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MULTIFREQUENCY ANALYSIS OF NONLINEAR CIRCUITS

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ABSTRACT

All electrical circuits are inherently nonlinear and frequently their design requires nonlinear circuit analysis. In addition, some systems have large multi-frequency excitation, and this paper considers the analysis of such circuits. In this paper a new frequency domain analysis method, based on functional minimization, is developed. Results are presented for the analysis of a diode mixer and our method is compared to those of others.

INTRODUCTION

All electrical circuits and physical systems are inherently nonlinear as a result of the very nature of the electronic components used. In most analog electrical circuits the nonlinearity becomes evident as a departure from ideal performance as the input signal becomes large. The offending signal may not be just the desired information carrier but may be an undesired signal resulting, in the case of communication receivers, from a nearby transmitter. In some systems one or more nonlinear devices form the basis of operation of the system. This is the case with diode mixers where typically an information carrier at a high frequency is coupled, by way of the nonlinear diode, to a large local oscillator signal to produce a low frequency signal that is more conveniently processed. When the circuit nonlinearity is significant, the design process requires an analysis procedure which accommodates large input signals.

This paper presents a power-series-based numerical nonlinear analysis technique that can be used with multifrequency large-signal excitation. The convergence properties of the method are excellent and are illustrated via an analysis of a pumped waveguide diode mixer and by comparing our method with those of earlier workers. The research reported here was driven by the need for computer aided design tools for microwave monolithic integrated circuits wherein device nonlinearities are significant, active components operate near their frequency limits, and it is not feasible to breadboard circuits.

REVIEW

The analysis of nonlinear systems is by no means a new area of science or mathematics. Since the end of the 19 th century Van der Pol, and others, have investigated various forms of second order nonlinear differential equations as many simple but important nonlinear systems can be so described. Initially graphical techniques were used to solve the equations with a periodic excitation. Now analogue computers, or digital computers using numerical integration, are used and higher order nonlinear differential equations can be handled. However, it is difficult to formulate a single differential equation for a system containing more than two or three elements (even if only one of the elements is nonlinear), or solve a system of coupled differential equations.

Numerical nonlinear analysis methods can be classed as either time domain or frequency domain depending on how the nonlinear element is handled. Time domain methods generally use numerical integration or, where possible, calculate the instantaneous value of the output from the instantaneous value of the input. An example of a computer aided analysis technique using this approach is the popular computer program "SPICE". Unfortunately analysis can be very slow and convergence problems can be significant. Several specialized circuit analyses treat nonlinear elements in the time domain yet analyze the linear part of the circuit in the frequency domain using numerically efficient nodal techniques (e.g. [1]-[4]). With these analyses the conversion between the frequency domain solution of the linear embedding network, and the time domain solution of the nonlinearity, is accomplished using fast Fourier transforms (FFT's). These time domain methods are limited to systems with only harmonically-related (or in the case of Ushida and Chua's analysis [4] nearly harmonically-related) components, as then the response of the system is periodic for a few cycles of the input components. With all these methods convergence is still a problem and analysis of a nonlinear circuit requires a skilled operator with knowledge of both circuit operation and of the convergence properties of the numerical analysis package being used.

Frequency domain nonlinear analyses use functional expansions of the input-output characteristic of the nonlinear element. Generally, the function itself is the summation of basis functions of the input, and the response due to each functional component of the expansion are summed to yield the total response of the system. Perhaps the most general method is that of Antonov and Ponkratov [5] who derived a formula for the output of a system described by the functional relation $u(t) = F(x(t))$ where $x(t)$ is a sum of sinusoids, and $F(.)$ is a function which can be expressed as a possibly infinite sum of orthonormal functions. Their output formula involved multiple infinite sums of integer order Bessel functions. As a result of the Bessel functions, convergence of the summations will be slow with poor numerical accuracy.

Two other frequency domain nonlinear analysis methods, using power series and Volterra series, can be viewed as special cases of the system described by Antonov and Ponkratov in that they use subsets of the infinite set of orthonormal functions. The simplest functional expansion is the representation of $u(t)$ as a power series in $x(t)$. Conventional power series (i.e. a power series with real coefficients and not incorporating time delays) expansions can only be used with frequency independent systems with single valued input-output characteristics (that is, without hysteresis). Other basis function expansions, such as the expansion of the Shockley diode equation in terms of Bessel functions [6], [7], have been used but these are generally restricted to systems with particular idealized input-output characteristics.

In 1930 Volterra introduced functional expansions that could be used with a large class of nonlinear systems. His work was developed further by Weiner in the 1950's for the expansion of functionals in terms of orthogonal polynomial series. Weiners functional expansions, now known as Volterra nonlinear transfer functions, while having a form similar to that of a power series, can handle frequency dependent systems with single valued input-output characteristics. Unfortunately, Volterra nonlinear transfer function analyses are restricted to weakly nonlinear systems because of the algebraic complexity of determining Volterra nonlinear transfer functions of higher order than 3 (as required with more strongly nonlinear systems or large signals). Because of this, systems are usually described by fixed, typically third order, Volterra series, although no indication of the error involved in doing this is available. For more strongly nonlinear systems, or large signals, the number of terms that must be considered becomes very large and the analysis becomes unwieldy. However the great importance of Volterra series analysis is that it can be systematically used to analyze fairly complex systems with possibly non-commensurable (that is not linearly related) frequencies of the input components.

Yet another frequency domain nonlinear analysis was introduced by Steer and Khan [8], who used a power series expansion of the input-output characteristics of a nonlinear system. In the method of Steer and Khan the output $u(t)$ of a system having an N component multi-frequency input

$$x(t) = \sum_{k=1}^{N} x_k(t) = \sum_{k=1}^{N} |X_k| . \cos(\omega_k t + \varphi_k)$$

is [8]

$$u(t) = A \sum_{\ell=0}^{\infty} \left[a_\ell \left\{ \sum_{k=1}^{N} b_k x_k(t) \right\}^{\ell} \right] \qquad (1)$$

where $u(t)$ is the output of the system; ℓ is the order of the power series terms; A_i and $a_{\ell,i}$ are complex coefficients; and $b_{k,i}$ is a real coefficient. Note that $|X_k|$ is the peak magnitude of an input sinusoid so that a dc input component has $\omega_k = 0$ and $\varphi_k = 0$ or π radians. The complex coefficients are defined as introducing a phase shift in the frequency components of the output. The rigorous definition of the complex coefficients is embodied in the following algebraic formula for the output of the power series. The phasor of the q th component of u of frequency ω_q is given by [8]

$$U_q = \sum_{n=0}^{\infty} \sum_{\substack{n_1,\ldots,n_N \\ |n_1|+\ldots+|n_N|=n}} U'_q \qquad (2)$$

where $\omega_q = \sum_{k=1}^{N} n_k \omega_k$ (a set of n_k's defines an intermodulation product),

$$U'_q = \mathrm{Re}\left[\epsilon_n \left\{ \prod_{k=1}^{N} X_k'^{\,|n_k|} \right\} . T \right]_{\omega_q}$$

is a single intermodulation product.

$$T = A \sum_{\sigma=0}^{\infty} \frac{(n+2\sigma)!}{2^{(n+2\sigma)}} . z$$

$$z = \sum_{\substack{S_1,\ldots,S_N \\ S_1+\ldots+S_N = \sigma}} a_{n+2\sigma} \left\{ \prod_{k=1}^{N} \left[\frac{|X_k|^{2S_k}}{S_k!(|n_k|+S_k)!} \right] \prod_{k=1}^{N} (b_k)^{(|n_k|+2S_k)} \right\}$$

X_k is the phasor of x_k,

$X'_k = X_k$ if $n_k \geq 0$

$= X_k^*$ if $n_k < 0$,

ϵ_n ($\epsilon_n = 1, n=0; \epsilon_n = 2, n \neq 0$) is the Neumann factor,

n is the order of intermodulation.

and $\mathrm{Re}[.]_{\omega_q}$ is defined such that it is ignored for $\omega_q \neq 0$ but for $\omega_q = 0$ the real part of the expression in square brackets is taken. This algebraic formula (together with the harmonic balance iteration procedure of Hicks and Khan [1]) has previously been used to successfully analyze a resistive mixer [9]. While convergence could always be obtained with a large input signal in addition to the local oscillator excitation, considerable operator intuition was required to successfully select the convergence parameters described in [1].

DEVELOPMENT OF METHOD

For our purposes, a current controlled nonlinear element embedded in a linear circuit can be viewed as single frequency Thevenin equivalent circuits separated from each other and the nonlinear elements by ideal bandpass filters, as shown in figure 1. E_q, Z_q, I_q and V_q are all phasor quantities of the q th component of frequency ω_q. The time-domain total current and voltage at the nonlinear element are given by

$$i = \sum_{q=1}^{N} \mathrm{Re}\left[I_q e^{j\omega_q t} \right]$$

and

$$v = \sum_{q=1}^{N} \left[\mathrm{Re}(V_q e^{j\omega_q t}) \right]$$

Since the nonlinear element is current-controlled, we can write $V_q = fn(I_1,\ldots,I_q,\ldots,I_N)$ where we consider N frequency components. Under steady state conditions $V_q - E_q + I_q Z_q = 0$ for all q. Alternatively, for a voltage-controlled nonlinear element we use the multifrequency Norton equivalent circuit of figure 2, where $I_q = fn'(V_1,\ldots,V_q,\ldots,V_N)$. Now under steady state conditions $I_q - I_{gq} + V_q Y_q = 0$ for all q.

For both equivalent circuits, the steady state condition can be obtained by minimizing the function

$$P = \sum_{q=1}^{N} |Q_q|^2$$

where

$$Q_q = U_q - G_q + X_q W_q$$

and U_q is the nonlinear term, G_q is the source term, X_q is the nonlinear input, and W_q is the immittance for the q th component. In the Thevenin equivalent circuit, the source $G_q = E_q$, immittance $W_q = Z_q$, nonlinear input $X_q = I_q$, and nonlinear output $U_q = V_q$, for the qth frequency component. The Norton circuit has $G_q = I_{gq}$, $W_q = Y_q$, and $X_q = V_q$ and $U_q = I_q$.

Minimization of P using efficient minization techniques (e.g. [10]) requires evaluation of P as a function of the X's as well as partial derivatives of P with respect to the amplitude and phase of the X's. In the following we present algebraic formulas for P and its derivatives with respect to the amplitude, X_m, and phase, φ_m of the m th component of the input to the nonlinearity are given by

$$\frac{\partial P}{\partial |X_m|} = \sum_{q=1}^{N} \left(\frac{\partial Q_q Q_q^*}{\partial |X_m|} \right) = \left\{ \sum_{q=1}^{N} 2\ \mathrm{Re}\left[Q_q^* \frac{\partial Q_q}{\partial |X_m|} \right] \right\}$$

and

$$\frac{\partial P}{\partial \varphi_m} = \sum_{q=1}^{N} 2\ \mathrm{Re}\left[Q_q^* \frac{\partial Q_q}{\partial \varphi_m} \right]$$

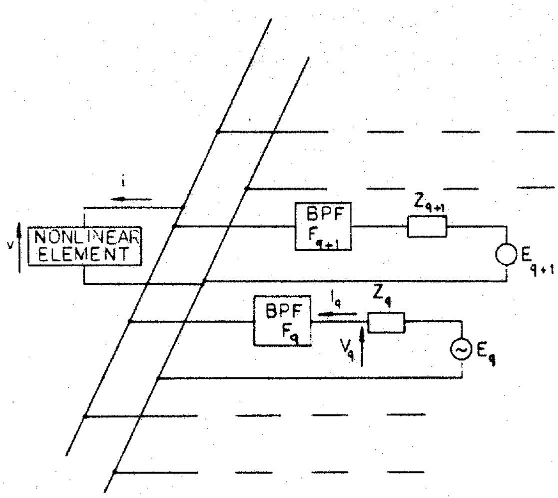

Figure 1
Multifrequency Thevenin equivalent circuit of a current-controlled nonlinear element embedded in a linear circuit.

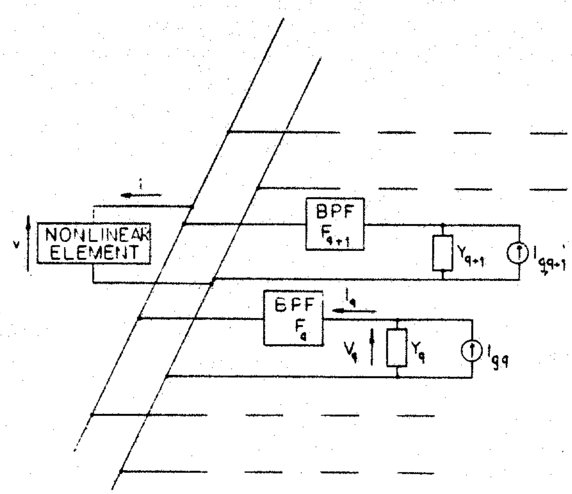

Figure 2
Multifrequency Norton equivalent circuit of a voltage-controlled nonlinear element emebedded in a linear circuit.

where

$$\frac{\partial Q_q}{\partial |X_m|} = \frac{\partial U_q}{\partial |X_m|} + \frac{\partial X_q}{\partial |X_m|} \cdot W_q$$

$$\frac{\partial X_q}{\partial |X_m|} = \begin{cases} 0 & q \neq m \\ \dfrac{X_q}{|X_q|} & q = m \end{cases}$$

$$\frac{\partial Q_q}{\partial \varphi_m} = \frac{\partial U_q}{\partial \varphi_m} + \frac{\partial X_q}{\partial \varphi_m}$$

and

$$\frac{\partial X_q}{\partial \varphi_m} = \begin{cases} 0 & q \neq m \\ jX_q & q = m \end{cases}$$

The derivatives of U_q with respect to $|X_m|$ and φ_m are obtained by differentiating (2) [11]

$$\frac{\partial U_q}{\partial |X_m|} = \sum_{n=0}^{\infty} \sum_{\substack{n_1,\ldots,n_N \\ |n_1|+\ldots+|n_N|=n}} \frac{\partial U_q'}{\partial |X_m|}$$

and

$$\frac{\partial U_q}{\partial \varphi_m} = \sum_{n=0}^{\infty} \sum_{\substack{n_1,\ldots,n_N \\ |n_1|+\ldots+|n_N|=n}} \frac{\partial U_q'}{\partial \varphi_m}$$

where

$$\frac{\partial U_q'}{\partial \varphi_m} = jn_m U_q'$$

$$\frac{\partial U_q'}{\partial |X_m|} = \frac{n_m}{x_m} \cdot U_q'$$

$$+ \varepsilon_n \prod_{k=1}^{N} \left\{ X_k'^{\,|nk|} \cdot \sum_{\sigma=0}^{\infty} \left[a(n+2\sigma) \frac{(n+2\sigma)!}{2^{(n+2\sigma)}} \cdot \frac{\partial z}{\partial |X_m|} \right] \right\}$$

and

$$\frac{\partial z}{\partial |X_m|} =$$

$$\sum_{\substack{S_1,\ldots,S_N \\ S_1+\ldots+S_N = \sigma}} \left(\prod_{\substack{k=1 \\ k\neq m}}^{N} \left[\frac{|X_k|^{2S_k}}{S_k!(|n_k|+S_k)!} \; \frac{2S_m |X_k|^{2S_m-1}}{S_m!(|n_m|+S_m)!} \right] \right.$$

$$\left. \cdot \left\{ \sum_{i=1}^{I} \left[A_i \cdot a_{n+2\sigma,i} \cdot R_{n+2\sigma,i} \prod_{k=1}^{N} (b_{k,i})^{(|n_k|+2S_k)} \right] \right\} \right)$$

DISCUSSION

As an example of the analysis procedure developed above, we consider the pumped waveguide diode mixer, figure 3, first analyzed by Kerr [3], and then by Hicks and Khan [1] to show the speed advantage of their method over Kerr's approach. More recently the same circuit was analyzed by Camacho-Penalosa [2] to show that, under weak pumping conditions, their method offers significant advantages over Hicks and Khan's method. However under large pumping conditions Camacho-Penalosa's method is inferior to that of Hicks and Khan. These three methods are concerned with the algorithm used in the harmonic balance iteration between the time domain solution of the diode and the frequency domain solution of the linear circuit. Our method represents a significant departure from this approach as we treat both the diode and the linear embedding circuit in the frequency domain. The number of iterations required for our method, as a function of DC diode current, is given in figure 4 where it is compared to the analyses of Hicks and Khan and of Camacho-Penalosa. On the whole our method requires significantly fewer iterations, convergence was always obtained, and the number of iterations required to obtain a solution virtually independent of the initial voltage estimate at the diode.

In comparing our method with the time domain techniques it should be realized that our method requires more computation per iteration. Nevertheless, as a result of the excellent convergence properties of our method, little intuitive understanding of the nonlinear analysis process required to carry out an analysis. Furthermore, since our method can be extended to larger circuits and design specifications can be incorporated in the objective function, our method is ideally suited to the computer aided design of large nonlinear circuits.

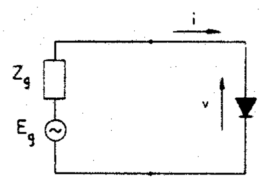

Figure 3
Equivalent circuit of Kerr's waveguide mixer with $i = i_0[\exp(\alpha v)-1]$, $i_0 = 5$ nA and $\alpha = 40\ V^{-1}$. $Z_g(f)$ for the 16 harmonics considered in the analysis can be found in [3].

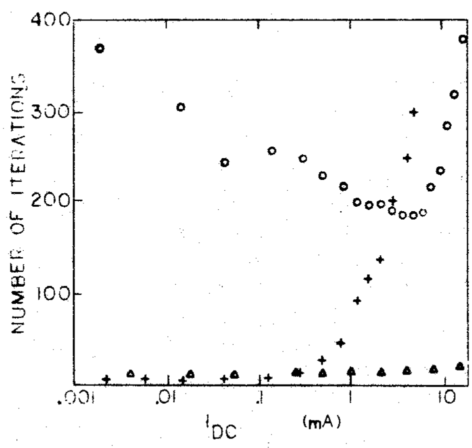

Figure 4
Number of iterations required to solve Kerr's waveguide diode mixer versus dc diode current (o - Hicks and Khan's method, + Camacho-Penalosa's method, ▲ - method proposed here). Convergence is deemed to have been achieved when the circuit voltages were calculated to within 1% of their final values. The initial voltage across the diode was taken as zero.

CONCLUSION

In this paper we presented a frequency domain method of nonlinear analysis which operates by minimizing an objective function derived from Kirchoff's current and voltage laws. As an example a diode mixer was considered and the method was seen to have excellent convergence characteristics. Since the method can be extended to handle more sophisticated circuits and design specifications can be incorporated in the objective function, the technique presented here is ideally suited to the computer aided design of large nonlinear circuits.

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