Efficient MOM-Based Generalized Scattering Matrix Method for the Integrated Circuit and Multilayered Structures in Waveguide

Ahmed I. Khalil, *Student Member, IEEE*, Alexander B. Yakovlev, *Member, IEEE*, and Michael B. Steer, *Fellow, IEEE*

Abstract— **The generalized scattering matrix (GSM) approach is proposed to analyze transverse multilayered structures with circuit ports in a metal waveguide. The Kummer transformation is applied to accelerate slowly converging double series expansions of Green's functions that occur in evaluating the impedance matrix elements. In this transformation the quasi-static part is extracted and evaluated to speed up the solution process resulting in a dramatic reduction of terms in a double series summation. The formulation incorporates electrical ports as an integral part of the GSM formulation and so that the resulting model can be integrated with circuit analysis.**

I. Introduction

The Generalized Scattering Matrix (GSM) method has been widely used to model passive waveguide structures. It enables the partitioning of a complex structure into much simpler separable waveguide sections. Each waveguide section is described by its GSM that takes into account evanescent as well as propagating modes. The whole structure is then modeled by cascading the individual GSMs using simple matrix operations.

The problem of modeling multilayered structures with ports in a shielded environment can be analyzed by characterizing each layer using a GSM with ports and then cascading this matrix with neighboring layers to obtain the composite GSM of a complete structure such as that shown in Fig 1. In its common implementation, the MOM uses subdomain basis functions of currents and voltages which are determined between cells. The MOM technique is used here to compute a port impedance matrix in the solution process. The major contributions of the work presented here are

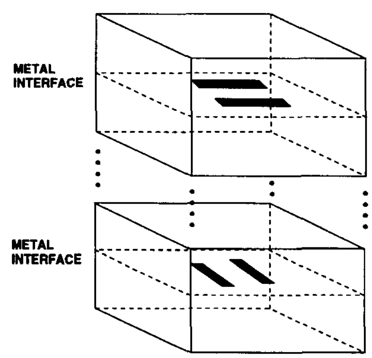

Fig. 1. A multilayer structure in metal waveguide showing cascaded blocks

- Introduction of circuit ports (ports with voltages and currents) into the GSM formulation. This supports the integrated circuit-field analysis of structures in waveguide.
- Efficient computation of the method of moments

This work was supported by the U.S. Army Research Office through Clemson University as a Multidisciplinary Research Initiative on Quasi-Optics, agreement number DAAG55-97-K-0132.

A. I. Khalil and A. B. Yakovlev are with the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27695-7914, USA.

M. B. Steer is with the Institute of Microwaves and Photonics, School of Electronic and Electrical Engineering, University of Leeds, Leeds, LS2 9JT, United Kingdom.

impedance matrix elements using Kummer transformation.

- Computing the GSM for a large number of modes without computing the induced currents as an intermediate step.

II. GSM for Metal Layer

The characterization of the metalized interface is developed by separately considering mode to mode, port to port, and port to mode interactions [1]. When applied to a single metalized interface, the method of moments leads to the matrix equation

$$[Z + Z_L][I] = [V] \qquad (1)$$

where the jith element of the impedance matrix $[Z]$ is

$$Z_{ji} = -\int\int\int\int \overline{B}_j(r).\overline{\overline{G}}_e(r,r')\cdot\overline{B}_i(r')dsds' \qquad (2)$$

the jth port voltage

$$V_j = \int\int \overline{B}_j(r).\overline{E}^i(r)ds \qquad (3)$$

and the load impedance

$$Z_L = diag(Z_{L1}...Z_{Li}...Z_{LN}) \qquad (4)$$

where B_j is the jth basis function, G_e is an electric-type double series Green's function, and Z_{Li} is the loading impedance at port i.

In order to speed up the computation of Z_{ji} an efficient technique based on the Kummer transformation has been applied in [2] to accelerate slowly converging series. This technique is utilized here to the Green's function components, leading to their transformation so that a quasi-static part (G_{ij}^{QS}) is extracted. The Green's function is then

$$G_{ij} = (G_{ij} - G_{ij}^{QS}) + G_{ij}^{QS}\ . \qquad (5)$$

The quasi-static component needs to be evaluated only once per frequency scan and the frequency dependent component $(G_{ij} - G_{ij}^{QS})$ is now fast convergent.

To construct the GSM efficiently it is essential to treat the incident field as being composed of a summation of waveguide modes rather than considering a single mode one at a time [3]. For an incident field propagating in the positive z direction from medium 1 into medium 2 at the interface

$$\overline{E}^i(r) = \sum_{l=1}^{L_{max}} a_l^1\ (1 + R_l)\ \overline{e}_l^+\ exp(-\Gamma_l^1 z) \qquad (6)$$

where Γ_l^1 is the propagation constant of mode l corresponding to medium 1, and R_l is the reflection coefficient of mode l. Substituting (6) into (3) the current described by (1) is written in terms of the modal vector $\mathbf{a}_1^1$ as

$$[I] = [Z + Z_L]^{-1}[W^+][U + R][\mathbf{a}_1^1] \qquad (7)$$

the elements of the $[W^+]$ matrix are given by

$$W_{ji}^+ = \int\int \overline{e}_i^+.\overline{B}_j ds$$

where U is the identity matrix and R is a diagonal matrix with diagonal elements being the modal reflection coefficients. Using the above expression for the current the scattering coefficients can be written as

$$[S_{11}^1] = -\frac{1}{2}[W^+]^T[Y][W^+][U + R] + [R] \qquad (8)$$

Similar expressions can be obtained for the other modal reflection coefficients.

The interaction between an incident mode and a port can be described using the concept of generalized power waves [1]. First assume that port k is terminated by an arbitrary impedance Z_{Lk}. Since the scattering parameters are normally given with reference to a 50 Ω system it is appropriate to set Z_{Lk} to $R_0 = 50\ \Omega$. The scattering matrix relating port to mode interaction is given by:

$$[S_{31}^1] = -[R_0]^{\frac{1}{2}}[Z + Z_L]^{-1}[W^+][U + R] \qquad (9)$$

III. Cascade Connection of Scattering Matrices

The technique of the previous section develops a GSM for a single interface at a transverse plane (with respect to the direction of propagation) in a metal waveguide. A multilayer structure such as that shown in Fig. 1 is modeled by cascading the GSMs of individual layers and propagation matrices. Each propagation matrix describes translation of the mode coefficients from one transverse plane to another through a homogeneous medium. The modeling of a two-layer structure with the layers separated by a waveguide section is illustrated in Fig. 2. The analysis proceeds by computing the GSM of the first layer $[S^{(1)}]$ and then evaluating a propagation matrix $[P]$ describing the waveguide section. Finally, computation of the GSM of the second layer $[S^{(2)}]$ enables cascading of $[S^{(1)}]$, $[P]$ and $[S^{(2)}]$ to obtain the composite GSM.

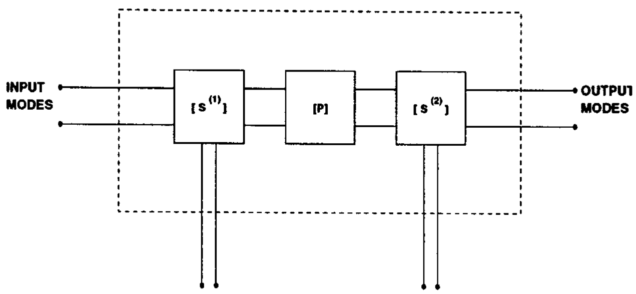

Fig. 2. Block diagram for cascading GSMs with ports

IV. Results and Discussion

The GSM-MOM method developed here can be used for analyzing multiple layers of arbitrarily shaped metalization. Numerical results have been obtained for two examples.

In the first example, a wide resonant strip embedded in a rectangular waveguide (with geometry shown in the inset of Fig. 5) is considered. The analysis of convergence and percentage error of the impedance matrix elements for the accelerated and direct double series summation are given in Figs. 3 and 4, respectively. It is shown that the error of 0.5% is obtained for 160 terms used in the accelerated summation procedure in comparison with 3000 terms required in the direct double series summation to reach the same error. The computation time is almost directly proportional to the number of terms in the summation and so the speed-up is approximately a factor of 20. Also, it can be difficult to determine when a sufficient number of terms is used to reach convergence by the direct method. Numerical results for the normalized susceptance of a wide strip agree well with the measured data provided in [4] (Fig. 5).

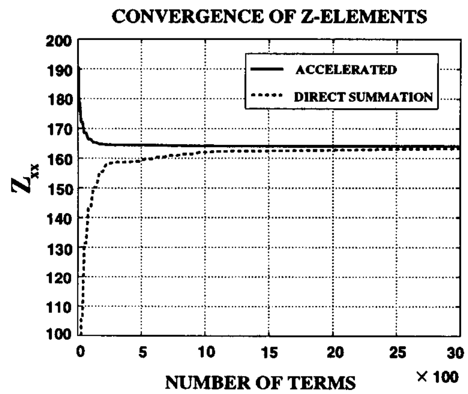

Fig. 3. Convergence of Z_{xx} matrix elements.

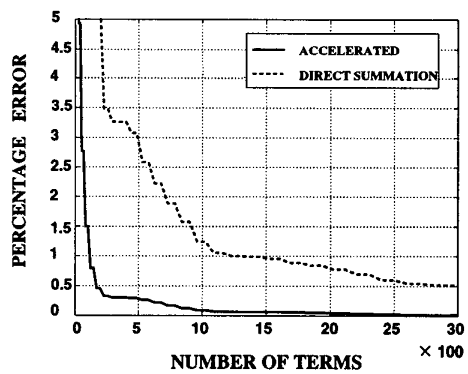

Fig. 4. Percentage error in the convergence of Z_{xx} .

The second example is of the shielded microstrip filter shown in Fig. 6. The filter is in a metal box of dimensions $92 \times 92 \times 11.4$ mm ($a \times b \times c$). The substrate height is 1.57 mm and it has a relative permitivity of 2.33. The top and bottom covers are perfect conductors and hence their GSMs

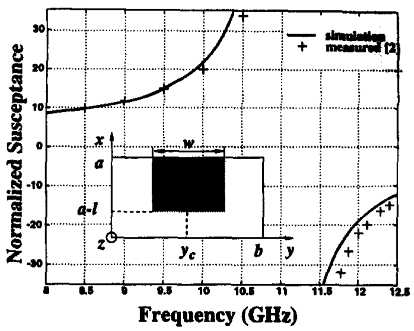

Fig. 5. Normalized susceptance, $a = 0.4$ inch, $b = 0.9$ inch, $w = 0.280$ inch, $\ell = 0.365$ inch, $y_c = b/2$.

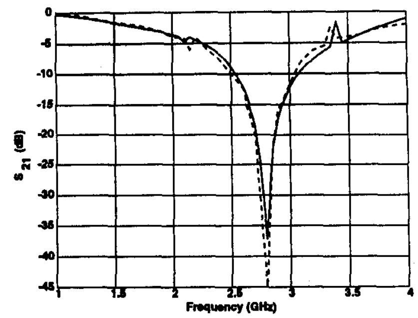

Fig. 7. Scattering parameter S_{21}: solid line is by MOM-GSM; doted line is from [5]

are diagonal matrices with -1 as diagonal elements. The intermediate layer is a metal layer with ports and the excitation ports are modeled by the delta-gap voltage model proposed in [5]. The number of modes considered in the GSM for the cover layers is 287. After cascading the three layers the modes are augmented. The final scattering matrix has rank two representing the circuit ports of the filter. The transmission coefficient S_{21} is calculated in Fig. 7 and compares favorably with previously reported results [5]. The work presented here is being used with the local reference node concept [6] to model waveguide-based spatial power combining circuits where the coupling of active devices in the waveguide environment is known to have significant effects on stability and performance.

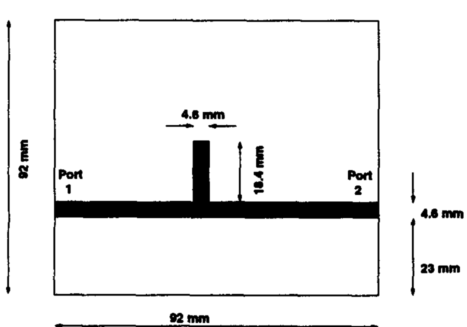

Fig. 6. Geometry of the microstrip stub filter.

REFERENCES

[1] L. Epp and R. Smith,"A generalized scattering matrix approach for analysis of quasi-optical grids and de-embedding of device parameters," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, May 1996, pp. 760-769.

[2] G. V. Eleftheriades, J. R. Mosig, and M. Guglielmi,"A fast integral equation technique for shielded planar circuits defined on nonuniform meshes," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, Dec. 1996, pp. 2293-2296.

[3] C. Wan and J. Encinar,"Efficient computation of generalized scattering matrix for analyzing multilayered periodic structures," *IEEE Trans. Antennas Propagat.*, Vol. 43, Nov. 1995, pp. 1233-1242

[4] H.-C. B. Chu and K. Chang,"Analysis of a wide resonant strip in waveguide," *IEEE Trans. Microwave Theory Tech.*, Vol. 40, March 1992, pp. 495-498.

[5] G. Eleftheriades and J. Mosig,"On the network characterization of planar passive circuits using the method of moments," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, March 1996, pp. 438-445.

[6] A. I. Khalil and M. B. Steer,"Circuit Theory for Spatially Distributed Microwave Circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. 46, Oct. 1998, pp. 1500-1502.