A NEW FREQUENCY DOMAIN APPROACH TO THE ANALYSIS OF NONLINEAR MICROWAVE CIRCUITS

George W. Rhyne and Michael B. Steer

Department of Electrical and Computer Engineering
North Carolina State University

ABSTRACT

Design of Microwave Monolithic Integrated Circuits (MMIC's) is currently limited by the lack of suitable analysis and computer aided design tools capable of efficiently handling large-scale analog nonlinear circuits. In this paper we address this problem and present a new multi-frequency technique for the analysis of nonlinear circuits. A novel feature of the method is that nonlinear analysis is performed entirely in the frequency domain using a generalized power series. We show how this technique can be applied to the analysis of MESFET circuits and, as an example, we consider gain saturation in a GaAs MESFET amplifier.

I INTRODUCTION

Great progress is being made in the application of GaAs MESFET's to the amplification, frequency multiplication, upconversion, generation and mixing of microwave signals. This is particularly so in the area of Microwave Monolithic Integrated Circuits (MMIC's). The rapidly improving performance of MMIC's has led, in many applications, to the replacement of two terminal devices at frequencies through Ka band (40 GHz). The principal advantages MMIC's offer are excellent input-output isolation and inherent wide bandwidth. At low microwave frequencies these circuits are commonly designed using linearized device models. Typically designs are bread-boarded and circuit adjustments are made to match actual and desired performance. This procedure circumvents the problem of active device variations and inaccuracies in circuit simulation. This approach is impractical for the design of MMIC's where fabrication costs are high and MESFET nonlinearities significantly affect circuit performance, even for such linear applications as amplification. The trend towards replacing passive elements with active loads, and of achieving high power amplification using amplifier circuits other than class A, further complicates circuit synthesis. Thus for the precise simulation and design of MMIC's an accurate large signal analysis method is required.

Existing nonlinear analyses can be placed in three categories

- those that operate entirely in the time-domain using numerical integration (e.g. SPICE-like analyses). These analyses are time-consuming since they make no distinction between linear and nonlinear elements. As well, these types of analyses are not suited to the simulation of circuits with sinusoidal excitation because the computation time required to obtain the steady state solution for a circuit is prohibitive.
- those operating entirely in the frequency domain (e.g. Volterra series analyses [1]). Until now these analyses have been limited to mildly nonlinear systems.
- those that iterate, using Fourier techniques, between time domain solutions of the nonlinear sub-circuit and frequency domain solutions of the linear embedding circuit (e.g. [2, 3]. The use of Fourier techniques requires that the time domain solutions be periodic (or for the technique presented in [2] very closely periodic) as only then can the fourier components of the current and voltage waveforms be extracted.

This paper presents an improved large signal nonlinear analysis with the following attributes:

(1) it operates entirely in the frequency domain so that large signal multi-frequency excitation can be handled. The method makes use of the numerically efficient frequency domain analysis of linear circuits and, by treating the nonlinearity in the frequency domain, can handle large signal multifrequency excitation so that intermodulation distortion can be simulated.

(2) current and voltage derivatives are immediately available with the proposed method so that efficient optimization techniques can be used to determine the state of a circuit. It is this attribute which results in the numerically efficient simulation of nonlinear circuits.

(3) it is suitable for inclusion in computer aided design schemes.

(4) it is anticipated that, when the analysis technique has been further developed, less skill will be required to use the analysis program than is required with those nonlinear analyses that use traditional harmonic balance iteration schemes.

In this paper, we apply our method to the simulation of MESFET circuits. In particular, we present results for the simulation of gain saturation in a single stage GaAs MESFET amplifier.

0149-645X/85/0000-0401

II THEORETICAL DEVELOPMENT

Popular nonlinear analysis progams such as SPICE analyze a large nonlinear circuit using numerical integration of every element in the circuit. Many other techniques, including our analysis method, segment a large nonlinear circuit into linear and nonlinear subcircuits, figure 1. In our analysis, the linear subcircuit is treated in the frequency domain using numerically efficient nodal techniques. Treatment of the nonlinear subcircuit is based on a generalized power series [1,2] (a power series with time delays and complex coefficients) expansion of the current-voltage characteristics of the internal nonlinearities. Previously we have developed an algebraic *formula* for a phasor of the output of a generalized power series with a multi-frequency input [1]. The *formula* is a function of the phasors of the frequency components of the input and so, a time domain description (the power series) of a nonlinear component is converted into a frequency domain description (the *formula*). As well, additional formulas for the derivatives of each of the output phasors with respect to the amplitudes and phases of the frequency components of the input are available [6]. The circuit analysis we propose uses the phasor and derivative formulas in minimizing an objective function derived from the harmonic balance error of the nonlinear subcircuit. The objective function approach is attractive in that it is well suited to our longer term goal of developing computer aided design tools for MMIC's since design specifications can be incorporated into the objective function. Our objective function, O, is derived from the error in Kirchoff's current law applied at each frequency considered and at each nonlinear subcircuit node

$$O = \sum_{i=1}^{N} \left\{ \sum_{q=1}^{Q} \left| I_{q,i} \right|^2 \right\}$$

where $I_{q,i}$ is the total current flowing away from node q at the i^{th} frequency component, there are Q nodes, and N frequencies are being considered. The current contributions

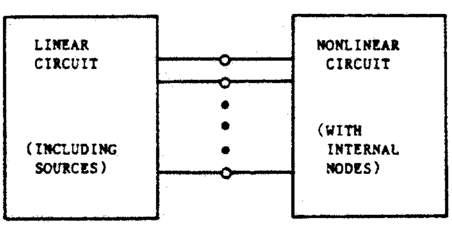

Figure 1
A large nonlinear circuit segmented into linear and nonlinear subcircuits. The linear subcircuit includes any voltage sources. The nonlinear subcircuit may have internal nodes in addition to nodes shared with the linear subcircuit. (Note that we ignore the internal nodes of the linear subcircuit.)

from the nonlinear subcircuit are nonlinear functions of the voltages at the nodes whereas the current contributions from the linear subcircuit are easily calculated using linear frequency-domain circuit techniques. When the proper circuit conditions have been determined, O will be zero as then Kirchoff's current law is satisfied. Thus, by minimizing O with respect to the node voltages, the state of the circuit is determined.

This objective function can be viewed as a collection of squares of nonlinear equations, each one dependent on a set of variables, and each one to be minimized. One technique to minimize such a set of nonlinear equations is Newton's method. Given a set of P nonlinear equations (the vector $\mathbf{F}(\mathbf{x})$), dependent on a set of variables (the vector $\mathbf{x}$), this method seeks to find the minimum of $\sum_{i=1}^{P} F_i^2(\mathbf{x})$ with respect to $\mathbf{x}$ using the iterative procedure

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{J}^{-1}(\mathbf{x}_i)\mathbf{F}(\mathbf{x}_i)$$

where $\mathbf{J}$ is the Jacobian matrix with elements

$$\left[\mathbf{J}(\mathbf{x}_i)\right]_{j,k} = \frac{\partial F_j(\mathbf{x}_i)}{\partial(\mathbf{x}_i)_k}$$

In our case, F_i is the KCL error at a node and $\mathbf{x}$ is the vector of phasor node voltages. To calculate the Jacobian, we need derivatives of current with respect to voltage for every combination of node, frequency, and circuit element. These derivatives are readily available for the linear subcircuit as the y-parameters of the linear network. The derivatives of the current contributions of the nonlinear elements are available since the elements are described by a generalized power series. In [3]. these derivatives are calculated with respect to the magnitude and phase of the node voltages . It is not appropriate to use these derivatives in the optimization process as the current phasors are, in general, strongly nonlinear functions of the magnitudes and phases of the voltage components even for components in the nonlinear subcircuit that are almost linear. This causes needless convergence problems. For example, the derivative of the current through a linear admittance (Y) with respect to the magnitude of the phasor voltage across it is

$$\frac{\partial I_k}{\partial |V_j|} = \begin{cases} Y\left(\frac{V_k}{|V_k|}\right) & j = k \\ 0 & j \neq k \end{cases}$$

and the derivative with respect to the phase of the voltage is

$$\frac{\partial I_k}{\partial \phi_j} = \begin{cases} jV_k Y & j = k \\ 0 & j \neq k \end{cases}$$

where the phasor voltage at the k th frequency is

$$V_k = |V_k| e^{j\phi_k}$$

and I_k is the phasor current at the k th frequency. This added nonlinearity needlessly complicates minimization. If real and imaginary components of the phasors are used as variables, the derivatives become simpler. The derivative of the current through the linear admittance with respect to the real component of the phasor voltage across it is

$$\frac{\partial I_k}{\partial \mathrm{Re}(V_j)} = \begin{cases} Y & j = k \\ 0 & j \neq k \end{cases}$$

while the derivative with respect to the imaginary component is

$$\frac{\partial I_k}{\partial \mathrm{Im}(V_j)} = \begin{cases} jY & j = k \\ 0 & j \neq k \end{cases}$$

Another benefit of using the real and imaginary components as variables is that they are dimensionally similar quantities, unlike the magnitude and phase components. Also, the real and imaginary components are not constrained to be positive numbers. Thus, the convergence of the minimization process is improved by using real and imaginary components of the phasor voltages as the variables to be optimized. The derivatives calculated for the nonlinear elements with respect to magnitude and phase can be easily modified to relate to real and imaginary components in the following manner

$$\frac{\partial I_k}{\partial \mathrm{Re}(V_j)} = \frac{\partial I_k}{\partial |V_j|}\frac{\partial |V_j|}{\partial \mathrm{Re}(V_j)} + \frac{\partial I_k}{\partial \phi_j}\frac{\partial \phi_j}{\partial \mathrm{Re}(V_j)}$$

and

$$\frac{\partial I_k}{\partial \mathrm{Im}(V_j)} = \frac{\partial I_k}{\partial |V_j|}\frac{\partial |V_j|}{\partial \mathrm{Im}(V_j)} + \frac{\partial I_k}{\partial \phi_j}\frac{\partial \phi_j}{\partial \mathrm{Im}(V_j)}$$

where k and j are frequency indices. Thus, because the nonlinear elements are described by power series, derivative information is available which is necessary for efficient optimization. An algorithm based on this concept is shown in figure 2.

III MESFET SIMULATION

This nonlinear circuit analysis procedure can be applied to simulation of a MESFET embedded in a linear circuit. In this case, the nonlinear subcircuit consists of the model for the transistor's nonlinearities. The linear subcircuit consists of device parasitics, the embedding circuit, and any signal sources. The MESFET's nonlinearities are modelled using a collection of nonlinear resistors, capacitors, and a current source as shown in figure 3. In general, all of the elements shown can be nonlinear functions of voltage, including the gate-to-drain capacitance which is frequently ignored or linearized. Such a model can predict a wide range of effects including forward bias gate current and gate-to-drain breakdown, as well as the usual drain current characteristics. Each of the nonlinear two terminal elements are described by generalized power series expansions relating current and voltage. These expansions can include time delays which are necessary for accurate modelling of the transconductance and breakdown effects. The power series can be found by analytical means or by curve fitting to experimental data. The objective function for this application consists of the sum of currents squared at each of the four nodes of the nonlinear subcircuit (at each frequency of interest). This model can be used to investigate gain compression and intermodulation distortion in large signal amplifiers, oscillators, and mixers.

IV GAIN COMPRESSION IN A MESFET AMPLIFIER

A computer program based on our analysis technique has been written and, as an example, was used to investigate gain compression in a single stage GaAs MESFET amplifier in a common source configuration, figure 4. The linear subcircuit consists of the parasitic resistances and inductances of the transistor model along with the source and load impedances of the external circuit, whereas the nonlinear subcircuit comprises the transistor model shown

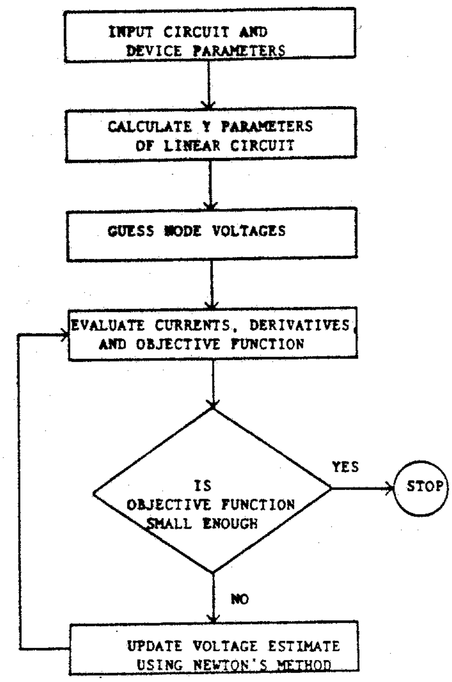

Figure 3
Model Used to Simulate MESFET Nonlinearities

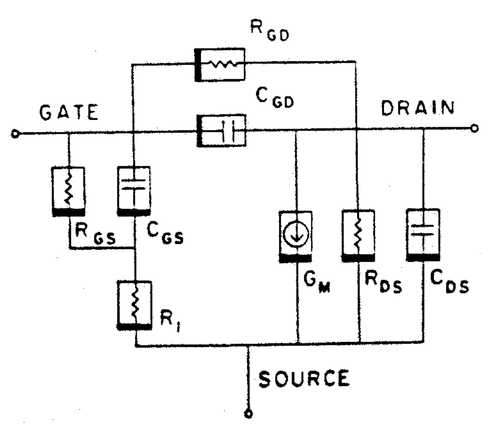

Figure 2
Flowchart for Nonlinear Analysis Algorithm

in figure 3. In this example, the transconductance, the gate-source resistance, and the gate-drain resistance are treated as being nonlinear. A power series representation of the transconductance was found by a least squares curve fit to a set of drain current characteristics. The gate-source resistance and the gate-drain resistance were assumed to be exponential functions of the respective voltages across the resistances. The accuracy of the circuit simulation was enhanced by using Chebyshev economization of the power series expansions for the exponential characteristics. Although any number of harmonics could have been included in the circuit simulation, we limited ourselves to the fundamental and the second harmonic. Numerical nonlinear analysis of the MESFET circuit yielded the gain compression results shown in figure 5 where the fundamental output power as well as the second harmonic power appearing at the amplifier output port is plotted as a function of the input RF power. In this simulation, the fundamental and one harmonic frequency were considered. In general, any number of harmonic frequencies can be considered. Also, multiple signals may be applied and the intermodulation products considered. Thus, this model is capable of simulating large signal effects that have previously been difficult to simulate.

V CONCLUSION

This paper presented a new algorithm for analyzing nonlinear circuits. Unlike previous methods, this analysis operates entirely in the frequency domain. Advantages of this approach are that it can handle large nonlinear circuits, non-harmonically related signals can be considered, and circuit design specifications can be simply incorporated into the analysis. Thus the analysis has the potential of being a key ingredient in the development of computer aided design tools for microwave monolithic integrated circuits.

VI REFERENCES

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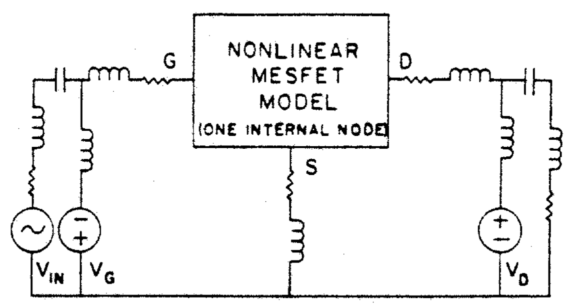

Figure 4
Single Stage Common Source MESFET Amplifier

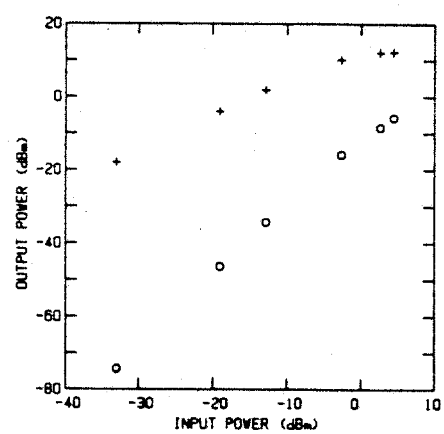

Figure 5
Gain Compression Characteristics of Common Source Amplifier with 15 dB small signal gain. Shown are the output powers contained in the fundamental frequency (10 GHz - +) and in the second harmonic (20 GHz - O).