at  $F_{MAX}$ , but are not equivalent at all frequencies, they cannot have the same frequency roll-off as is predicted by first order theories when  $|\mathbf{k}| > 1$  [9, 10, 11].

Variation in the gain roll-off rate can also be seen analyticly for transistor amplifiers. Previously [13] we developed an analytic expression for the unilateral gain of a common source transistor with the circuit model of Fig. 1 but without the parasitics  $R_S$ ,  $R_D$ ,  $R_G$ ,  $R_{GD}$ , and  $C_{DS}$  (the remaining circuit models the intrinsic transistor). We showed that the transistor has a four pole response

$$U = \left[ \frac{g_{m0}^2 R_{DS}}{4 C_{GS} R_{I} (C_{GS} - C_{DC} g_{m0} R_{DS})} \right] \left[ \frac{1}{\omega^2 (1 - p^2 \omega^2)} \right]$$
(4)

where

$$p^{2} = \frac{(R_{I}^{2}C_{GS})(C_{DC} + C_{GS})^{2} + \frac{1}{2}C_{DC}g_{m0}R_{DS}\tau^{2}}{(C_{DC}g_{m0}R_{DS} - C_{GS})}$$
(5)

At frequencies much less than  $(2\pi R_I C_{GS})^{-1}$  and ignoring  $C_{DC}$ ,  $G_U$  reduces to [9, 10]

$$U = \frac{g_{m0}^2 R_{DS}}{4\omega^2 C_{GS}^2 R_I} \tag{6}$$

<sup>&</sup>lt;sup>1</sup>This applies to all active devices which are not unconditionally stable at low frequencies and for which k is continuous. Alternatively we could argue that  $|S_{21}/S_{12}|$  monotonicly reduces with frequency for practical devices. The effect of these arguments is to force the "+" solution to occur at a lower frequency than the "-" solution.

<sup>&</sup>lt;sup>2</sup>This is also true for the maximum unilateral transducer power gain.

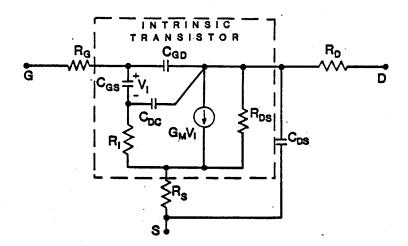


Fig. 1. Circuit model of monolithicly integrated MESFET, HEMT or PBT.  $G_{\rm M} = G_{\rm M0} {\rm e}^{-{\rm j}\omega\tau}$ .  $R_{\rm G}$ ,  $R_{\rm D}$  and  $R_{\rm S}$  are parasitic resistances resulting largely from bulk semiconductor resistance and metallization resistance.  $R_{\rm I}$  is the resistance of the semiconductor channel.  $C_{\rm GD}$  and  $C_{\rm DS}$ , are essentially parasitic fringing capacitances.

This is the commonly used expression for U and rolls off at 6 dB/octave because of the  $1/\omega^2$  term. However with  $C_{DC}$  in the expression for  $G_U$ , (4), there is an additional 6 dB/octave roll-off at higher frequencies due to the complex pole-pair contained in the  $1/(1-p^2\omega^2)$  term. The complex-conjugate poles are at frequency

$$f_{\mathbf{p}} = \frac{1}{2\pi |\mathbf{p}|} \tag{7}$$

As will be seen this pole frequency is typically below for millimeter-wave transistors  $f_{max}$  so that the pole has a limiting effect on frequency performance.

## III MILLIMETER-WAVE PERFORMANCE OF ACTUAL TRANSISTORS

In this section we investigate the millimeter-wave performance of state-of-the-art HEMT [8] (transistor T1), MESFET [3] (transistor T2) and [2] (transistor T3), and PBT [4] (transistor T4) transistors. Our calculated results are based on models developed by fitting S-parameter measurements between 2 and 18 GHz to the circuit of Fig. 1 yielding the element values of the model are given in table 1.

In Fig. 2 we present a comparison of U calculated using the model of the intrinsic transistor with that calculated using the multiple pole formula (4). Correlation is excellent with the pole frequency calculated using the model being 89 GHz compared to 86 GHz obtained from (7).

The calculated  $G_{MS}$  and  $G_{U}$  versus frequency responses of the transistor are plotted in Fig. 3-5. The frequency response of the transistors below 20 GHz conforms to measurements and to first order theory which predicts a 6 dB per octave roll-off of  $G_{U}$  and a 3 dB per octave roll-off of  $G_{MS}$  [9, 10]. However in the millimeter-wave region the roll-off in  $G_{U}$  approaches 12 dB per octave because of the pole-pair identified in the previous section. The frequency of the  $G_{U}$  resonance is just the frequency of the pole-pair.

TRANSISTOR DATA				
DEVICE	T1	T2	T3	T4
SOURCE	Camnitz,[5]	Feng,[3]	Maki,[2]	Bozler,[4]
STRUCTURE	0.33x300µm	0.3x150μm	$0.25$ x $60$ $\mu$ m	$8x40\mu m^3$
TYPE	HEMT	MESFET	MESFET	PBT <sup>4</sup>
PARAMETER				
$R_{s}(\Omega)$	1.7	2.39	4.55	1.3
$R_{\mathbf{G}}(\Omega)$	$2^{1}$	2.9	1.46	1.3
$R_{\mathbf{D}}(\Omega)$	1.72	$2.39^{2}$	6.7	O <sub>2</sub>
$R_{I}(\Omega)$	1.12	0.94	2.69	O <sub>2</sub>
$R_{DS}(\Omega)$	72.3	258	556	432
C <sub>GS</sub> (pF)	0.34	0.147	0.071	0.97
C <sub>GD</sub> (pF)	0.063	0.009	0.001	0.015
$C_{DS}(pF)$	0.139	$0.05^{2}$	0.025	0.036
$C_{DC}(pF)$	0.12	$0.03^{2}$	0.011	O <sub>2</sub>
$G_{M0}$ (mS)	171	25.8	15.2	181
τ (ps)	3.62	32	1.25	4.27
f <sub>max</sub>			· .	`
@1 GHz	148	132	400	430
Calculated	73	68	97	125

Table 1. Transistor data for state-of-the-art HEMT, MESFET and PBT transistors. The parameters refer to the model in Fig. 1.  $f_{max}$ 's calculated using the model and extrapolated using the unilateral gain calculated at 1 GHz are presented for the transistor models. Most of the elements in the models were obtained from S-parameter measurements between 2 and 18 GHz but, where necessary, were estimated based on the parameters of similar devices. Based on expected [5] reduction of  $R_G$  from  $7\Omega$  (actual) to  $2\Omega$ . Estimated. The PBT has a structure very different from that of a HEMT or MESFET. For PBT's it is usual to use the subscripts c, b and e (for collector base and emitter) in place of s, g and d (for source, gate and drain). Estimate not available. Estimate, based on geometry. Estimate, obtained from [14]. Estimate of domain capacitance not available.

Extrapolated (from  $G_U$  at 1 GHz at a roll-off of 6 dB per octave) and calculated (using the model) values of  $f_{max}$  are compared in table 1. Extrapolated  $f_{max}$ 's are up to 400 GHz, considerably greater than the calculated  $f_{max}$ 's of around 100 GHz. These results have significant implications for interpretations of  $f_{max}$  based on low frequency gain measurements and extrapolation using the 6 dB per octave roll-off assumption. Such an extrapolation leads to erroneously high predictions of the millimeter-wave performance of microwave transistors. The use of models to calculate the millimeter-wave performance of microwave transistors is a method of extrapolation considerably more accurate than linear extrapolation of measured or calculated low frequency gain. Essentially this is because the models are based on physical understanding of the transistor, in addition to measurements.

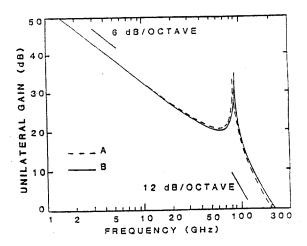


Fig. 2. Calculated  $G_U$  for the intrinsic transistor of the MESFET of Maki et al. [2]. (a)  $G_U$  calculated using the formula, (5), developed here. (b)  $G_U$  calculated using the intrinsic transistor circuit model.

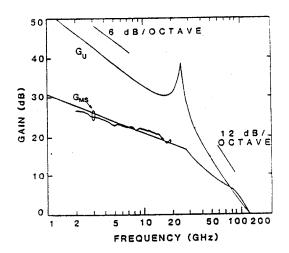


Fig 4. Calculated and measured [4] maximum stable gain, G<sub>MS</sub>, and calculated Mason's gain, U, responses for the PBT transistor T4.

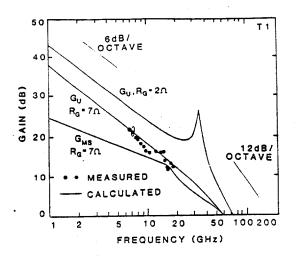


Fig. 3. Gain response of the HEMT transistor T1. Measured [5] and calculated Mason's gain, U, and the calculated maximum stable gain,  $G_{MS}$ , are plotted against frequency for  $R_G = 7\Omega$ .  $G_U$  is also plotted for  $R_G = 2\Omega$ , the expected [5] improvement in  $R_G$ .

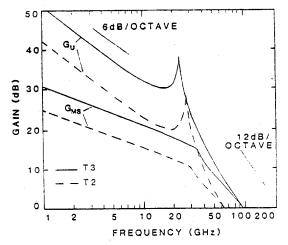


Fig. 5. Calculated Mason's gain, U, and maximum stable gain,  $G_{MS}$ , responses for the MESFET transistors T2 and T3.

## PARAMETRIC STUDY

A parametric study of Maki's MESFET was undertaken and results are presented in Fig. 6. These results are typical of those for all of the transistors we have examined. In Fig. 6, curve B is the calculated U versus frequency response of the actual transistor model. The other curves are the calculated responses with one element at a time eliminated from the model. The most significant observation is that there is no simple relation between low frequency gain and  $f_{max}$ . For example, elimination of the domain capacitance, curve E, reduces the low frequency gain of the transistor by eliminating a positive feedback path internal to the transistor. However this has virtually no effect on  $f_{max}$  Similarly eliminating  $\tau$ , curve A, increases the low frequency gain but  $f_{max}$  is unaltered. Significant increases in  $f_{max}$  result from the elimination of a parasitic, either  $C_{DS}$  or  $R_D$ . Notably, elimination of either of these elements has little effect on low frequency gain.

## **CONCLUSION**

The major contribution of this work is showing that low microwave frequency gain measurements and the commonly used 6 dB/octave gain roll-off assumption cannot be used to predict the millimeter-wave performance of microwave transistors. It was also shown that  $f_{max}$  can be increased significantly by reducing drain parasitics.

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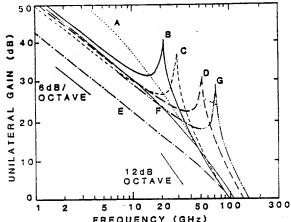


Fig. 6. Calculated Mason's gain, U, responses of the MESFET transistor T3 with one element at a time eliminated from the transistor model. Curve B is the response of the actual transistor. The other curves are for the actual transistor but for curve A,  $\tau = 0$  ( $f_{max} = 96$  GHz); curve C,  $C_{GD} = 0$  ( $f_{max} = 97$  GHz); curve D,  $C_{DS} = 0$  ( $f_{max} = 140$  GHz); curve E,  $C_{DC} = 0$  ( $f_{max} = 97$  GHz); curve F,  $R_S = 0$  ( $f_{max} = 111$  GHz); and for curve G,  $R_D = 0$  ( $f_{max} = 163$  GHz).

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