

# On the Capacity of Vector Antenna MIMO Systems

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**Abstract** — Most wireless communication systems currently employ single- or dual-polarized antennas which measure one or two components of the received electromagnetic (EM) signal. In this paper, we investigate the performance gains achievable by using “vector antennas” that can detect or excite up to six independent degrees of freedom.

Recently, it has been suggested in [1] that wireless capacity can be improved in rich scattering environments by exploiting additional components of the EM field. In this paper, we present a simple space-time propagation model for the electric and magnetic fields which is based on the discrete multipath model proposed in [2]. We note that this model is accurate for small dipoles and loops, but nevertheless provides a good approximation to practical sized antenna structures. Consider a six-element antenna structure with three colocated orthogonal dipoles and three orthogonal loops which we refer to as a “vector antenna”. For a system with  $t$  vector antennas at the transmitter and  $r$  vector antennas at the receiver, the input-output system equation can be written as

$$\mathbf{Y}(t) = \sum_{l=1}^L B_r(\theta_l^R, \varphi_l^R) D_l B_t^T(\theta_l^T, \varphi_l^T) \mathbf{X}(t - \tau_l) + \mathbf{N}(t),$$

where  $\mathbf{X}(t)$  is the transmitted signal vector,  $\mathbf{Y}(t)$  is the received signal,  $L$  is number of dominant multipath components,  $\tau_l$  is the delay associated with the  $l$ -th path,  $D_l$  is a  $2 \times 2$  complex matrix that reflect the change in amplitude, phase and polarization experienced by the electric field vector in the  $l$ -th path.  $B_r(\theta, \varphi) = \mathbf{d}_r(\theta, \varphi) \otimes B(\theta, \varphi)$ , where  $B(\theta, \varphi)$  is the  $6 \times 2$  array response matrix for an electric field excitation (superscripts  $T$  and  $R$  indicate transmitter and receiver respectively), and  $\mathbf{d}(\theta, \varphi)$  is the classical narrowband response vector for a linear array.  $(\theta_l, \varphi_l)$  and  $(\theta_l^T, \varphi_l^T)$  are orientations of the  $l$ -th scatterer with respect to the transmit and receive arrays respectively.  $\mathbf{N}(t)$  is the additive noise process.

Assuming that delay spread is negligible compared to the symbol duration and that the noise process is spatio-temporally white, the input-output equation for the  $n$ -th symbol interval can be written as  $\mathbf{Y}_n = \sqrt{\rho}/tm H \mathbf{X}_n + N_n$ , where  $H = (1/cL) \sum_l B_l^R D_l B_l^T$ , normalization constant  $c$  is chosen such that  $E[HH^\dagger] = m\rho$ . Here  $m(\leq 6)$  corresponds to the number of elements of an antenna structure with a subset of the elements of 6-element vector antenna. A lower bound to the ergodic capacity (assuming perfect channel knowledge at the receiver) can be obtained with circularly symmetric complex Gaussian input vector  $\mathbf{X}_n$ , and the lower bound can be written as  $C_l \stackrel{\text{def}}{=} \log \det(I + (\rho/mt) H H^\dagger)$ .

The rank of the matrix  $H$  determines the capacity scaling with respect to SNR. We study the impact of choice of the

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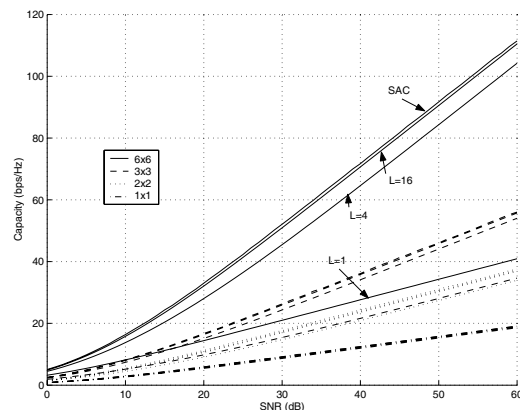


Fig. 1:  $C_l$  versus number of multipaths for 1-, 2-, 3- and 6-element vector antennas

antenna elements and propagation conditions on  $C_l$ . Fig.1 contains the plot of  $C_l$  for  $m = 1, 2, 3$  and 6 antenna elements and different number of multipaths.

**Proposition 1** For 6-element vector antenna both at the transmit and receive to have a full rank channel, ( $\text{rank}(H) = 6$ ), it is sufficient to have nonsingular polarization transformation matrix  $\mathcal{D}$ , and two multipaths such that the scatterer orientations with respect to the transmitter (receiver) correspond to distinct spatial angles.

**Proposition 2** For 6-element vector antenna at both the transmit and receive, with the entries of  $D_l$  taken to be i.i.d circularly symmetric complex Gaussian with zero mean and unit variance, as  $L \rightarrow \infty$ , the entries of  $H$  converge in distribution to a  $6 \times 6$  i.i.d circularly symmetric complex Gaussian system.

One can conclude that: (1) Systems employing polarimetric antennas approach an i.i.d system in performance under rich multipath, (2) Fewer scatterers are sufficient to provide full capacity scaling for vector antennas as compared to their scalar counterparts, (3) Subsets of the six-element vector antenna achieve full-scaling under the same channel conditions for which the six-element vector antennas achieve full-scaling. A notable subset is the planar 3-element colocated antenna structure with two orthogonal dipoles and a loop, described in [3]

## REFERENCES

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