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Inversion theory as applied to the optimisation of conductivity profile for PML absorbing boundary condition for FDTD code

G. Lazzi and O.P. Gandhi

Indexing terms: Finite-difference time domain method, Electromagnetic wave absorption

A theoretical approach to obtain optimised profiles of the conductivity for the perfect matched layer (PML) absorbing boundary conditions is presented. The analytical expression recently introduced by the authors to estimate the error caused by the use of the PML backed by a metal plane as the absorbing boundary condition, has been minimised with respect to the conductivity profile following an inverse theory approach. The algorithm uses the Frechet derivative of the theoretical error expression, together with a simple regularisation technique in order to avoid the quasi-singularity intrinsic in the problem formulation. Results show that, following this approach, it is possible to easily obtain the conductivity profiles that render the performances of the absorbing boundary conditions comparable to, or better than, the best performance obtained for just a particular order of a more classic parabolic-like profile.

Introduction: Recently, the effectiveness of the perfect matched layer (PML) [1] as a superior absorbing boundary condition (ABC) for the finite-difference time-domain method has been proven convincingly [2 – 5]. This ABC gives errors that are approximately three to four orders of magnitude lower than the traditional Mur second order [6] or retarded time [7] ABCs. It is however necessary to carefully select the profile of the conductivities of the PML layers. In [5], we have derived an approximate expression to estimate the reflection of the PML boundary condition as used in the FDTD code. This reflection can be approximately described as a composite of two effects: the reflection of the PML material backed by a metal plane and a second order error intrinsic in the FDTD linear approximation of the derivatives. The total error E has been estimated, therefore, with the following formula:

$$E = e^{-2Z_0 \int_{0.5\delta}^{(m-0.5)\delta} \sigma(x)dx} + \frac{(\delta/2)^2}{6} \int_{-0.5\delta}^{(m-0.5)\delta} \frac{\partial^3}{\partial x^3} e^{-Z_0 \sigma(x) x} dx \quad (1)$$

where Z_0 is the impedance of a vacuum, δ is the cell size, m is the number of PML layers, and the limits of the integration have been chosen for a better matching with the stair-step approximation of the FDTD.

In [5], this equation has been used to find the best order for a parabolic-like profile of the conductivities, given by

$$\sigma_i = \sigma_0 \frac{i^{n+1} - (i-1)^{n+1}}{n+1} \quad (2)$$

where i is the cell number in the PML layer ($i > 0$) and n is the order of variation of the profile of the conductivity. In this Letter, we assume the conductivities of the PML layers as unknowns, and try to minimise the error expressed by eqn. 1.

Theory: To solve the inverse problem expressed by eqn. 1, we have used the Newton method as a nonlinear inversion technique. This method is similar to the steepest descent method, but generally it has a faster convergence to the solution. Defining m as the number of PML layers, we can introduce a vector σ of the conductivities, with dimension m. A vector of perturbations $\mathbf{d}\sigma$ allow us to define the component l of the Frechet derivative of eqn. 1 corresponding to a variation of the component l of $\mathbf{d}\sigma$ [8]:

$$F_\sigma(l) = \frac{E(\sigma + \mathbf{d}\sigma) - E(\sigma)}{\sqrt{\mathbf{d}\sigma^T \cdot \mathbf{d}\sigma}} \quad (3)$$

The function in eqn. 1 can therefore be locally linearised as follows:

$$E(\sigma + \mathbf{d}\sigma) = E(\sigma) + \mathbf{F}_\sigma \cdot \mathbf{d}\sigma \quad (4)$$

A minimum of the function E can be found by minimising the norm of eqn. 4. Imposing the differential of the norm of eqn. 4 equal to 0, we obtain

$$\mathbf{d}\sigma = -\frac{\mathbf{F}_\sigma E(\sigma)}{\mathbf{F}\sigma^T \cdot \mathbf{F}_\sigma} \quad (5)$$

The introduction of a regularisation factor α is necessary due to the quasi-singularity of the problem. Eqn. 5 may then be rewritten as follows:

$$\mathbf{d}\sigma = -\frac{\mathbf{F}_\sigma E(\sigma)}{\mathbf{F}_\sigma^T \cdot \mathbf{F}_\sigma + \alpha \mathbf{I}} \quad (6)$$

with $\mathbf{I}$ an identity matrix of dimensions $m \times m$.

Following this approach, a simple computer program can be written to iteratively calculate the Frechet derivative $\mathbf{F}_\sigma$ and the variation $\mathbf{d}\sigma$ until convergence is reached.

Results: As discussed in [5], eqn. 1 is a simple 1D description of the behaviour of the PML ABC. Therefore, the performance of the absorbing boundaries can be improved by altering by a scaling factor λ, the profile of the conductivities as obtained by the procedure described above. For this reason, two variables affect the result of the optimisation procedure: the scaling factor λ and the regularisation factor α.

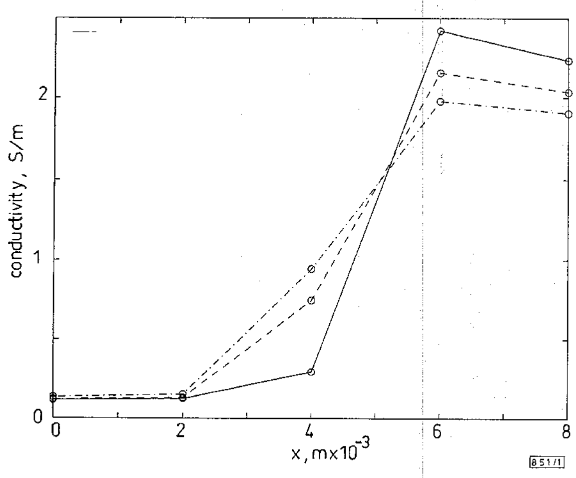

Fig. 1 *Profiles of electric conductivities against distance from air-PML interface for various regularisation factors* α *(4 layer PML)*

——— $\alpha = 0.0005$
– – – – $\alpha = 0.0010$
· – · – · $\alpha = 0.0015$

For comparison, the same problem considered in [5] has been considered here. A Hertzian dipole sinusoidally excited with an E-field amplitude of 0.1V/m at a frequency of 835MHz has been placed at the centre of a 50×50×50 cells grid of resolution 2×2×3mm. Each simulation has been run for 120 time steps, and the average of the local error as defined in [9] relative to the results obtained by using a 'big mesh', has been considered. In the following, we will consider just the case of four layer PML as an example of the theory described above. Fig. 1 shows examples of the profiles of the electric conductivities against distance x from the air-PML interface for various regularisation factors α. Fig. 2

shows the average of the local error obtained by using the new profiles against regularisation factor α and for two different multiplication factors λ. For comparison, in the same graph we give the minimum local error obtained for the parabolic-like profile (with order $n = 4.6$), as well as the local error that has been obtained with order $n = 2$ [5]. As it is possible to see, the PML with the new profiles performs extremely well for all of the chosen regularisation factors, having for a certain multiplication factor ($\lambda = 0.55$), a performance even slightly better than the best obtained for just a particular order ($n = 4.6$) of the parabolic-like profile. The performance is therefore broadband with respect to the regularisation and the multiplication factors.

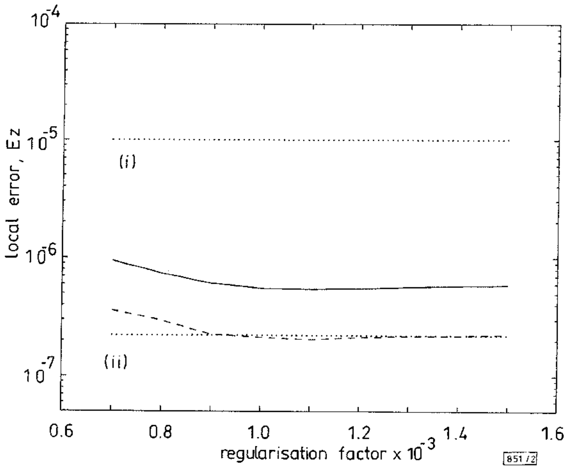

Fig. 2 *Average of local error obtained by using new profiles against regularisation factor α and for two different multiplication factors λ*

(i) parab.-like, n = 2.0
(ii) parab.-like, n = 4.6
——— λ = 1.0
- - - - λ = 0.55

Conclusions: An efficient approach to determine alternative profiles of the conductivities to be used for effective performances of the PML ABC for the FDTD code has been presented. It is shown that by using an inverse theory approach it is possible to design highly absorbing profiles of conductivities to be used for the PML layers. Further investigations in this area should be carried out with other characterisations of the PML error, leading to the possibility of an optimum design of the needed boundary conditions for any situation.

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G. Lazzi and O.P. Gandhi (*Department of Electrical Engineering, 3280 Merrill Engineering building, University of Utah, Salt Lake City, Utah 84112, USA*)

E-mail: lazzi@ee.utah.edu

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Miniature MMIC star double balanced mixer using lumped dual balun

Hwann-Kaeo Chiou, Yu-Ru Juang and Hao-Hsiung Lin

Indexing terms: MMIC, Mixers (circuits), Baluns

The authors present the design and performance of a very compact MMIC star mixer. The mixer is composed of four diodes and two lumped dual baluns which are realised using two high-lowpass out-of-phase power splitters in parallel. The MMIC star mixer shows wide bandwidth, high P1dB, high third order intercept (IP3) point and high port-to-port isolations both in the applications of up and down converter. The chip size of this star mixer is much smaller than those of the conventional mixers. To the best of the authors' knowledge, it is the smallest size ever reported for passive mixer operated at S-band.

Introduction: The star double balanced mixer (DBM) is an excessively complex version of DBMs. The advantage of star mixers over the traditional ring DBMs is mainly on their low IF parasitic inductance which enables a much wider IF bandwidth to be achievable. In star mixers, the IF signal is direct coupled, which allows the applications on phase detection or direct baseband conversion. Furthermore, the circuit size can be greatly reduced because the IF diplexing filter and external DC return path are no longer needed. So far, very few studies regarded star mixers [1 – 3] because they need a complicated dual balun structure, which is much more difficult on circuit realisation than the conventional single balun. Previous star mixers were designed using hybrid-based implementations. Their distributed balun utilised either vertical or horizontal couple stripe-lines to form a Marchand balun. Consequently, these mixers become bulky or nonplanar, and cannot be adopted in MMIC processes. In comparison to these hybrid star mixers, very few MMIC star mixers have been presented. A Ka-band MMIC star mixer with excellent performances using a coplanar Marchand-like balun was first reported by Maas *et al.* [4]. However, even in the Ka-band, the chip size is 2.4 × 2.8 mm², still too large for commercial applications. In addition, the distributed baluns would become an impractical way to realise the balun in star mixers in the lower frequency range. In this Letter, an S-band MMIC star mixer using a lumped dual balun is first proposed. With the chip dimension significantly reduced to 0.8 × 0.8 mm², the mixer still has all the feasibilities that the conventional star mixer has. It would certainly find wide applications in the low microwave band.

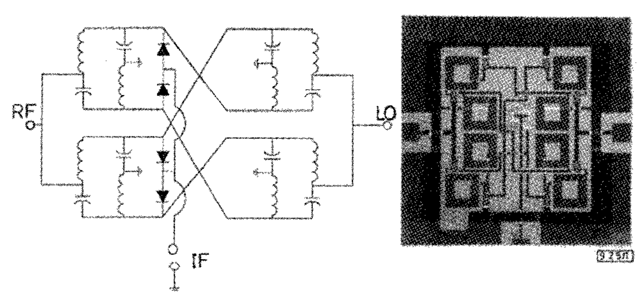

Fig. 1 *Circuit schematic diagram and photograph of lumped high-low-pass star mixer*