Autocorrelation Analysis of Distortion Generated from Bandpass Nonlinear Circuits

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Abstract

Estimation of spectral regrowth generated by a digitally modulated carrier passed through a nonlinear RF circuit is analyzed using a formulation of the output autocorrelation function. The estimation is based on developing an analytical expression for the output power spectrum when the nonlinearity is modeled a complex power series model extracted from measured amplitude-to-amplitude (AM-AM) and amplitude-to-phase (AM-PM) characteristics. Comparisons are presented of measured versus predicted ACPR values for a CDMA amplifier.

Introduction

Wireless technology is one of the fastest growing fields in circuit development today. Digital systems now dominate the cellular phone industry bringing the promise of mobile data and internet capabilities. Code Division Multiple Access (CDMA) is one of the premier digital technologies for current and future generation wireless systems because of its superior performance and capacity. Many systems use linear modulation techniques such as BPSK, QPSK, O-QPSK, D-QPSK, HPSK, etc. These linear modulation schemes exhibit amplitude modulation of the carrier envelope and require highly linear amplifiers to prevent system performance degradation from excessive intermodulation distortion. Unfortunately, the linearity requirement restricts amplifiers to operate below the maximum output power capacity thereby reducing the operating efficiency and available talk time of a wireless handset.

The importance of quantifying linearity and efficiency of wireless systems and circuits has provoked a review of different methods to simulate and measure distortion generated by wireless integrated circuits such as power amplifiers and transmitter integrated circuits. Traditional distortion analysis and measurement from two sinusoidal tones is not adequate to describe the distortion generated from digital signals. Digital modulation signals are random in nature, and exhibit a different amplitude distribution than sinusoidal signals. Exact analytical expressions for modulated waveforms are complicated, and difficult to work with, for high order nonlinearities. Simulation is often used as an alternative to analysis [1-2]; however, full circuit simulation is time and resource intensive requiring many hours to simulate a moderate number of transmitted symbols.

This paper presents an autocorrelation analysis of the distortion generated when a modulated carrier is passed through a bandpass nonlinearity model of a nonlinear circuit. The bandpass nonlinear model is formulated as a complex power series representation of the nonlinear gain characteristic of the carrier also known as the amplitude-to-amplitude (AM-AM) and amplitude-to-phase (AM-PM) distortion characteristic. A power series representation permits an analytical analysis of the output autocorrelation function of a modulated carrier passed through the model. The general autocorrelation function for the output signal is derived for a modulated carrier passed through the model. A special case of a complex gaussian input signal is analyzed yielding a closed form expression of the output autocorrelation function. Finally, measured distortion data is compared to simulated distortion based upon the measured AM-AM, AM-PM data and the autocorrelation model.

Bandpass Nonlinearity Analysis

A wireless digital communication signal is generated from a quadrature modulator as shown in Figure 1. Inphase and quadrature carrier signals are mixed respectively with two analog input signals representing the inphase and quadrature symbol components of the data. The mixed signals are summed together to form the quadrature modulated carrier signal.

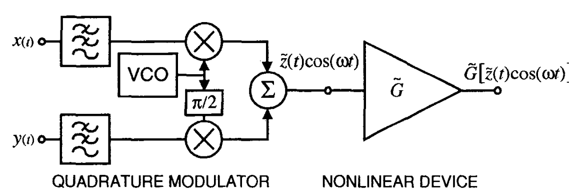

Figure 1: Block diagram of quadrature modulator.

The development presented here is an extension of earlier related work [3], on transformation of a gaussian signal, to the nonlinear transformation of general time domain signals. Consider the complex envelope representation of an amplitude and phase modulated carrier, $w(t)$, with carrier frequency ω_c,

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$$w(t) = A(t)\cos[\omega_c t + \theta(t)] \\ = \frac{1}{2}\tilde{z}(t)e^{j\omega_c t} + \frac{1}{2}\tilde{z}^*(t)e^{-j\omega_c t} \quad (1)$$

where $A(t)$ and $\theta(t)$ are the amplitude and phase components of the modulation. Carrier modulation is contained in the complex envelope, $z(t)$, the quadrature signal and is represented in either polar or rectangular form

$$\tilde{z}(t) = A(t)e^{j\theta(t)} = x(t) + jy(t) \quad (2)$$

The modulated carrier signal is applied to a nonlinear circuit with a nonlinear gain characteristic, $G[w(t)]$. The nonlinear gain characteristic is assumed to be a bandpass nonlinearity containing no significant memory within the bandwidth of the modulation [4]. Thus, the AM-AM and AM-PM nonlinearities respond instantaneously to amplitude changes from the modulated carrier signal. It is important to note that the AM-AM and AM-PM response represents the transfer characteristic of the input to the desired output frequency. A complex power series expansion is used to model the instantaneous AM-AM and AM-PM characteristics

$$\tilde{G}[w(t)] = \tilde{a}_1 w(t) + \tilde{a}_3 w(t)^3 + \tilde{a}_5 w(t)^5 + \ldots + \tilde{a}_N w(t)^N . \quad (3)$$

An odd power series is used because only odd terms contribute to the fundamental input/output characteristic. To simplify the analysis, a binomial expansion is used to compute the m^{th} power of $w(t)$ yielding

$$w^m(t) = \frac{1}{2^m}\sum_{k=0}^{m}\binom{m}{k}[\tilde{z}(t)]^k[\tilde{z}^*(t)]^{m-k}e^{j\omega_c(2k-m)t} . \quad (4)$$

Consider now only the complex envelope terms centered at the carrier frequency (this is usually referred as the first zonal filter at the output of the nonlinearity). This implies 2k-m=±1 for odd n only. Then the complex envelope of (4) around the fundamental becomes

$$w^m(t) = \frac{1}{2^{m-1}}\binom{m}{\frac{m+1}{2}}[\tilde{z}(t)]^{\frac{m+1}{2}}[\tilde{z}^*(t)]^{\frac{m-1}{2}} \quad (5)$$

It is convenient to rewrite (5) in terms of a sequence of odd powers by substituting m=2n+1

$$w^{2n+1}(t) = \frac{1}{2^{2n}}\binom{2n+1}{n+1}[\tilde{z}(t)]^{n+1}[\tilde{z}^*(t)]^n \quad (6)$$

Combining (6) with the complex gain expression (3) yields the fundamental envelope output signal

$$\tilde{G}[\tilde{z}(t)] = \sum_{n=0}^{(N-1)/2}\frac{\tilde{a}_{2n+1}}{2^{2n}}\binom{2n+1}{n+1}\tilde{z}(t)^{n+1}[\tilde{z}^*(t)]^n \quad (7)$$

This expression describes the complex envelope, of the first harmonic, of a modulated carrier signal passed through a bandpass nonlinear circuit described by a complex power series.

Autocorrelation Analysis of Nonlinear Systems

The power spectral density of a signal is commonly estimated by computing a Fourier transform of the output signal; however, an alternative approach is to first estimate the autocorrelation function of the signal followed by a Fourier transform

$$\tilde{S}_{zz}(f) = \int_{-\infty}^{\infty}\tilde{\Re}_{zz}(\tau)e^{-j\omega\tau}d\tau . \quad (8)$$

Autocorrelation analysis has the advantage of providing a method to compute each component of the output spectrum in terms of the input signal and the nonlinear model. The time average autocorrelation function is defined as the convolution of a signal with its complex conjugate

$$\tilde{\Re}_{zz}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tilde{z}(t)\tilde{z}^*(t+\tau)dt . \quad (9)$$

In practice a discrete time estimate of the autocorrelation function is used from a realization of the input signal

$$\hat{\Re}_{zz}(m) = \frac{1}{K}\sum_{k=|m|}^{K-1}\tilde{z}(k)\tilde{z}^*(k+m), m = -1,-2,\ldots,1-K . \quad (10)$$

An autocorrelation function estimate of a IS-95 CDMA [5] reverse link signal is shown in Figure 2.

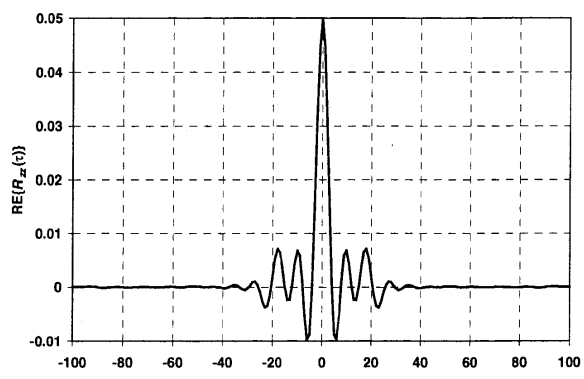

Figure 2: Autocorrelation estimate for IS-95 signal.

The output autocorrelation function of the model is computed from

$$\tilde{\Re}_{gg}(\tau) = \frac{1}{2T}\int_{-T}^{T}\tilde{G}[\tilde{z}(t)]\tilde{G}^*[\tilde{z}(t+\tau)]dt . \quad (11)$$

Expanding (11) by substituting (7) leads to

$$\tilde{\Re}_{gg}(\tau) =$$
$$|\tilde{a}_1|^2 \tilde{\Re}_{z_1z_1}(\tau) + \frac{3}{4}\left[\tilde{a}_1\tilde{a}_3^*\tilde{\Re}_{z_1z_3}(\tau) + \tilde{a}_1^*\tilde{a}_3\tilde{\Re}_{z_3z_1}(\tau)\right] +$$
$$\frac{9}{16}|\tilde{a}_3|^2 \tilde{\Re}_{z_3z_3}(\tau) + \frac{5}{8}\left[\tilde{a}_1\tilde{a}_5^*\tilde{\Re}_{z_1z_5}(\tau) + \tilde{a}_1^*\tilde{a}_5\tilde{\Re}_{z_5z_1}(\tau)\right] +$$
$$\frac{15}{32}\left[\tilde{a}_3\tilde{a}_5^*\tilde{\Re}_{z_3z_5}(\tau) + \tilde{a}_3^*\tilde{a}_5\tilde{\Re}_{z_5z_3}(\tau)\right] + \frac{25}{64}|\tilde{a}_5|^2 \tilde{\Re}_{z_5z_5}(\tau) + \ldots \quad (12)$$

where

$$\tilde{\Re}_{z_nz_m}(\tau) = \frac{1}{2T}\int_{-T}^{T} \tilde{z}_1^{\frac{n+1}{2}} \left(\tilde{z}_1^*\right)^{\frac{n-1}{2}} \tilde{z}_2^{\frac{m-1}{2}} \left(\tilde{z}_2^*\right)^{\frac{m+1}{2}} dt$$

and

$$\tilde{z}_1 = \tilde{z}(t) \quad and \quad \tilde{z}_2 = \tilde{z}(t+\tau).$$

The outut power spectrum is obtained from the Fourier transform of (12)

$$\tilde{S}_{gg}(f) =$$
$$|\tilde{a}_1|^2 \tilde{S}_{11}(f) + \frac{3}{4}\left[\tilde{a}_1\tilde{a}_3^*\tilde{S}_{13}(f) + \tilde{a}_1^*\tilde{a}_3\tilde{S}_{31}(f)\right] +$$
$$\frac{9}{16}|\tilde{a}_3|^2 \tilde{S}_{33}(f) + \frac{5}{8}\left[\tilde{a}_1\tilde{a}_5^*\tilde{S}_{15}(f) + \tilde{a}_1^*\tilde{a}_5\tilde{S}_{51}(f)\right] +$$
$$\frac{15}{32}\left[\tilde{a}_3\tilde{a}_5^*\tilde{S}_{35}(f) + \tilde{a}_3^*\tilde{a}_5\tilde{S}_{53}(f)\right] + \frac{25}{64}|\tilde{a}_5|^2 \tilde{S}_{55}(f) + \ldots \quad (13)$$

where

$$\tilde{S}_{nm}(f) = \int_{-\infty}^{\infty} \tilde{\Re}_{z_nz_m}(\tau) e^{-j\omega\tau} d\tau.$$

Therefore, the output spectrum is a sum of the Fourier transforms from each component of the autocorrelation function weighted by the appropriate power series coefficients. For a particular modulation waveform, the individual autocorrelation and spectrum terms are computed only once and stored in a file. At run time, the spectral components are read, then scaled by the power series coefficients, and summed to yield the output spectrum.

Analysis of Complex Gaussian Signals

The previous discussion derived the time average output autocorrelation function of a general time domain signal. The autocorrelation functions of several important communication signals are best described using statistical properties instead of a realization of the signal. Ergodicity is a property of signals where the statistical and time average properties are identical. The autocorrelation function of an ergodic signal can be calculated from either the time domain signal or by using statistical moments

$$\tilde{\Re}_{zz}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \tilde{z}(t)\tilde{z}^*(t+\tau)dt = E\left[\tilde{z}(t)\tilde{z}^*(t+\tau)\right] \quad (14)$$

where $E[]$ is the expectation operator denoting the outcome of a moment calculation of the operand. The statistical properties of real and complex gaussian signals are well known. The moments involving permutations of two complex gaussian signals are

$$E\left[\tilde{z}_1\tilde{z}_2^*\right] = \tilde{\Re}_{z_1z_2}(\tau),\ E\left[\tilde{z}_1^*\tilde{z}_2\right] = \tilde{\Re}_{z_1z_2}^*(\tau),$$
$$E\left[\tilde{z}_1\tilde{z}_1^*\right] = \tilde{\Re}_{z_1z_2}(0),\ E\left[\tilde{z}_2\tilde{z}_2^*\right] = \tilde{\Re}_{z_1z_2}^*(0),$$
$$E\left[\tilde{z}_1\tilde{z}_1\right] = E\left[\tilde{z}_2\tilde{z}_2\right] = E\left[\tilde{z}_1\tilde{z}_2\right] = 0. \quad (15)$$

The moments of multiple complex gaussian signal are [6]

$$E\left[\tilde{z}_1\tilde{z}_2\ldots\tilde{z}_s\tilde{z}_1^*\tilde{z}_2^*\ldots\tilde{z}_t^*\right]$$
$$= \begin{cases} 0 & , s \neq t \\ \sum_{\pi} E\left[\tilde{z}_{\pi(1)}\tilde{z}_1^*\right]E\left[\tilde{z}_{\pi(2)}\tilde{z}_2^*\right]\ldots E\left[\tilde{z}_{\pi(s)}\tilde{z}_t^*\right] & , s = t \end{cases} \quad (16)$$

where s and t denote the set of complex gaussian random processes, and π is a permutation of the set of integers {1, 2, ..., s}. The expectation for the case of a carrier modulated by a single complex gaussian signal is

$$\tilde{\Re}_{z_nz_m}(\tau) = E\left[\tilde{z}_1^{\frac{n+1}{2}} \left(\tilde{z}_1^*\right)^{\frac{n-1}{2}} \tilde{z}_2^{\frac{m-1}{2}} \left(\tilde{z}_2^*\right)^{\frac{m+1}{2}}\right]. \quad (17)$$

The output autocorrelation function is obtained by calculating each of the moments using (16). After computing several of the moments and collecting like power terms, it can be shown that the autocorrelation function terms follow the following pattern

$$\tilde{R}_{gg}^{2k+1}(\tau) = \tilde{R}_{zz}^{k+1}(\tau)\left[\tilde{R}_{zz}^*(\tau)\right]^{k-1} \left|\sum_{n=k}^{(N-1)/2} \frac{\tilde{a}_{2n+1}(2n+1)!}{2^{2n}(n-k)!} R_{zo}^{n-k}\right|^2 \quad (18)$$

where

$$R_{zo} = \tilde{R}_{zz}(\tau = 0).$$

The output autocorrelation function is the sum of all terms

$$\tilde{R}_{gg}(\tau) = \sum_{k=0}^{(N-1)/2} \tilde{R}_{gg}^{2k+1}(\tau). \quad (19)$$

The output autocorrelation function is a closed form expression in terms of autocorrelation of the input signal and the sum of the input power weighted by the power series coefficients. Note that there are only N autocorrelation terms as compared to N^2 for the general time domain case. The output power spectrum is the Fourier transform of the autocorrelation function

$$\tilde{S}_{gg}(f) = \sum_{k=0}^{(N-1)/2} \tilde{S}_{gg}^{2k+1}(f) \quad (20)$$

where

$$\tilde{S}_{gg}^{2k+1}(f) = \left|\sum_{n=k}^{(N-1)/2} \frac{\tilde{a}_{2n+1}}{2^{2n}} \frac{(2n+1)!}{(n-k)!} R_{zo}^{n-k}\right|^2 F\left\{\tilde{R}_{zz}^{k+1}(\tau)\left[\tilde{R}_{zz}^*(\tau)\right]^{k-1}\right\}$$

and

$$F\left\{\tilde{R}_{zz}^{k+1}(\tau)\left[\tilde{R}_{zz}^{*}(\tau)\right]^{k-1}\right\}=\int_{-\infty}^{\infty}\tilde{R}_{zz}^{k+1}(\tau)\left[\tilde{R}_{zz}^{*}(\tau)\right]^{k-1}e^{-j2\pi f\tau}d\tau .$$

Use of the moment theorem yielded a closed form expression for the output spectrum and greatly reduced the number of terms in the calculation.

Measured Results

The general time domain autocorrelation function method was used to calculate the output power spectrum of a integrated RF amplifier. The amplifier is a GaAs MESFET 900MHz driver amplifier [7]. The AM-AM, AM-PM characteristics were measured using a vector network analyzer to measure the complex gain as a function of input power. The measured AM-AM, AM-PM characteristics are shown in Figure 3. A complex power series of order N=13 was fitted to the measured data using a least squares solution to the over determined system of equations using the measured data and (7) with a sinusoid input signal. The resulting fit is also shown in Figure 3 as a dashed line.

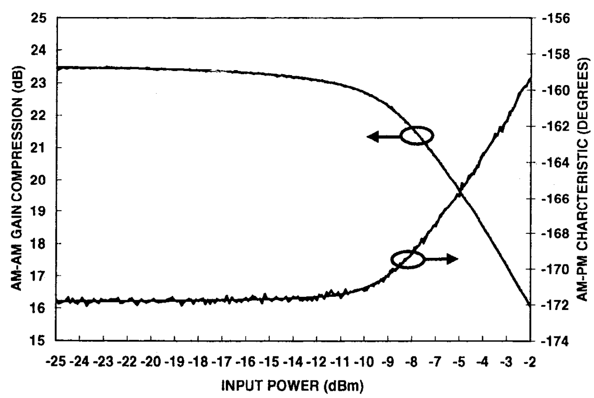

Figure 3: Measured and modeled AM-AM, AM-PM characteristics.

A carrier modulated with a single CDMA reverse link single is applied to the amplifier circuit and the output distortion measured using a spectrum analyzer. Adjacent channel power ratio (ACPR) is the standard distortion measurement for CDMA transmitters. ACPR is the ratio, in decibels, of the distortion power, in a 30kHz bandwidth offset by ±885KHz, and the desired channel power, in a 1.23MHz bandwidth. The measured ACPR is shown in Figure 4.

The ACPR was simulated using (13) and the power series coefficients. The individual autocorrelation terms are estimated using (10) from a realization of 2^{16} CDMA symbols. The power spectrum terms of (13) were obtained by taking the Fourier transform of each autocorrelation term. The spectrum terms were saved to a file for use at run time. The ACPR program reads in the power spectrum terms, weights them by the power series coefficients and the input power, sums up the terms, and computes ACPR from the resulting power spectrum. This process is repeated for each input power level. The simulated ACPR results agree well with the measured data as shown in Figure 4. The ACPR plateaus at lower output power because of the finite rejection of the CDMA baseband filter used by the waveform generator. The computation time was 38 seconds on a Pentium III 450MHz workstation for 101 input power points.

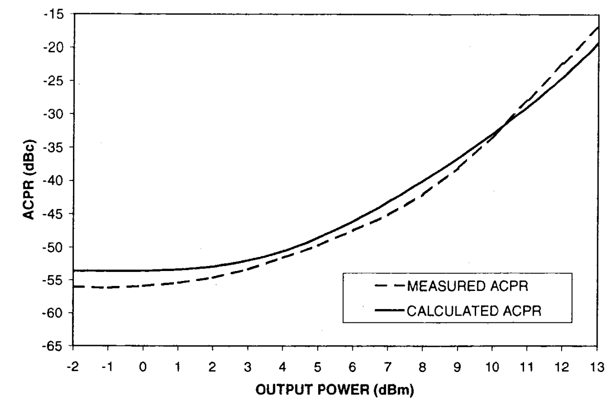

Figure 4: Measured and simulated ACPR.

Conclusions

A method for simulating the distortion generated by passing a modulated carrier through a bandpass nonlinear circuit was presented. The output autocorrelation function is a sum of moments of the input signal weighted by a set of complex power series coefficients representing the complex gain characteristic of a bandpass nonlinear circuit. The formulation accounts for the spectral contribution of each term in the power series model thus providing insight into how to reduce distortion through manipulation of the power series coefficients. A closed form expression for the output autocorrelation function was obtained for the special case when the input signal is a complex gaussian random variable.

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