Characterization of In-Band Distortion in RF Front-Ends Using Multi-Sine Excitation

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***Abstract* -Multi-sine signals are useful for approximating communication signals when analyzing distortion generated by nonlinear circuits. In this work the correlated and uncorrelated components of distortion are derived using multi-sine signals with random phases. Distortion analysis of two-tone and four-tone random phase signals demonstrates the in-band distortion characteristics of multi-tone signals. Simulated in-band distortion for a sixteen-tone random phase signal is verified against measured in-band distortion characteristics of a CDMA forward link signal.**

I. Introduction

Multi-sine stimuli are often used to model the behavior of nonlinear systems because of the simplicity of analysis using discrete tones with nonlinear models. Considerable research has been undertaken on the use of multi-sine signals to represent digitally modulated communication signals for characterizing out of band distortion of non-linear circuits [1]–[3]; however, multi-tone signals have not been widely used to analyze in-band distortion. In-band distortion is important in wireless systems because it contributes directly to signal to noise ratio (SNR) degradation when digitally modulated signals are applied to nonlinear power amplifiers [4]. Measurement of SNR degradation is often quantified by signal quality metrics such as error vector magnitude and rho.

The difficulty with characterizing in-band distortion is the identification of the nonlinear terms which are responsible for distortion inside the main band of the input signal spectrum. This is because it is usually hard to represent nonlinearity as pure linear and pure distortion noise terms within the signal main bandwidth. The reason for this is that part of the nonlinear output is not totally uncorrelated from the linear output and hence they add to the useful component causing gain compression or enhancement. The remaining part of the nonlinear output is the effective in-band distortion. Thus both the correlated and uncorrelated components of the nonlinear output contribute to the degradation of system SNR in different ways.

In this paper we provide an approach to the characterization of in-band distortion using multi-sine excitation. It will be shown than traditional two-tone test is inadequate for the estimation of in-band distortion. We show that multi-sine signals with random phases enable the evaluation of in-band distortion of real communcation signals. The shape of the uncorrelated distortion spectrum depends on the randomness of the initial phases and this has different effect on in-band and out-of-band distortions. The randomness of the phases of the multiple tones is key to understanding distortion in terms of random signal distortion measures. It provides a basis for the approximation of communication signals distortion by the distortion of multi-sine signals with random phases. Estimated in-band distortion of multi-sine signals is verified by measured in-band distortion of CDMA signals using feed-forward cancelation.

II. Behavioral Model

A memoryless nonlinearity is characterized by an envelope power series model as

$$\tilde{y}(t) = \sum_{n=1}^{N} \tilde{w}_n = \sum_{n=1}^{N} b_n \tilde{w} |\tilde{w}|^{n-1} \tag{1}$$

These transfer characteristics represent the envelop relationship between the input and the output waveforms. In this model the linear and nonlinear branches may be statistically correlated. In [4] we developed an orthogonalization procedure to convert the model (1) into a model with with uncorrelated outputs so that distortion can be distinguished from the useful components of the output. Using this approach for a third order nonlinearity we define a new set of outputs:

$$\tilde{y}_c(t) = b_1 \tilde{w}(t) + \alpha \tilde{w}(t)$$

which is correlated to the input signal and

$$\tilde{y}_u(t) = b_3 \tilde{w}(t) |\tilde{w}(t)|^2 - \alpha \tilde{w}(t)$$

which represent the uncorrelated output. The coefficient α is a complex coefficient that represents the fraction of the

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cubic term that is correlated with the linear response. It follows that the output autocorrelation function can now be written as a sum of the autocorrelation functions of the uncorelated compnents $\tilde{y}_c$ and $\tilde{y}_u$ as [4]

$$R_{\tilde{y}\tilde{y}}(\tau) = R_{\tilde{y}_c\tilde{y}_c}(\tau) + R_{\tilde{y}_u\tilde{y}_u}(\tau) \tag{2}$$

and hence, the coefficient α is defined as

$$\alpha = \frac{b_3 R^*_{\tilde{w}_1\tilde{w}_3}(0)}{R^*_{\tilde{w}\tilde{w}}(0)} \tag{3}$$

The output Power Spectral density (PSD) can be found from the Fourier transform of (2):

$$S_{\tilde{y}\tilde{y}}(f) = S_{\tilde{y}_c\tilde{y}_c}(f) + S_{\tilde{y}_u\tilde{y}_u}(f)$$

Note that this formulation enables the output spectrum to be analyzed into a useful and an uncorrelated distortion components.

III. Multi-Sine Signals with Random Phases

Consider a multi-sine input signal $w(t)$ consisting of the sum of K tones where K is an even number and applied to the nonlinear amplifier so that

$$w(t) \quad = \quad \sum_{k=-K}^{K} \frac{A}{2} \cos\left(\omega_c t + \left(k - \frac{1}{2}\right)\omega_m t + \phi_k\right)$$

where $\omega_m = \omega_k - \omega_{-k}$ is the frequency separation between the input tones assuming a uniform frequency separation and even number of tones. The phases ϕ_k are assumed to be random phases uniformly distributed in $[0, 2\pi]$. The total input to the nonlinear system can be written as a complex conjugate pair as

$$w(t) = \sum_{k=-K}^{K} \frac{1}{2}\tilde{w}_k(t) e^{j\omega_c t}$$

it follows that $\tilde{w}_k = A\cos\left(\left(k - \frac{1}{2}\right)\omega_m t + \theta_{1k}\right) e^{j\theta_{2k}}$ where $\theta_{2k} = (\phi_k + \phi_{-k})/2$ and $\theta_{1k} = (\phi_k - \phi_{-k})/2$ and hence,

$$\tilde{w}(t) = \sum_{k=1}^{K} A\cos\left(\left(k - \frac{1}{2}\right)\omega_m t + \theta_{1k}\right) e^{j\theta_{2k}}$$

for the special case where $\phi_k = -\phi_{-k}$ it follows that

$$\tilde{w}(t) = \sum_{k=1}^{K} A\cos\left(\left(k - \frac{1}{2}\right)\omega_m t + \theta_{1k}\right) \tag{4}$$

In the following subsections we consider the cases of two-tone and four-tone signals with random phase. The output spectrum which results from the Fourier transform of the autocorrelation function partitioned into correlated and uncorrelated components is used to estimate the effective in-band distortion.

A. Two-Tone Excitation

For a two-tone input each with amplitude $A/2$, the input $w(t)$ can be expressed in complex envelope form using (4) as

$$\tilde{w}(t) \quad = \quad A\cos\left(\frac{\omega_m t}{2} + \theta_1\right)$$

Now using the orthogonalization procedure, the correlation coefficients α can be found using (3):

$$\alpha = b_3 \frac{E[\tilde{w}_3(t)\tilde{w}_1^*(t)]}{E[\tilde{w}_1(t)\tilde{w}_1^*(t)]} = \frac{3A^2 b_3}{4}$$

and it follows that

$$R_{\tilde{y}_c\tilde{y}_c}(\tau) = \frac{|b_1 + (3A^2/4)b_3|^2 A^2}{2} \cos\left(\frac{\omega_m \tau}{2}\right)$$

and

$$R_{\tilde{y}_u\tilde{y}_u}(\tau) = \frac{|b_3|^2 A^6}{32} \cos\left(\frac{3\omega_m \tau}{2}\right)$$

The output autocorrelation function of a two-tone input consists of components at integer multiples of intermodulation frequency ω_m. The uncorrelated part of the output consists of the out-of-band intermodulation products. Note that a two tone test cannot predict the uncorrelated in-band distortion since the only distortion terms that result as uncorrelated components are the uncorrelated intermodulation components which lie outside the frequency band of the input tones.

B. Four-Tone Excitation

For a four-tone input each with amplitude $A/2$, the input $w(t)$ can be expressed in complex envelope form using (4) as

$$\tilde{w}(t) = A\cos\left(\frac{\omega_m t}{2} + \theta_{11}\right) + A\cos\left(\frac{3\omega_m t}{2} + \theta_{12}\right)$$

the correlation coefficients for a third-order system can be found using can be found using (3) to be $\alpha = 9A^2 b_3/4$ and hence

$$R_{\tilde{y}_c\tilde{y}_c}(\tau) = \frac{|b_1 + (9A^2/4)b_3|^2 A^2}{2} \times \left[\cos\left(\frac{\omega_m \tau}{2}\right) + \cos\left(\frac{3\omega_m \tau}{2}\right)\right]$$

and

$$R_{\tilde{y}_u\tilde{y}_u}(\tau) \quad = \quad \frac{|b_3|^2 A^6}{32}[9\cos\left(\frac{\omega_m \tau}{2}\right) + \cos\left(\frac{3\omega_m \tau}{2}\right) + 18\cos\left(\frac{5\omega_m \tau}{2}\right) + 9\cos\left(\frac{7\omega_m \tau}{2}\right) + \cos\left(\frac{9\omega_m \tau}{2}\right)] \tag{5}$$

Note that with four tones the uncorrelated distortion is manifested as components at the fundamental in addition to the intermodulation components. The in-band distortion is therefore represented by the components $9\cos(\omega_m\tau/2) + \cos(3\omega_m\tau/2)$. Note that there is a difference of about 9.5 dB in the power levels of the in-band and out-of-band components.

Discussion

In this section we have shown the application of the orthogonal behavioral model using different multi-sine signals. Single and two-tone signals are inadequate to model the in-band uncorrelated distortion. This is also intuitive since a single tone excitation of a nonlinear system results only in gain compression and not distortion. In the case of two-tone excitation, the output consists of compressed output at the fundamental frequencies and uncorrelated out of band intermodulation components. In the case of K tones where $K > 2$, the uncorrelated distortion consist of components at the fundamental frequencies in addition to out of band components. The analysis of multi-sine signals for $K > 4$ can be developed using the same approach however is more involved than the above cases. Increasing the number of tones increases the accuracy of representing a fully loaded CDMA signal by a multi-sine signal. This is because according to the central limit theorem the sum of a large number of random processes approaches the Gaussian distribution which is a common model of CDMA signals. Therefore, the approximation of communication signals with multi-sine signals improves with increasing the number of tones used. Increasing the number of tones however means a higher simulation complexity. In the next section we show by simulations that in-band distortion of a CDMA signal can be approximated by that of a 16-tone signal with uniformly distributed random phases.

IV. Simulation of CDMA Signal Distortion By Multi-Sine Signals

In this section we use multi-sine signals with random phases to simulate in-band distortion of CDMA signals. The phases of the input tones were generated using the Matlab uniform random number generator. The initial phases were generated and an average is taken for multiple runs. The number of runs depend on the number of tones because as the number of tones increases the probability of having a uniformly distributed phases increases. It was found that distortion of 16-tone signal with random phases converges to that of a Forward-link IS-95 signal as shown in Fig. 1. The figure shows the total output and the uncorrelated distortion spectra of a 16-tone signal with random phases compared to those of a Forward-link IS-95 spectrum. The 16-tone signal was designed to have a total bandwidth equal to the bandwidth of an IS-95 CDMA signal (1.2288 MHz) and the tones were equally spaced. An estimate of in-band distortion can be obtained with less accuracy when using a smaller number of tones. A maximum of 10 runs for the randomization of the input phases was needed to reach convergence. Fig 2 shows the total output and the uncorrelated distortion spectra of a 16-tone signal with fixed phases (zero initial phases) compared to those of a Forward-link IS-95 spectrum. Figs 1 and 2 also show that he shape of the uncorrelated spectrum depends on the number of tones and their initial phases. It is clear that in-band distortion is minimum when the phases are aligned while the out-of-band distortion is at its maximum. With random input phases, the uncorrelated distortion has an almost flat spectrum with in-band distortion is at its maximum while the out-of-band distortion is at its minimum. Fig 3 shows in-band distortion as a function of input power of multi-sine signals and a CDMA signal and compared to measured in-band distortion of the CDMA signal. The figure shows a good agreement in the estimation of in-band distortion of the random phase multi-sine and CDMA signals and with measured data of a CDMA signal. The measured in-band distortion was done using feed-forward cancellation.

V. Conclusion

We have developed an analysis of multi-sine signals in a nonlinear system. The analysis is used for the estimation of in-band distortion of communication signals. We have shown that traditional two-tone test is inadequate for the estimation of in-band distoriton. It was also shown that Multi-sine signals (four tones and more) with random phases can be used to estimate in-band distortion in real communication signals. This is significant because multi-sine signals are easy to simulate and require less simulation time complexity. The accuracy of the simulations using multi-sine signals depends on the number of tones and the randomization of the initial phases.

References

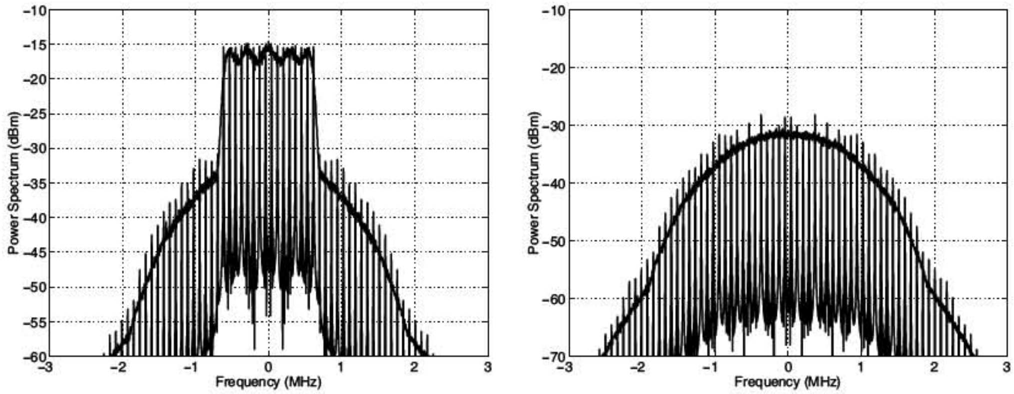
[1] K. A. Remley, "Multi-sine excitation for ACPR measurements" *Int. Microwave Symposium*, June 2003, pp. 2141–2144.

[2] J.C. Pedro, N. B. Carvalho, "On the use of multi-tone technique for assessing RF components, intermodulation distortion," *IEEE Trans. Microwave Theory Techn.*, vol. 52, pp. 2393–2402, Dec. 1999.

[3] N. Boulejfen, A. Harguem and F. M. Ghannouchi, "New closed-form expression for the prediction of multi-tone intermodulatiion distortion in fifth-order nonlinear rf circuits/systems," *IEEE Trans. Microwave Theory Techn.*, vol. 52, pp. 121–132, Jan. 2004.

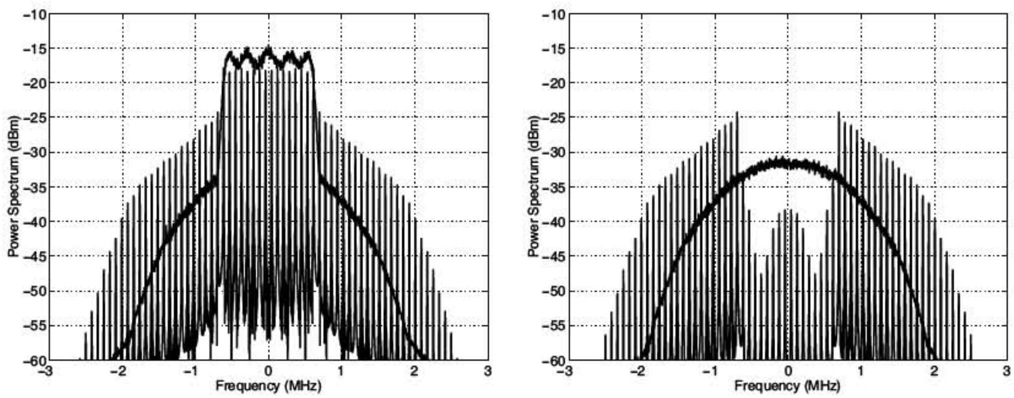
[4] K. M. Gharaibeh, K. Gard and M. B. Steer, "The impact of nonlinear amplification on the performance of CDMA systems," *Radio and Wireless Conference*, Sept. 2004, pp. 83–86.

[5] J. S. Bendat, *Nonlinear System Analysis and Identification*, John Wiley and Sons, NY 1990.



(a) (b)

Fig. 1. Simulated output spectrum and uncorrelated distortion spectrum of random phase 16-tone signal and an IS-95 signal; (a) total spectrum, (b) uncorrelated distortion spectrum (the bold line represents the CDMA signal).



(a) (b)

Fig. 2. Simulated output spectrum and uncorrelated distortion spectrum of phase aligned 16-tone signal and an IS-95 signal; (a) Output spectrum and (b) uncorrelated distortion spectrum (the bold line represents the CDMA signal)

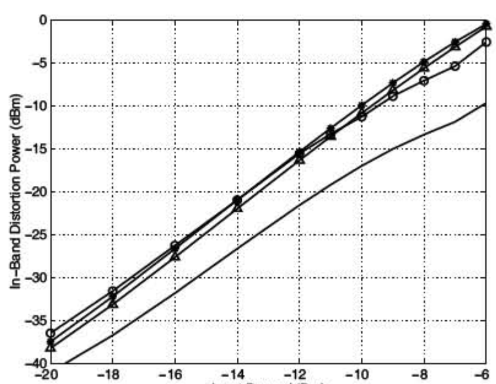


Fig. 3. In-Band distortion vs. input power; $*$: random phase 16-tone signal, $\triangle$: simulated CDMA signal, $\circ$: measured CDMA signal and solid: phase aligned 16-tone signal.