Spectral Regrowth in Microwave Amplifiers Using Transformation of Signal Statistics

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Abstract- *A method is presented for the estimation of spectral regrowth generated by a digitally modulated carrier passed through a nonlinear RF circuit. The estimation is based on developing an analytical expression for the autocorrelation of the output signal as a function of the input statistics. A complex power series model extracted from measured AM-AM and AM-PM data is used. Comparisons of measured and predicted ACPR values for a CDMA system are made.*

I. INTRODUCTION

The availability of accurate methods for the estimation of spectral regrowth is of particular interest to those involved in the design of cellular and personal communications systems. Here nonlinear devices, especially the power amplifiers, generate co-channel and adjacent channel interference due to sideband regrowth. Stringent regulatory emission requirements limit interference with neighboring channels, directly affecting the efficiency of microwave power amplifiers.

Several different methods have been proposed to characterize nonlinear distortion and spectral leakage to adjacent channels [1-3]: in cellular and PCS systems, sideband regrowth or ACPR (adjacent channel power ratio) is usually the favored method.

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Performance specifications required to control adjacent channel distortion profoundly affect RF active component design, hence the current interest in developing schemes to accurately predict the ACPR of an RF amplifier in a timely manner.

This paper describes a method of predicting spectral regrowth based on the nonlinear transformation of the amplitude statistics of the input signal. The output power spectrum is estimated as an analytical expression that describes the transformation of a complex gaussian signal when passed through a quadrature nonlinearity. The quadrature nonlinearity is modeled as a complex power series representation of the AM-AM and AM-PM characteristics of the device. The estimated spectrum can then be used to predict ACPR.

The major contribution of this paper is the characterization of a digitally modulated signal by its autocorrelation functions, and its use with a behavioral nonlinear model to obtain the statistics of the amplified signal. From this, Adjacent Channel Power Ratio (ACPR) is obtained. Predicted ACPR compares very favorably with measurements.

II. OUTPUT AUTOCORRELATION OF A COMPLEX GAUSSIAN SIGNAL PASSED THROUGH A NONLINEAR DEVICE

A block diagram of the target quadrature modulator cascaded with a nonlinear element is shown in Fig. 1. Fig. 1(a) shows the actual RF modulation scheme at the carrier frequency, while Fig. 1(b) represents the corresponding baseband equivalent. The procedure followed here is to calculate the autocorrelation function of the output of the baseband equivalent system by applying an input signal, characterized as a complex auto-

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correlation function, to a model of the nonlinearity described by a complex power series. This is then used to predict output power spectrum and ACPR in the RF system.

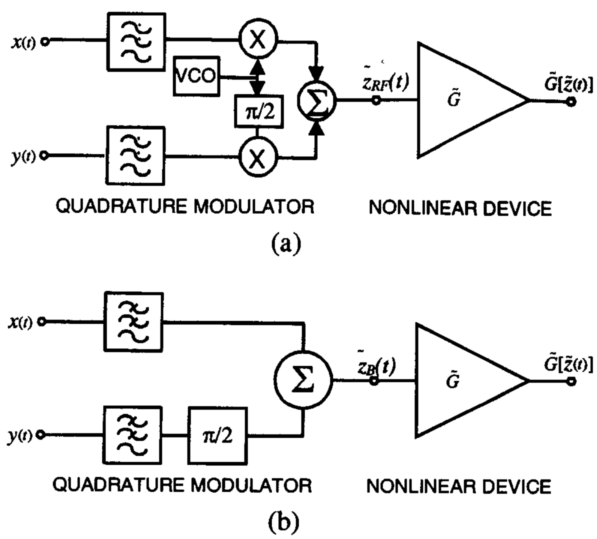

Figure 1. Quadrature modulated nonlinear amplifier
(a) RF system (b) baseband equivalent

The digital quadrature modulated signal, $z(t)$, can be modeled as the quadrature addition of two filtered, zero mean, normally distributed random processes $x(t)$ and $y(t)$ as:

$$\tilde{z}(t) = x(t) + jy(t)$$

The autocorrelation function of the quadrature process described above is:

$$\tilde{R}_{zz}(\tau) = E[\tilde{z}(t)\tilde{z}^*(t+\tau)] = E[\tilde{z}_1\tilde{z}_2^{\;*}] \tag{1}$$

where * denotes the complex conjugate. This can be expanded to [8]:

$$\begin{aligned} \tilde{R}_{zz}(\tau) &= E[\tilde{z}(t)\tilde{z}^*(t+\tau)] = E[\tilde{z}_1\tilde{z}_2^{\;*}] \\ &= E[(x_1 + jy_1)(x_2 - jy_2)] \\ &= E[x_1x_2 + y_1y_2 + j(x_2y_1 - x_1y_2)] \\ &= 2R_{xx}(\tau) - j2R_{xy}(\tau) \end{aligned} \tag{2}$$

The input autocorrelation function is thus complex with real and imaginary parts that are even and odd functions, respectively, about $\tau = 0$. The total power of the input signal $z(t)$ is given by the autocorrelation function evaluated at $\tau = 0$. The input signal is now characterized. Consider now the nonlinear device as a complex power series description of a quadrature nonlinearity [1,2]:

$$\tilde{G}(z) = \tilde{a}_1\tilde{z} + \tilde{a}_3\tilde{z}^3 + \tilde{a}_5\tilde{z}^5 + \ldots + \tilde{a}_n\tilde{z}^n \tag{3}$$

It is important to realize that (3) represents a general nonlinear transfer characteristic, not gain expression. An expression for gain can be obtained from (3) using generalized power series analysis, as described in [1]. The output of the nonlinear system described by the power series model (3), excited by an N component multifrequency input, can be expressed as a sum of intermodulation products, and then computed by multinomial expansion. Restricting the analysis to first order intermodulation, the component of the output signal extracted is that part correlated with the input signal. The output is also a single tone, and can be shown to be

$$Y_1 = \tilde{a}_1X_1 + \sum_{\alpha=1}^{\alpha_m} b_{1+2\alpha}|X_1|^{2\alpha}X_1 \tag{4}$$

which corresponds to the standard nonlinear gain expression. This expression relates a phasor at the input (X_1) to the phasor at the output (Y_1), and corresponds to the complex gain measured in the AM-AM and AM-PM characterization. The new coefficients, b_i, represent the relationship between the envelope behavioral model and the baseband equivalent behavioral model (a complex power series for instantaneous quantities).

On the other hand, the autocorrelation of the signal at the output of the nonlinear device can be computed (for the fifth order case) as:

$$\begin{aligned} \tilde{R}_{gg}(\tau) = \; & E[\tilde{G}(\tilde{z}_1)\tilde{G}^*(\tilde{z}_2)] \\ = \; & E[\{\tilde{a}_1\tilde{z}_1 + \tilde{a}_3\tilde{z}_1^{\;3} + \tilde{a}_5\tilde{z}_1^{\;5}\}\{\tilde{a}_1^{\;*}\tilde{z}_2^{\;*} + \tilde{a}_3^{\;*}(\tilde{z}_2^{\;*})^3 \\ & + \tilde{a}_5^*(\tilde{z}_2^{\;*})^5\}] \\ = \; & |\tilde{a}_1|^2E[\tilde{z}_1\tilde{z}_2^*] + \tilde{a}_1\tilde{a}_3^*E[\tilde{z}_1(\tilde{z}_2^3)^*] + \tilde{a}_1\tilde{a}_5^*E[\tilde{z}_1(\tilde{z}_2^5)^*] + \\ & \tilde{a}_1^*\tilde{a}_3E[\tilde{z}_1^3\tilde{z}_2^*] + |\tilde{a}_3|^2E[\tilde{z}_1^3(\tilde{z}_2^3)^*] + \tilde{a}_3\tilde{a}_5^*E[\tilde{z}_1^3(\tilde{z}_2^5)^*] + \\ & \tilde{a}_1^*\tilde{a}_5E[\tilde{z}_1^5\tilde{z}_2^*] + \tilde{a}_3^*\tilde{a}_5E[\tilde{z}_1^5(\tilde{z}_2^3)^*] + |\tilde{a}_5|^2E[\tilde{z}_1^5(\tilde{z}_2^5)^*] + \ldots \end{aligned} \tag{5}$$

Expanding $R_{gg}(\tau)$ results in many algebraically intensive moment calculations involving $x(t)$ and $y(t)$. Fortunately, a previous result [4] can be used to calculate each of the moments in (5), by using the following theorem for the moments of stationary complex gaussian random variables:

a) if s≠t, then

$$E[\tilde{z}_1\tilde{z}_2...\tilde{z}_s\tilde{z}_1^*\tilde{z}_2^*...\tilde{z}_t^*]=0 \quad (6)$$

b) if $s=t$, then

$$E[\tilde{z}_1\tilde{z}_2...\tilde{z}_s\tilde{z}_1^*\tilde{z}_2^*...\tilde{z}_t^*] = \sum_{\pi} E[\tilde{z}_{\pi(1)}\tilde{z}_1^*]E[\tilde{z}_{\pi(2)}\tilde{z}_2^*]...E[\tilde{z}_{\pi(s)}\tilde{z}_t^*] \quad (7)$$

where s and t denote the set of complex gaussian random processes, and π is a permutation of the set of integers {1, 2, ... , s}. Application of (7) yields:

$$E[(\tilde{z}_1\tilde{z}_2^*)^n] = n!E[\tilde{z}_1\tilde{z}_2^*]^n = n!\tilde{R}_{zz}^n(\tau) \quad (8)$$

The moments in (5) are calculated by combining (6), (7) and (8), yielding:

$$\tilde{R}_{gg}(\tau)=|\tilde{a}_1|^2 A\tilde{R}_{zz}(\tau)+3!|\tilde{a}_3|^2 A^3\tilde{R}_{zz}^3(\tau)+... \quad (9)$$
$$+5!|\tilde{a}_5|^2 A^5\tilde{R}_{zz}^5(\tau)+...=\sum_{\substack{n=1\\ n\ odd}}^{N} n!|\tilde{a}_n|^2 A^n\tilde{R}_{zz}^n(\tau)$$

where A is a power scaling variable used to set the input power level. The output autocorrelation function is thus defined by powers of the input autocorrelation function, an input scaling factor, and the power series coefficient magnitudes. The estimated output power spectrum is the Fourier transform of the autocorrelation function (9). The output power spectrum is therefore given by the sum of the individual spectra of each term in the power series, scaled by its corresponding coefficient and input power level. Hence only the input autocorrelation function, $R_{zz}(\tau)$, and the complex power series description are necessary to compute the output power spectrum of the nonlinear device.

III. Signal Characterization

Evaluation of the output power spectrum requires an estimate of the input autocorrelation function, $R_{zz}(\tau)$, of the complex input process $z(t)$. A closed form analytical expression for this is usually difficult to derive. However, good estimates can be obtained from simulation using long, independent data sequences passed through a quadrature modulator. Input autocorrelation is estimated by applying a sliding discrete correlator to the gaussian input streams $x(t)$ and $y(t)$, after passing them through a hard limiter (sign function) and an FIR baseband filter. This generates an input stream as defined by the CDMA IS-95 standard [7]. To evaluate the performance of the proposed technique, a 13th-order complex power series was fitted to AM-AM and AM-PM data from a 900 MHz CDMA power amplifier [5]. The quality of the fit is illustrated in Figures 2 and 3, where measured AM-AM and AM-PM data is plotted on solid lines, and interpolated values from the power series model are shown on dashed lines.

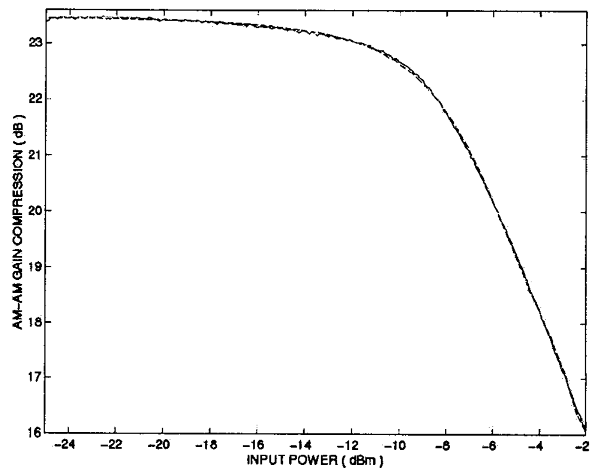

Figure 2. Measured and predicted AM-AM characteristics

This yielded the coefficients of the resulting power series model, (3). This enables the autocorrelation function of the output, (9), to be evaluated. Estimated spectra for several output power levels are shown in Figure 4.

The calculation outlined above uses a complex envelope baseband representation of the signal. This introduces a factor of 2 in the power spectral density of uncorrelated terms [6]. For this reason, the spectral components corresponding to nonlinear distortion (terms in Equation 5 with n >1) have been weighted by a factor of 0.5. ACPR was then calculated as described in [1] and compared with ACPR measurements. Results are shown in Figure 5.

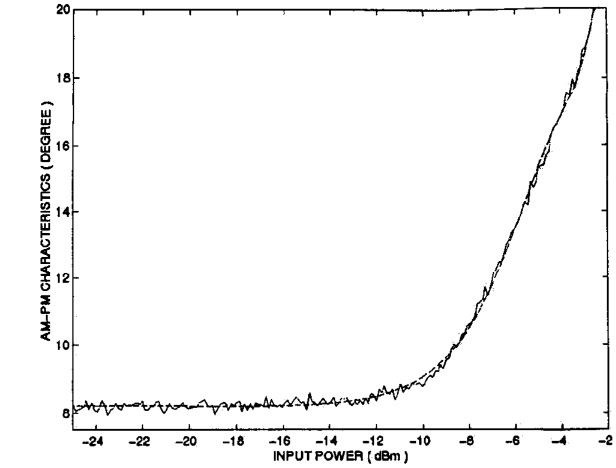

Figure 3. Measured and predicted AM-PM data

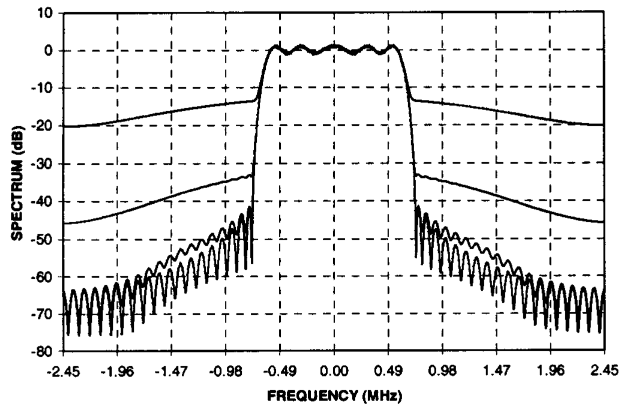

Figure 4. Spectrum estimate based on the proposed method. 0, 5, 10 and 13dBm output power.

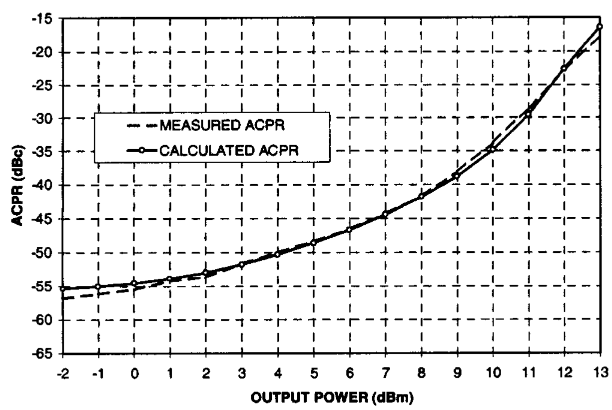

Figure 5. Measured vs. predicted spectral regrowth generated by a digitally modulated signal.

IV. Conclusion

A method for the estimation of spectral regrowth of digitally modulated signals passed through a nonlinear amplifier, modeled by a complex power series, has been presented. The proposed method takes advantage of statistical properties of the moments of complex gaussian random variables to develop a simple expression for the autocorrelation of the output stream, hence providing an output spectrum estimate from which ACPR can be easily calculated. Since the behavioral model is derived from single-tone envelope characterizations, it describes only odd order intermodulation. Envelope termination effects which result from second order non-linearities are not considered.

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