# **Electromagnetic Modeling of an Aperture-Coupled** Patch Array in the N-Port Layered Waveguide for Spatial Power Combining Applications

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#### Abstract

An integral equation formulation is proposed for the analysis of waveguide-based aperture-coupled patch arrays for use in spatial power combining systems. A method of moments discretization is used to reduce a coupled set of integral equations to a matrix system resulting in the Generalized Scattering Matrix (GSM) for the Nport waveguide transition. In addition, the GSM for a rectangular waveguide taper is obtained using a mode-matching technique. Receiving and transmitting waveguide modules are cascaded to obtain the GSM of the entire passive structure. The proposed electromagnetic algorithm has been incorporated in a modeling environment for the analysis of complete waveguide-based aperture-coupled patch amplifier arrays, including passive elements and active devices.

#### Introduction

In recent years, an increased interest in open and waveguide-based spatial power combining systems [1,2] has created opportunities for the modeling and design of efficient high-power combiners and for a better understanding of power combining mechanisms in free space. This includes full-wave electromagnetic modeling of passive guided-wave structures, field-circuit interfacing techniques (i.e. active devices coupled into the electromagnetic environment), and the modeling of multimoding, coupling and radiation effects. In this paper, we present a full-wave integral equation formulation of an aperture-

coupled patch array in the layered N-port waveguide transition. This is developed for the analysis of a complete waveguide-based amplifier array, which includes receiving and transmitting waveguide tapers, N-port patch-to-slot waveguide transitions, and amplifier networks [3]. The analysis of the N-port waveguide transition (geometry shown in Fig. 1) is based on the method of moments integral equation formulation for electric and magnetic currents induced on the surface of the patch array and slot apertures coupled to single-mode waveguides. The GSM of the transition is obtained for all propagating and evanescent TE and TM modes, taking into account a coupling of all ports. A rectangular waveguide taper (shown in Fig. 1), approximated by double-plane stepped junctions, is analyzed using the mode-matching technique similar to [4]. The GSMs of waveguide tapers and N-port waveguide transitions are cascaded to obtain the overall response of the entire passive structure. Numerical results are presented for a single patch-to-slot waveguide transition, two cascaded waveguide-based patch-to-slot transitions (patch-slot-patch), and two cascaded 2  $\times$ 3 patch-to-slot arrays with rectangular waveguide tapers.

#### Theory

Consider the N-port waveguide transition (Fig. 1), which contains a patch array  $S^i_m$  at the interface  $S_d$  of adjacent dielectric layers with permittivities  $\epsilon_1$  and  $\epsilon_2$  in the overmoded waveguide (regions  $V_1$  and  $V_2$ , respectively), slot apertures  $S_s^j$  at the ground plane  $S_g$ , and N-1 single-mode waveguides coupled to the patch array through an array of slots (regions  $V_s^j$  of the dielectric permittivity  $\epsilon_3$ ).

A coupled set of integral equations is obtained by enforcing the boundary condition on the tangential components of the electric field on the conductive surfaces  $S_m^i$ 

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Figure 1: Geometry of a waveguide-based aperture-coupled patch array in the N-port layered waveguide. A rectangular waveguide transition (taper) is used for the excitation by the dominant mode.

at z = 0 and the continuity condition on the tangential components of the magnetic field on the magnetic surfaces (slot apertures)  $S_j^i$  at  $z = \tau$  (similar formulations have been presented in [5]). We are particularly interested in the electric field rep-resentation in the region  $V_1$  and the magnetic field representation in regions  $V_2$  and  $V_s^j$  (to satisfy boundary and continuity conditions). The total electric field in the region  $V_1$  is obtained in the integral form in terms of the unknown electric current density  $\overline{\mathbf{J}}_i(\vec{r}')$  and magnetic current density  $\overline{\mathbf{M}}_j(\vec{r}')$  using the second vector-dyadic Green's theorem:

$$\overline{\mathbf{E}}_{1}^{tot}(\vec{r}) = \overline{\mathbf{E}}_{1}^{inc}(\vec{r}) - \jmath \omega \mu_{0} \sum_{i=1}^{N_{m}} \int_{S_{m}^{i}} \overline{\mathbf{J}}_{i}(\vec{r}') \cdot \overline{\mathbf{G}}_{e1}^{(11)}(\vec{r}',\vec{r}) dS' \\
+ \sum_{j=1}^{N_{s}} \int_{S_{s}^{j}} \overline{\mathbf{M}}_{j}(\vec{r}') \cdot [\nabla' \times \overline{\mathbf{G}}_{e1}^{(21)}(\vec{r}',\vec{r})] dS' .$$
(1)

The electric dyadic Green's functions of the third kind,  $\overline{\overline{G}}_{e1}^{(11)}(\vec{r},\vec{r}')$  and,  $\overline{\overline{G}}_{e1}^{(21)}(\vec{r},\vec{r}')$ , are obtained for regions  $V_1$  and  $V_2$  satisfying boundary and continuity conditions for the electric field on the surfaces  $S_M$ ,  $S_g$ , and on the interface  $S_d$  (see [5]). The total magnetic field in the region  $V_2$  due to induced electric  $\overline{J}_i(\vec{r}')$  and magnetic  $\overline{M}_j(\vec{r}')$  currents is obtained as follows:

$$\overline{\mathbf{H}}_{2}^{iot}(\vec{r}) = \overline{\mathbf{H}}_{2}^{inc}(\vec{r}) + \frac{\epsilon_{2}}{\epsilon_{1}} \sum_{i=1}^{N_{m}} \int_{S_{m}^{i}} \overline{\mathbf{J}}_{i}(\vec{r}') \cdot [\nabla' \times \overline{\overline{\mathbf{G}}}_{e2}^{(12)}(\vec{r}',\vec{r})] dS' 
+ \jmath \omega \epsilon_{0} \epsilon_{2} \sum_{j=1}^{N_{s}} \int_{S_{s}^{j}} \overline{\mathbf{M}}_{j}(\vec{r}') \cdot \overline{\overline{\mathbf{G}}}_{e2}^{(22)}(\vec{r}',\vec{r}) dS',$$
(2)

and in the regions  $V_s^j$  due to magnetic current  $\overline{\mathbf{M}}_j(\vec{r}')$ :

$$\overline{\mathbf{H}}_{3}^{tot}(\vec{r}) = -\jmath\omega\epsilon_{0}\epsilon_{3}\sum_{j=1}^{N_{s}}\int_{S_{s}^{j}}\overline{\mathbf{M}}_{j}(\vec{r}^{\,\prime})\cdot\overline{\mathbf{G}}_{e2}^{(j)}(\vec{r}^{\,\prime},\vec{r})]dS^{\prime}.$$
(3)

The electric dyadic Green's functions of the third kind,  $\overline{\mathbf{G}}_{e2}^{(12)}(\vec{r},\vec{r}')$ ,  $\overline{\mathbf{G}}_{e2}^{(22)}(\vec{r},\vec{r}')$  have been obtained for regions  $V_1$  and  $V_2$  satisfying boundary and continuity conditions for the magnetic field vector. The electric dyadic Green's functions of the second kind  $\overline{\overline{\mathbf{G}}}_{c2}^{(j)}(\vec{r},\vec{r}')$  are obtained for the semi-infinite waveguides (regions  $V_s^j$ )

terminated by a ground plane at  $z = \tau$  (similar formulations are shown in [5]).

The incident electric  $\overline{\mathbf{E}}_{1}^{inc}(\vec{r})$  and magnetic  $\overline{\mathbf{H}}_{2}^{inc}(\vec{r})$  fields are expressed in terms of eigenmode expansions, including propagating and evanescent TE and TM modes normalized by a unity power condition [5]. Note that the incident magnetic field in the regions  $V_{s}^{i}$  is similarly handled. The integral representations (1), (2), and (3) yield a coupled system of integral (functional) equations, which is discretized using the method of moments. The GSM for the N-port waveguide transition is obtained by relating the magnitudes of incident and scattered modes at each port, where the magnitudes of scattered modes are expressed in terms of current coefficients (in the current expansion) using a unity power normalization.

A rectangular waveguide taper (Fig. 1) is approximated by double-plane stepped junctions. Each junction is analyzed by the mode-matching technique similar to [4] The GSM of the entire linear taper is obtained by cascading GSMs of individual junctions. The accuracy of the GSM cascading algorithm depends on the number of TE and TM modes and steps used to simulate the tapered waveguide transition.

### **Results and Discussion**

The numerical results are presented here for several passive waveguide-based structures, including a single unit cell and an aperture-coupled patch array operating at X-band. Geometrical and material parameters used for the single patch-to-slot transition,  $2 \times 3$  aperture-coupled patch array, and a rectangular waveguide taper are given in [6] and [3], respectively. Fig. 2 demonstrates dispersion characteristics (magnitude and phase) for the reflection,  $S_{11}$ , and transmission,  $S_{21}$ , coefficients of the dominant TE<sub>10</sub> mode for the single patch-to-slot transition. This was used to obtain an overall response of two cascaded unit cells (patch-slot-patch transition). The results for the S-parameters of the entire structure are shown in Fig. 3. It can be seen that cascading of two unit cells results in the appearance of the second resonance at higher frequency (about 11 GHz). The numerical results were also generated for  $2 \times 3$  patch array and a rectangular waveguide taper. Magnitude and phase of the reflection and transmission coefficients of the dominant mode are presented in Fig. 4 for the entire structure consisting of the input and output waveguide tapers (used for feeding and collecting) and two aperture-coupled 2 imes 3 patch arrays (patch-slot-patch transition). Again, we observed two sharp resonances at 10.5 GHz and approximately 11.5 GHz.

The proposed methodology and electromagnetic algorithm will be used for the optimization of waveguide-based aperture-coupled patch amplifier arrays for use in spatial power combining.

## References

References
[1]. Active and Quasi-Optical Arrays for Solid-State Power Combining, Edited by R. York and Z. Popovič. New York, NY: John Wiley & Sons, 1997.
[2]. Active Antennas and Quasi-Optical Arrays. Edited by A. Mortazawi, T. Itoh and J. Harvey. New York, NY: IEEE Press, 1999.
[3]. A.B. Yakovlev, S. Ortiz, M. Ozkar, A. Mortazawi, and M.B. Steer, "Electromagnetic modeling and experimental verification of a complete waveguide-based aperture-coupled patch amplifier array," *IEEE Int. Microwave Symp. Dig.*, June 2000, (to be published).
[4]. H. Patzelt and F. Arndt, "Double-plane steps in rectangular waveguides and their application for transformers, irises, and filters," *IEEE Trans. Microwave Theory Tech.*, vol. 30, May 1982, pp. 771-776.
[5]. A. B. Yakovlev, A. I. Khalil, C. W. Hicks, A. Mortazawi, and M. B. Steer, "The generalized scattering matrix of closely spaced strip and slot layers in waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 48, Jan. 2000 (in Press).
[6]. S. Ortiz and A. Mortazawi, "A perpendicular aperture-fed patch antenna for quasi-optical amplifier arrays," *IEEE AP-S Int. Symp.*, July 1999, pp. 2386-2389.

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Figure 2: Magnitude and phase of the reflection and transmission coefficients of a single patch-to-slot waveguide transition.



Figure 3: Magnitude and phase of the reflection and transmission coefficients of two cascaded waveguide-based single patch-to-slot transitions.



Figure 4: Magnitude and phase of the reflection and transmission coefficients of two cascaded 2  $\times$  3 patch-to-slot arrays with rectangular waveguide tapers.

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