Efficient Method-of-Moments Formulation for the Modeling of Planar Conductive Layers in a Shielded Guided-Wave Structure

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***Abstract*— An electric-field integral-equation formulation discretized via the method of moments (MoM) is proposed for the analysis of arbitrarily shaped planar conductive layers in a shielded guided-wave structure. The method results in a generalized scattering matrix (GSM) for the planar structure and can be used with other GSM's, derived using this or other techniques, to model cascaded structures in waveguide. The Kummer transformation is applied to accelerate slowly converging double series expansions of impedance matrix elements obtained in the MoM solution. In this transformation, the quasi-static part associated with a singularity of the electric-type Green's function in the region of a conductive layer is extracted and evaluated in terms of modified Bessel functions, resulting in a dramatic reduction of terms in a double series summation. The proposed technique permits the modeling of a variety of conductive frequency-selective surfaces, including quasi-optical grids and patch arrays for application to spatial power combining.**

***Index Terms*—Acceleration techniques, electromagnetic analysis, Green's functions, layered waveguide, method of moments, planar conductive layers.**

I. Introduction

Shielded guided-wave structures are becoming an essential part of millimeter and submillimeter-wave systems with the application of micromachining technology as an alternative to expensive and time-intensive mechanical machining, and also with the development of waveguide-based spatial power combining systems [1], [2]. Periodic grid structures in waveguides, waveguide-based strip and microstrip filters, patch arrays, densely packaged passive elements and devices of microwave integrated circuits, and waveguide-based spatial power combiners are among the structures that can be categorized as planar conductive layers in a guided-wave environment. The work described in this paper is part of a project to model waveguide-based spatial power combiners. These are arranged as cascaded blocks in the transverse plane of the waveguide. Some of these blocks can be modeled as planar conductive layers, others as sections of open waveguide, while others require more complicated three-dimensional modeling, such as obtained using the finite-element method. Numerical electromagnetic (EM) analysis of these structures can be performed using almost any EM technique. However, efficient use of memory and the ability to reuse unmodified EM characterizations in iterative design are obtained using the generalized scattering matrix (GSM) approach. In the GSM method, each block is represented by a matrix that relates the coefficients of forward and backward propagating waveguide modes at the two sides of each block. These matrices are cascaded to arrive at the overall response of a multiblock sequence. The work presented here focuses on efficient formulation of the GSM matrix for a single planar conductive layer in waveguide. The approach is based on an integral-equation formulation (electric field or mixed potential) discretized via a method-of-moments (MoM) solution for the electric current induced on the surface of planar conductive layers. This eliminates the need to discretize the entire shielded structure or to discretize the entire volume of the structure. In this formulation, the planar conductors are generally discretized into cells and localized (termed "subdomain") basis functions are used to model the surface current density discretization.

An analysis of a narrow capacitive strip in a waveguide is provided in [3], and later, a variational form for the susceptance was obtained for a wide resonant strip [4] using an MoM discretization, yielding a characterization of the current distribution and voltages at gaps in the metallization. This makes MoM formulations particularly attractive when integrating EM modeling with circuit modeling, as voltage and current are used in both modeling domains. Several developments are related to this requirement. A conductive diaphragm in a rectangular waveguide has been analyzed by MoM with a dyadic Green's function formulation [5]. Also, the mutual impedance between thin metal probes positioned in a rectangular waveguide has been calculated in [6] using the reaction concept. Finally, numerical and experimental studies of thin metallic posts located in rectangular waveguides is provided in [7], based on the MoM solution for the current distribution. Piecewise sinusoidal or pulse basis functions were used in all cases for the current discretization.

The integral-equation formulation for the unknown electric current leads to the Fredholm integral equation of the first kind with a singular kernel (system of singular integral or integrodifferential equations), which is associated with a primary part of a Green's function in the region of a conductive

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layer (surface of a planar metallization where a point of observation and source point are located as a result of a boundary condition for the electric-field components). The most common representation of a Green's function (the dyadic Green's function components) for guided-wave problems is in the form of a double infinite series expansion over a complete system of eigenfunctions of a Sturm–Liouville operator (usually, Helmholtz differential operator), where a singularity of a primary part is implicitly introduced in a series expansion. A singularity of the electric-type Green's function is not integrable in the region of metallization. In most cases, we deal with slowly converging double infinite series, which occur in the impedance matrix elements as a result of MoM discretization (a Galerkin method for a complete basis). Several attempts have been made to alter the summations and, thus, obtain a faster converging double series. Transformation of a double series expansion into a contour complex integral to which the residue theorem was applied was developed by Hashemi-Yeganeh [8]. This method leads to the computation of a few single summations of fast converging series. Park and Nam [9], in considering a shielded planar multilayered structure, transformed a scalar Green's function into a static image series that was evaluated using the Ewald method. It was pointed out that the final form of the Green's function converges rapidly with a small number of terms in a series summation.

To speed up the process of impedance matrix fill, several endeavors have been made in dividing a matrix operator into frequency-dependent and frequency-independent parts. Spectral operator expansion technique was introduced by Jansen and Sauer [10] for a high-speed EM simulation of three-dimensional (3-D) interconnects for microwave integrated circuit/monolithic microwave integrated circuit (MIC/MMIC) computer-aided design (CAD). Eleftheriades *et al.* [11] pioneered a procedure that partitions a potential Green's function into an asymptotic (frequency independent) part and dynamic part, where the asymptotic part was converted to a rapidly converging series summation. Basically, the extraction and evaluation of the static part is associated with a reduction of a singular matrix operator as the analog of a singular integral operator. We found this technique to be the most efficient one for the computation of the slow converging series occurring in the work presented in this paper. In addition, the method is flexible, enabling different basis functions to be used in the MoM formulation and without the geometrical restrictions imposed by the other methods. This idea of extraction of the asymptotic part of the Green's function and, consequently, the impedance matrix with its analytic evaluation, has been recently implemented by Park and Balanis for antenna and open microstrip discontinuity problems [12], [13].

The purpose of this paper is to present an efficient electric-field integral-equation formulation with an MoM discretization, which results in the GSM of arbitrarily shaped planar metal layers enclosed in a layered shielded environment. Electric dyadic Green's functions of the third kind are derived and implemented in integral equations for the unknown electric current induced on the metal surface. The quasi-static parts of the Green's functions and impedance matrix elements are extracted and evaluated based on the ideas proposed in [11], leading to a dramatic reduction of computational time in the MoM matrix fill. Numerical results are obtained for strip discontinuities with a complicated geometry showing the effectiveness of the proposed approach. The work has been incorporated in a GSM modeling scheme, which retains circuit ports, for the modeling of waveguide enclosed microwave structures, particularly spatial power combiners [14].

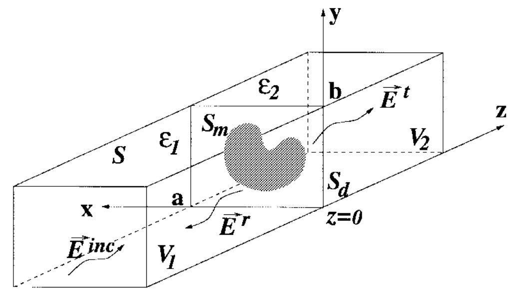

Fig. 1. Geometry of a planar arbitrarily shaped metallization in a shielded layered guided-wave structure.

II. Theory

Consider the rectangular waveguide shown in Fig. 1 with two dielectric layers adjacent to the current-carrying transverse interface S_d. The total electric field $\vec{E}$ in the volume V_1 with dielectric permittivity ϵ_1 is characterized by a total incident electric field (direct from an impressed current source and reflected from the dielectric interface in the absence of the metal surface) $\vec{E}^{\mathrm{inc}}$ and a scattered (reflected) field due to induced electric current $\vec{E}^r$, ($\vec{E} = \vec{E}^{\mathrm{inc}} + \vec{E}^r$). The total electric field in region V_2 with ϵ_2 is represented by the scattered (transmitted) field $\vec{E}^t$. (Note that an incident field from region V_2 is similarly handled.) An arbitrarily shaped metallization S_m, ($S_m \subset S_d$) is located on the interface at $z = 0$.

The electric-field integral-equation formulation is obtained by enforcing a boundary condition on the tangential components of the electric field on the conducting surface S_m

$$\jmath\omega\mu_0 \hat{z} \times \int_{S_m} \overline{\overline{\boldsymbol{G}}}(\vec{r}, \vec{r}\,') \cdot \overline{\boldsymbol{J}}(\vec{r}\,')\, dS' = \hat{z} \times \overline{\boldsymbol{E}}^{\mathrm{inc}}(\vec{r}) \quad (1)$$

where the integral over S_m (after cross multiplication) yields the scattered electric field due to the conduction current $\overline{\boldsymbol{J}}(\vec{r}\,')$ on the metal surface, and $\overline{\overline{\boldsymbol{G}}}(\vec{r}, \vec{r}\,')$ is the electric dyadic Green's function of the third kind [15], obtained for a two-layered rectangular waveguide.

The total electric field introduced by the boundary condition (1) is expressed as a series eigenmode expansion, including both propagating and evanescent TE and TM modes

$$\begin{aligned}\overline{\boldsymbol{E}}^{\mathrm{inc}}(x, y, 0) = &\sum_{m=0}^{\infty} \sum_{m \neq n=0}^{\infty} a_{mn}^{\mathrm{TE}} \overline{\boldsymbol{e}}_{mn}^{+\mathrm{TE}}(x, y)\Big(1 + R_{mn}^{\mathrm{TE}}\Big) \\ &+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^{\mathrm{TM}} \overline{\boldsymbol{e}}_{mn}^{+\mathrm{TM}}(x, y)\Big(1 + R_{mn}^{\mathrm{TM}}\Big) \quad (2)\end{aligned}$$

where a_{mn}^{TE}, a_{mn}^{TM} are unknown magnitudes of all propagating and evanescent TE and TM modes, respectively. The electric vector functions $\overline{\boldsymbol{e}}_{mn}(x, y)$ satisfy a unity power normalization condition [16]. The reflection R_{mn} and transmission T_{mn}

coefficients are determined as the solution of the boundary value problem for a two-layered waveguide in the absence of planar metallization

$$
\begin{aligned}
R_{mn}^{\rm TE} &= \frac{\gamma_{mn}^{(1)} - \gamma_{mn}^{(2)}}{\gamma_{mn}^{(1)} + \gamma_{mn}^{(2)}} \\
T_{mn}^{\rm TE} &= \frac{2\sqrt{\gamma_{mn}^{(1)}\gamma_{mn}^{(2)}}}{\gamma_{mn}^{(1)} + \gamma_{mn}^{(2)}} \\
R_{mn}^{\rm TM} &= \frac{\epsilon_1\gamma_{mn}^{(2)} - \epsilon_2\gamma_{mn}^{(1)}}{\epsilon_1\gamma_{mn}^{(2)} + \epsilon_2\gamma_{mn}^{(1)}} \\
T_{mn}^{\rm TM} &= \frac{2\sqrt{\epsilon_1\gamma_{mn}^{(2)}\epsilon_2\gamma_{mn}^{(1)}}}{\epsilon_1\gamma_{mn}^{(2)} + \epsilon_2\gamma_{mn}^{(1)}}
\end{aligned} \tag{3}
$$

where $\gamma_{mn}^{(i)}, (i = 1, 2)$ is the propagation constant defined as

$$
\gamma_{mn}^{(i)} = \begin{cases} \jmath\sqrt{k_i^2 - k_c^2}, & k_i^2 > k_c^2 \\ \sqrt{k_c^2 - k_i^2}, & k_i^2 < k_c^2 \end{cases} \tag{4}
$$

and

$$
k_c^2 = k_x^2 + k_y^2 \quad k_x = \frac{m\pi}{a} \quad k_y = \frac{n\pi}{b} \quad k_i = \frac{2\pi}{\lambda_0}\sqrt{\epsilon_i}.
$$

The vector integral equation (1) is now reduced to a coupled set of scalar integral equations in terms of the unknown electric current components

$$
\begin{aligned}
&\jmath\omega\mu_0 \left\{ \int_{S_m} G_{xx}(\vec{r}, \vec{r}')J_x(\vec{r}')\, dS' \right. \\
&\qquad \left. + \int_{S_m} G_{xy}(\vec{r}, \vec{r}')J_y(\vec{r}')\, dS' \right\} = E_x^{\rm inc}(\vec{r}) \\
&\jmath\omega\mu_0 \left\{ \int_{S_m} G_{yx}(\vec{r}, \vec{r}')J_x(\vec{r}')\, dS' \right. \\
&\qquad \left. + \int_{S_m} G_{yy}(\vec{r}, \vec{r}')J_y(\vec{r}')\, dS' \right\} = E_y^{\rm inc}(\vec{r}).
\end{aligned} \tag{5}
$$

Note that after multiplying by the factor $\jmath\omega\mu_0$, the integral over S_m represents the scattered (reflected and transmitted) electric field E_i at $\vec{r}$ due to unit surface current density J_j at $\vec{r}'$, $(i, j = x, y)$.

Here, development of MoM proceeds by discretizing the current J_j using local overlapping piecewise sinusoidal basis and testing functions in the coupled set of (5), leading to a matrix system of linear equations

$$
\Big[Z(\omega)\Big][I] = [V]. \tag{6}
$$

Here, $Z(\omega)$ is the impedance matrix of all self and mutual interactions of the electric-field vector components with the components of current density vector, I is the vector of unknown coefficients of the current expansion, and V is the vector of the incident field tested with the x- and y-directed local functions, respectively.

The magnitudes $b_{mn}^{\rm TE}$ and $b_{mn}^{\rm TM}$ of TE and TM modes in a series eigenmode expansion of the reflected electric field at $z = 0$ are expressed in the following form [14], [16]:

$$
\begin{Bmatrix} b_{mn}^{\rm TE} \\ b_{mn}^{\rm TM} \end{Bmatrix} = -\tfrac{1}{2} \int_{S_m} \boldsymbol{\overline{J}} \cdot \begin{Bmatrix} \boldsymbol{\overline{e}}_{mn}^{+\rm TE}(1 + R_{mn}^{\rm TE}) \\ \boldsymbol{\overline{e}}_{mn}^{+\rm TM}(1 + R_{mn}^{\rm TM}) \end{Bmatrix} dS + \begin{Bmatrix} a_{mn}^{\rm TE} \\ a_{mn}^{\rm TM} \end{Bmatrix} \begin{Bmatrix} R_{mn}^{\rm TE} \\ R_{mn}^{\rm TM} \end{Bmatrix}. \tag{7}
$$

Following the procedure shown in details in [14], (6) and (7) are combined via the vector of current coefficients I, resulting in the generalized reflection coefficients of all propagating and evanescent TE and TM modes. It should be noted that this formulation in conjunction with a matrix approach does not require the calculation of the induced electric current in the explicit form in order to obtain the GSM. It serves as an intermediate result in a matrix procedure to relate magnitudes of incident and reflected modes. A similar procedure has been applied to obtain the generalized transmission coefficients.

The electric dyadic Green's functions introduced in (1) and (5) are obtained as the solution of the coupled set of vector wave equations [15]

$$
\begin{aligned}
&\nabla \times \nabla \times \overline{\overline{\boldsymbol{G}}}_{11}(\vec{r}, \vec{r}') - k_1^2\overline{\overline{\boldsymbol{G}}}_{11}(\vec{r}, \vec{r}') = \overline{\overline{\boldsymbol{I}}}\delta(\vec{r} - \vec{r}'), \\
&\qquad \vec{r}, \vec{r}' \in V_1 \\
&\nabla \times \nabla \times \overline{\overline{\boldsymbol{G}}}_{21}(\vec{r}, \vec{r}') - k_2^2\overline{\overline{\boldsymbol{G}}}_{21}(\vec{r}, \vec{r}') = 0, \\
&\qquad \vec{r} \in V_2;\ \vec{r}' \in V_1
\end{aligned} \tag{8}
$$

subject to two sets of boundary conditions. The first set is of the first kind on the surface of a conducting shield S

$$
\hat{n} \times \overline{\overline{\boldsymbol{G}}}_{i1}(\vec{r}, \vec{r}') = 0, \qquad \vec{r} \in S;\ i = 1, 2. \tag{9}
$$

The second set describes the mixed continuity conditions for the electric Green's dyadics of the third kind on the interface of adjacent layers in the absence of the metal surface S_m at $z = 0$

$$
\begin{aligned}
\hat{z} \times \overline{\overline{\boldsymbol{G}}}_{11}(\vec{r}, \vec{r}') &= \hat{z} \times \overline{\overline{\boldsymbol{G}}}_{21}(\vec{r}, \vec{r}'), \qquad \vec{r} \in S_d \\
\hat{z} \times \left(\nabla \times \overline{\overline{\boldsymbol{G}}}_{11}(\vec{r}, \vec{r}')\right) &= \hat{z} \times \left(\nabla \times \overline{\overline{\boldsymbol{G}}}_{21}(\vec{r}, \vec{r}')\right)
\end{aligned} \tag{10}
$$

where V_i $(i = 1, 2)$ are the waveguide regions, S_d is the surface of the interface at $z = 0$, and $\hat{n}$ is an outer normal to the surface S. It should be noted that the location of the δ sources in the above formulation is considered to be in the region V_1. Similarly, the boundary value problem for the electric Green's dyadics $\overline{\overline{\boldsymbol{G}}}_{22}(\vec{r}, \vec{r}')$ and $\overline{\overline{\boldsymbol{G}}}_{12}(\vec{r}, \vec{r}')$ can be formulated for δ sources positioned in the region V_2.

Solution of the boundary value problem (8)–(10) yields nine components of the electric Green's dyadics expressed in terms of double infinite series expansions over the complete system of eigenfunctions of the Helmholtz operator. Note that according to the system of integral equations, i.e., (5), we are primarily interested only in the transverse components of the Green's functions calculated on the interface at $z = z' = 0$. Due to the continuity equations of (10), the transverse components of the Green's functions $\overline{\overline{\boldsymbol{G}}}_{11}(\vec{r}, \vec{r}')$ and $\overline{\overline{\boldsymbol{G}}}_{21}(\vec{r}, \vec{r}')$ are equal on the interface, which yields the unique representations

$$
G_{ij}(x, y; x', y') = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varphi_{mn}^{i}(x, y)\, \varphi_{mn}^{j}(x', y') f_{mn}^{ij}. \tag{11}
$$

The functions $\varphi_{mn}^{i,j}(x, y), (i, j = x, y)$ represent a complete set of orthonormal eigenfunctions of the Helmholtz operator satisfying appropriate boundary conditions on the surface of a conducting shield S

$$\varphi^x_{mn}(x,y) = \sqrt{\frac{\epsilon_{0m}\epsilon_{0n}}{ab}}\ \cos(k_x x)\ \sin(k_y y)$$
$$\varphi^y_{mn}(x,y) = \sqrt{\frac{\epsilon_{0m}\epsilon_{0n}}{ab}}\ \sin(k_x x)\ \cos(k_y y) \qquad (12)$$

with $\epsilon_{0m}, \epsilon_{0n}$ being Newman indexes such that $\epsilon_{00} = 1$ and $\epsilon_{0m} = 2, m \neq 0$. The one-dimensional Green's functions $f^{ij}_{mn}(z,z')$ are obtained on the interface at $z = z' = 0$ as the solution of the second-order differential equation forced with a $\delta(z - z')$ function and satisfying appropriate boundary and continuity conditions

$$\begin{aligned} f^{xx}_{mn} &= \frac{k_y^2 - \gamma^{(1)}_{mn}\gamma^{(2)}_{mn}}{(\gamma^{(1)}_{mn} + \gamma^{(2)}_{mn})(k_x^2 + k_y^2 - \gamma^{(1)}_{mn}\gamma^{(2)}_{mn})} \\ f^{xy}_{mn} &= f^{yx}_{mn} \\ &= -\frac{k_x k_y}{(\gamma^{(1)}_{mn} + \gamma^{(2)}_{mn})(k_x^2 + k_y^2 - \gamma^{(1)}_{mn}\gamma^{(2)}_{mn})} \\ f^{yy}_{mn} &= \frac{k_x^2 - \gamma^{(1)}_{mn}\gamma^{(2)}_{mn}}{(\gamma^{(1)}_{mn} + \gamma^{(2)}_{mn})(k_x^2 + k_y^2 - \gamma^{(1)}_{mn}\gamma^{(2)}_{mn})}. \end{aligned} \qquad (13)$$

It is known that a double series expansion of Green's function components is slowly convergent (even divergent in the region of a metal layer) due to the presence of a singularity of the primary part implicitly involved in the double series expansion (11). An efficient technique based on the Kummer transformation [17] has been applied to accelerate slow convergent series [11] of a vector potential Green's function. This technique is applied here to the Green's function components, (11), leading to their transformation so that a quasi-static part (G^{QS}_{ij}) is extracted. The Green's function is then

$$G_{ij} = (G_{ij} - G^{QS}_{ij}) + G^{QS}_{ij} \qquad (14)$$

where G^{QS}_{ij} captures the asymptotic behavior of G_{ij} for large indexes m and n. As there is symmetry between G_{xx} and G_{yy} components $(x \leftrightarrow y, x' \leftrightarrow y', a \leftrightarrow b)$, and G_{xy} and G_{yx} components $(x \leftrightarrow x', y \leftrightarrow y')$, G^{QS}_{xx} and G^{QS}_{xy} only functions will be considered from here on. The asymptotic evaluation of G_{xx} and G_{xy} components yields

$$\begin{aligned} &G^{QS}_{xx}(x,y;x',y') \\ &= -\frac{1}{abk_0^2(\epsilon_1 + \epsilon_2)} \sum_{m=1}^{\infty} \left(\frac{m\pi}{a}\right)^2 \cos\left(\frac{m\pi}{a}\,x\right) \\ &\cdot \cos\left(\frac{m\pi}{a}\,x'\right) \sum_{n=1}^{\infty} \frac{4\sin\left(\frac{n\pi}{b}\,y\right)\,\sin\left(\frac{n\pi}{b}\,y'\right)}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}} \end{aligned} \qquad (15)$$

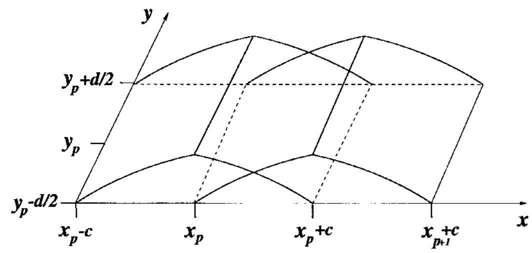

Fig. 2. Rectangular cells with the x-directed overlapping piecewise sinusoidal basis functions.

and

$$\begin{aligned} &G^{QS}_{xy}(x,y;x',y') \\ &= -\frac{1}{abk_0^2(\epsilon_1+\epsilon_2)} \sum_{n=1}^{\infty} \left(\frac{n\pi}{b}\right) \sin\left(\frac{n\pi}{b}\,y\right) \cos\left(\frac{n\pi}{b}\,y'\right) \\ &\cdot \sum_{m=1}^{\infty} \frac{4\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\,x\right)\,\sin\left(\frac{m\pi}{a}\,x'\right)}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}. \end{aligned} \qquad (16)$$

Note that the summations are frequency independent, hence, the quasi-static designation for this part of the Green's function. The second infinite summation in (15) has been transformed into a fast converging series of K_0, the zeroth-order modified Bessel functions of the second kind (details are shown in [11]). Similarly, the second infinite summation in (16) is obtained in terms of K_1, the first-order modified Bessel functions of the second kind, as follows:

$$\begin{aligned} &\sum_{m=1}^{\infty} \frac{4\left(\frac{m\pi}{a}\right)\ \cos\left(\frac{m\pi}{a}\,x\right)\ \sin\left(\frac{m\pi}{a}\,x'\right)}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}} \\ &= -\frac{2an}{b} \sum_{m=-\infty}^{\infty} \left\{K_1\left(\frac{n\pi}{b}\ (x - x' + 2ma)\right)\right. \\ &\qquad \left. - K_1\left(\frac{n\pi}{b}\ (x + x' + 2ma)\right)\right\}. \end{aligned} \qquad (17)$$

Only a few terms of the series in (17) are required to reach convergence due to the exponential decay of K_1, the modified Bessel functions. This property, together with the frequency independence of the summations, are the key attributes leading to computational speedup.

An MoM procedure in conjunction with the above transformation results in the following representation for the impedance matrix elements:

$$Z^{\mathrm{MoM}}_{ij}(\omega) = \jmath\omega\mu_0\left[(Z_{ij}(\omega) - Z^{QS}_{ij}) + Z^{QS}_{ij}\right] \qquad (18)$$

where $Z^{QS}_{ij}, i,j = x,y$ are the quasi-static impedance matrix elements obtained in the integral form for the rectangular cells with the x- and y-directed piecewise sinusoidal basis and testing functions. As an example, rectangular cells with the x-directed overlapping basis functions are shown in Fig. 2

and, Z_{xx}^{QS}, the impedance matrix element, is

$$Z_{xx}^{QS} = \int_{x_p-c}^{x_p+c} \int_{y_p-d/2}^{y_p+d/2} \int_{x_r-c}^{x_r+c} \int_{y_r-d/2}^{y_r+d/2} T_{xp}(x) \cdot G_{xx}^{QS}(x,y;x'y')T_{xr}(x')\, dx\, dy\, dx'\, dy' \quad (19)$$

with $T_{xp}(x)$ and $T_{xr}(x')$ being overlapping piecewise sinusoidal, locally determined, testing, and basis functions

$$T_{xp}(x) = \frac{\sin[k_s(c-|x-x_p|)]}{d\sin(k_s c)}, \qquad |x-x_p| \le c;\ |y-y_p| \le d/2.$$

A parameter $k_s = k_0/\sqrt{\epsilon_{\max}}$ determines in some sense a degree of smoothness of basis functions. Note that these functions are continuous with a discontinuous derivative at $x = x_p$ and, for small $k_s c$, they approach triangular basis functions $(\sin(k_s c) \approx k_s c)$.

The other impedance matrix elements Z_{xy}^{QS}, Z_{yx}^{QS}, and Z_{yy}^{QS} can be similarly obtained in the integral form for the x- and y-directed basis and testing functions with corresponding Green's function components. The problem of evaluation of the quasi-static impedance matrix elements Z_{xx}^{QS}, given by (19), together with the quasi-static Green's functions expression (15) is reduced to the calculation of a double integral over the y-domain

$$I_{xx}^{QS} = \int_{y_p-d/2}^{y_p+d/2} \int_{y_r-d/2}^{y_r+d/2} \left\{ K_0\left(\frac{m\pi}{a}(y-y'+2nb)\right) - K_0\left(\frac{m\pi}{a}\,(y+y'+2nb)\right)\right\} dy'\, dy. \quad (20)$$

This integral results from substituting for the basis functions and replacing the quasi-static part of the Green's function, using (15), in (19). A double integral of cosine functions with piecewise sinusoidal basis and testing functions over the x-domain is easily evaluated and will not be discussed here. A similar idea was followed in formulating the Z_{xy}^{QS} impedance matrix element. Calculation of a double integral over the x-domain is required for Z_{xy}^{QS} involving the integral

$$I_{xy}^{QS} = \int_{x_p-c}^{x_p+c} \sin\Big[k_s\big(c-|x-x_p|\big)\Big] \cdot \int_{x_r-c/2}^{x_r+c/2} \left\{K_1\left(\frac{n\pi}{b}\,(x-x'+2ma)\right) - K_1\left(\frac{n\pi}{b}\,(x+x'+2ma)\right)\right\} dx'\, dx. \quad (21)$$

Similar evaluations can be derived for the Z_{yx}^{QS}, Z_{yy}^{QS} impedance matrix elements considering the symmetry property discussed above.

The internal integral with respect to y' in (20) is simplified using a change of variable and then precomputed numerically and stored in a table. The outer integral is then calculated numerically by a Gauss quadrature using tabulated data of the previous integration. The internal integral with respect to x' in (21) yields the zeroth-order modified Bessel function of the second kind $K_0(x)$. The outer integration is also performed numerically, again using a Gauss quadrature.

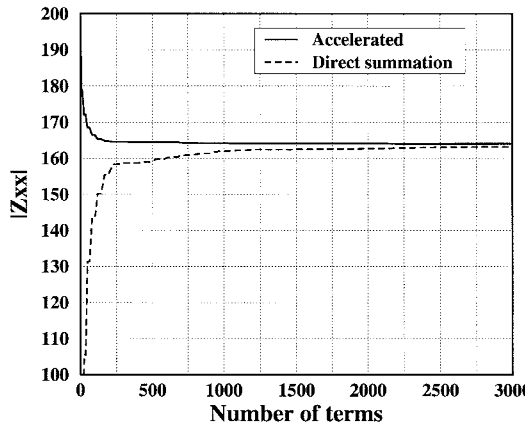

Fig. 3. Convergence of Z_{xx} matrix elements for the accelerated and direct double series summation.

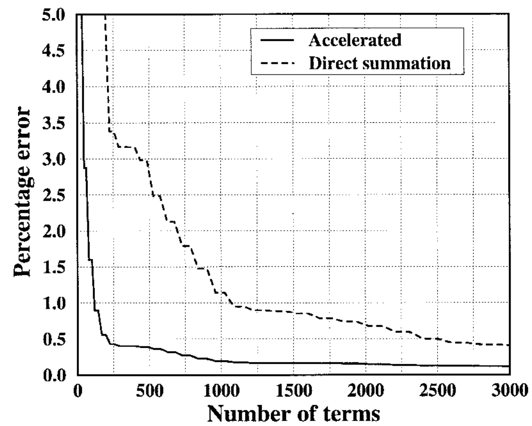

Fig. 4. Percentage error in the convergence of Z_{xx} matrix elements for the accelerated and direct double series summation.

III. Discussion

The Kummer transformation applied to the Green's functions and, consequently, to the impedance matrix elements, leads to efficient evaluation of the quasi-static part and results in a dramatic reduction of overall computation time. This is because it is frequency independent and only need be calculated once (in a frequency sweep), and this calculation is fast because of rapid convergence. Also, the frequency-dependent terms in the representation (18) $(Z_{ij}(\omega) - Z_{ij}^{QS})$ converge quickly as a result of the Kummer transformation.

To illustrate the proposed approach, resonant strip structures, including a wide resonant strip and patch array structure embedded in a rectangular waveguide, have been investigated numerically. In the first example, the convergence and percentage error of the impedance matrix elements for the accelerated and direct double series summation are demonstrated in Figs. 3 and 4, respectively, (with geometry shown in the inset of

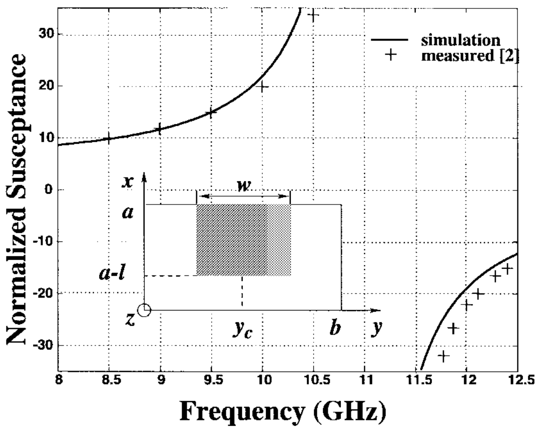

Fig. 5. Normalized susceptance of a wide resonant strip in waveguide: $a = 0.4$ in, $b = 0.9$ in, $w = 0.280$ in, $\ell = 0.365$ in, $y_c = b/2$.

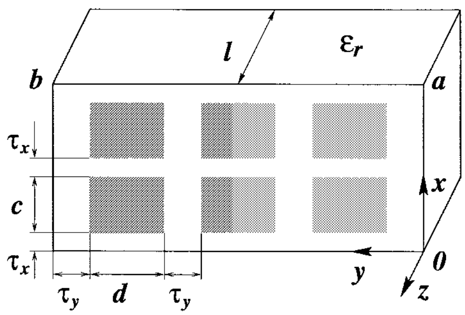

Fig. 6. Geometry of patch array supported by dielectric slab in a rectangular waveguide: $a = 1.0287$ cm, $b = 2.286$ cm, $\ell = 2.5$ cm, $\epsilon_r = 2.33$, $d = 0.4572$ cm, $c = 0.3429$ cm, $\tau_x = 0.1143$ cm, $\tau_y = 0.2286$ cm.

Fig. 5). The convergence error (Fig. 4) is calculated for the x-directed current element with $x_p = a/2$ and $y_p = b/2$ (the unit cell shown in Fig. 2 has dimensions $c = 0.2318$ cm and $d = 0.2371$ cm). The relative error is defined as $|Z_{xx} - Z_{xx}^{\infty}|/|Z_{xx}^{\infty}| \times 100$, where Z_{xx} represents the impedance matrix element either calculated as a direct summation or using the proposed acceleration technique, and Z_{xx}^{∞} is the value of Z_{xx} obtained for a large number of summation terms m and n in the direct summation. To generate results shown in Fig. 4, we used $m = n = 1500$ summation terms with the value of Z_{xx}^{∞} equal to $10.9283 - j163.5033$. It is shown that the error of 0.5% is obtained for 200 terms used in the accelerated summation procedure, in comparison with 2500 terms required in the direct double series summation to reach the same error. The computation time is almost directly proportional to the number of terms in the summation. Also, it can be difficult to determine when sufficient terms have been used with the direct method. Numerical results for the normalized susceptance of a wide strip agree well with the measured data provided in [4] (Fig. 5).

As another example, a resonant patch array supported by dielectric slab in a rectangular waveguide (Fig. 6) is analyzed for application in high-frequency EM and quasi-optical transmitting and receiving systems [1], [2]. The structure resonates at 8.6 GHz with a reflection coefficient of -26 dB (Fig. 7). The dispersion behavior of a phase angle is given in Fig. 8, showing the resonant properties of the structure.

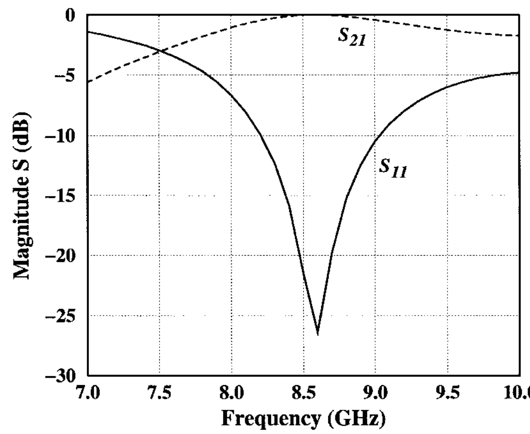

Fig. 7. Magnitude of S_{11} and S_{21} for the patch array embedded in a rectangular waveguide.

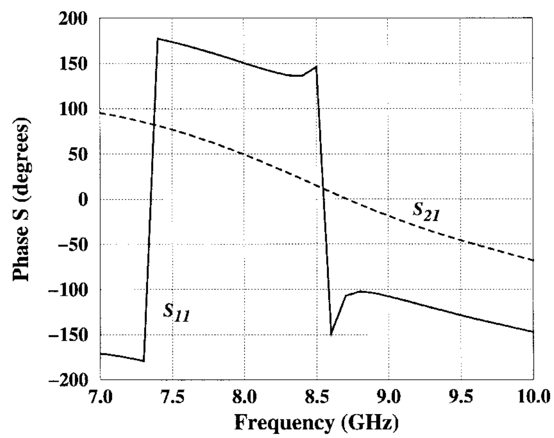

Fig. 8. Phase of S_{11} and S_{21} for the patch array embedded in a rectangular waveguide.

IV. Conclusion

An electric-field integral-equation formulation in conjunction with the acceleration procedure for a double series summation has been developed for the efficient analysis of various strip discontinuities embedded in layered guided-wave structures. The electric-type Green's function has been derived for a rectangular waveguide with a current-carrying interface of adjacent dielectric layers. The Kummer transformation with the evaluation of the quasi-static part of the Green's function is proposed to accelerate convergence of double series expansions in the representations for the impedance matrix elements. The numerical analysis of convergence and percentage error of Z-matrix elements is provided for the specific example of a wide strip in a rectangular waveguide. A patch array embedded in a rectangular waveguide has been modeled for applications in high-frequency EM and quasi-optical systems. The derivations here were incorporated in a GSM simulator, which extracts circuit ports where active devices would be located [14]. This extraction of circuit ports enables circuit–field interactions to be handled. The work can, therefore, be used to analyze structures consisting of multiple transverse layers of conductors in a possibly overmoded rectangular waveguide.

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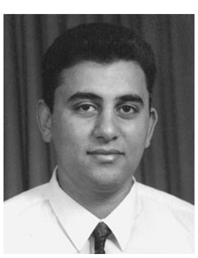

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