Empirical Statistical Analysis Of Planar Transmission Lines On PCBs Accounting For Manufacturing Variations

Michael Kay, Real Pomerleau
BNR Inc.
Research Triangle Park, North Carolina

Michael Steer
Electrical & Computer Engineering
North Carolina State University
Raleigh, North Carolina

Abstract

The effect of random process variations in Printed Circuit Board (PCB) manufacturing with controlled impedance can cause significant deviation from design intent. This paper will describe an empirical analysis which quantitatively describes the repeatability of a particular process of microstrip manufacturing. It will also address the problem of random manufacturing variations in PCB structures.

Introduction

The transmission line effects between high speed digital devices on a PCB is becoming a more significant design issue as the speed and size of processors and systems increase. Theoretical models and simulators have been created to predict the PCB transmission line effects, but little has been done to model the random impedance variations resulting from a manufacturing process. This paper addresses the problem of random manufacturing variations in PCB structures as well as describes an empirical technique to perform a statistical analysis of PCB lines. The technique will improve transmission line characterization by enabling a worst case analysis of transmission line effects to be done. This will allow the designer to minimize post-prototype problems associated with high speed PCBs concerning inconsistencies in a manufacturing process.

The description of PCB structures is obtained by measuring the Scattering (S) parameters and then converting them to frequency domain characteristic impedance (Z0) and propagation constant (gamma). A deviation in complex Z0 and gamma can be used to model line and discontinuity variations.

With the use of this technique, a better representation of Z0 and gamma can be obtained than by previous techniques. Presently, the most popular way to calculate Z0 and gamma is through the use of time domain reflectometry (TDR). TDR involves the use of an oscilloscope, and measuring intermediate voltages in the propagation of a step function through a transmission line. However, this process is subject to several significant errors:

1) The reflection caused by the connectors is not taken into consideration
2) Shunt and series losses also introduce uncompensated error to the TDR measurement
3) Z0 and gamma are only given as a constant instead of as a function of frequency.

In response to these errors, the technique described in this paper removes the effects of the connectors, characterizes loss, and returns impedance and gamma as a function of frequency, resulting in a much more accurate model than before.

Theory

A description of S-parameters is necessary to understand the basis of this analysis. S-parameters are used instead of a voltage and current parameter because the S-parameters describe the relative amplitude and phase of

CH2674-5/89/0000-0112$01.00©1989IEEE

forward and backward traveling waves. Because we measure transmission line effects of up to 10 GHz (in this paper) microwave measurement techniques must be used. Therefore, S-parameters are useful because they are directly measurable at these frequencies. Figure 1 describes how S-parameters are defined in a two-port network.

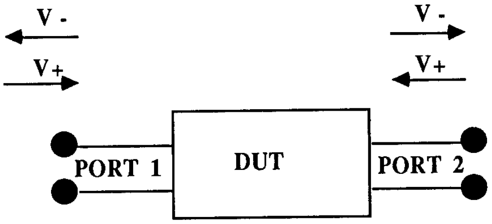


Figure 1: Description of Two-port S-parameters

V+ = Forward traveling wave (applied)
V- = Backward traveling wave (measured)

$V1- = (S_{11} * V_1+) + (S_{12} * V_2+)$
$V2- = (S_{21} * V_1+) + (S_{22} * V_2+)$

The automatic network analyzer (ANA) has two ports and returns four complex S-parameters for each frequency point. S_{11} is the port one reflection coefficient which describes the relative amplitude of the signal reflected back to port one as a function of frequency after a signal is applied. $S_{11} = V_1-/V_1+$ when port two is matched. S_{21} is the port one transmission coefficient, which describes the relative amplitude of signal that is transmitted from port one through to port two ($S_{21} = V_2-/V_1+$). S_{12} and S_{22} are measured the same way from port two. A magnitude and phase term is given for with each S-parameter.

The S-parameters of a straight line on a microstrip PCB are dependent mainly on four parameters of the board as shown in Figure 2.

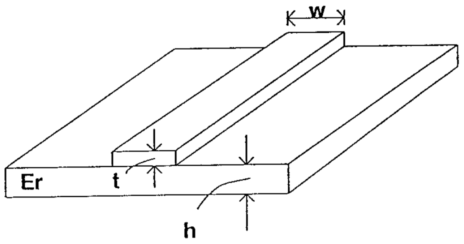


Figure 2: Microstrip board parameters.

W = Width of PCB line
T = Thickness of PCB line
H = Width of dielectric material (distance between line and ground plane)
Er= Dielectric constant of material

Each of the board parameters affect impedance differently. The actual amount that Z0 and gamma varies depends on which parameter is being considered.

Though S-parameters are a good way to measure a line, they are not the best way to describe it physically. A more intuitive description of a PCB line is obtained from the characteristic impedance and the propagation constant of the line which can be calculated from the S-parameters. Equations 1 and 2 were used to calculate Z0 and gamma from the S parameters (see reference [1] for derivation). Z0 is described in magnitude phase form and is independent of board length. Gamma contains a real and imaginary part. The real part, alpha, is the attenuation constant and the imaginary part, beta, is the phase constant, both of which are functions of line length. Z_m is the impedance to which the board was intended to be manufactured (50 ohms).

$$Zo = \frac{-b - \sqrt{b - 4ac}}{2a} \qquad (1)$$

$$a = (S_{11} - 1 - S_{11}S_{22} + S_{12}S_{21} + S_{22})$$

$$b = (2 \times Z_m)(S_{11} - S_{22})$$

$$c = Z_m^2(S_{11} + 1 + S_{11}S_{22} - S_{12}S_{21} + S_{22})$$

$$\text{Gamma} = \left(\frac{1}{\text{Length}}\right) \ln\left[A + \sqrt{A - 1}\right]$$

$$A = \left[\frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}\right] \qquad (2)$$

Since S-parameters are complex numbers, taking the square root causes phase discontinuity problems when implementing Equations 1 and 2 using Fortran 77. The problem occurs when the parameter passes through the negative real axis of the complex plane. For this reason, it is necessary to keep track of the absolute phase of each variable, instead of just considering it from -180 to +180 degrees.

In order to obtain a statistical description of Z0 and Gamma, in this paper, repeated S-parameter measurements were made of identical structures. These measurements were then converted to Z0 and Gamma. Then the mean and standard deviation were calculated as a function of frequency using Equations 3 and 4. The standard deviation describes the repeatability of manufacturing and measuring the line.

$$\overline{X} = \text{MEAN} = \frac{1}{N}\sum_{i=1}^{N} X_i \qquad (3)$$

$$\text{STANDARD DEVIATION} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (X_i - \overline{X})^2} \qquad (4)$$

In order to address the problem of manufacturing variation, the inherent variation of the test setup needs to be characterized. This setup consists of an HP 8510 network analyzer with precision 3.5 mm connectors. These connectors are attached to a microstrip-to-SMA adapter which is soldered to the PCB. Figure 3 shows the test setup.

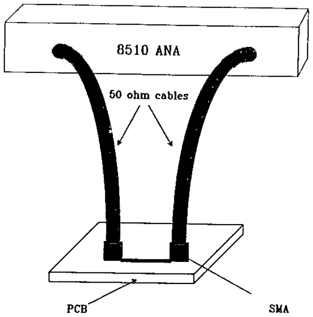


Figure 3: Test Setup

The repeatability of the connection between the precision 3.5mm connector and the microstrip-to-SMA adapter is then determined. Next we determine the repeatability of the connection of the adapter to the PCB involving solder joints that bonded two adapters to the test board.

Removing the errors caused by the measurement process will give us the manufacturing deviation only. When adding several random variables, the deviation of the sum is simply the square root of the sum of the deviations. Since soldering the SMA adapters to the PCB takes into account the variation caused by both the soldering of the SMAs and the connecting of the ANA cable, only this effect needs to be subtracted from the deviation caused by sampling different lines. Equation 5 gives the standard deviation of just the random manufacturing variations.

$$SD_3 = \sqrt{SD_2^2 - SD_1^2} \qquad (5)$$

SD_1 = standard deviation caused by measuring process

SD_2 = sta. dev. caused by measuring and manufacturing process

SD_3 = sta. dev. caused by just the manufacturing process

Method

This paper presents a method to empirically describe the manufacturing variations of a PCB structure. An eight mil microstrip line was chosen for the testing and was manufactured on a controlled impedance test board. The board impedance was manufactured to an intended impedance of 50 ohms based on existing formulas for the physical dimensions and manufacturing process. We measured a total of 20 microstrip lines on 10 separate test boards.

To measure the structures on these boards, the following equipment was used:

- An HP 8510 Network Analyzer, which produces an S-parameter matrix as a function of frequency
- Microstrip-to-SMA adapters soldered to the test boards
- A torque wrench to connect the coax

cable coming from the ANA to the SMA adapters.

For all measurements, we set the following parameters on the HP8510B:

- A frequency range of 45 MHz to 10 GHz
- 801 frequency points per measurement
- Power of 10 dBm
- 16 separate measurements were averaged in the ANA before storing them. Averaging improves dynamic range by reducing the effect of individual measurement deviations. Noise on an individual measurement is reduced by 20log(1/N) dB where, in this case N, is 16.

The description of the random deviation caused by just the manufacturing inconsistencies is made by:

1) Measuring all 20 structures and determining the variations about a mean
2) Choosing one structure to determine the variations of making connections from the ANA to the SMA adapter
3) Choosing one structure to determine the variations of soldering SMA adapters to PCBs
4) Removing the random deviation caused by the measurement process (2 and 3) from the measured deviation of all the structures (1). This is done by a process described in reference [3] called the "total sum of squares of deviations."

The ANA measures the S-parameters of the cables, connectors, and PCB structures, even though all we want is the effects of the structures. Therefore, the effects of the cables and connectors must be mathematically removed. The process of removing the effects of the cables is called calibration and in done within the ANA before the data is sent to the PC to be further analyzed and stored. The process of removing the effects of the connectors is called de-embedding which utilizes techniques described in reference [2]. After both of these processes are complete, we are left with S-parameters representing only the PCB structure.

Since S-parameters are not a particularly convenient way to model a PCB line or analyze it statistically, and Z0 and gamma are used to model PCB lines and discontinuities in the transmission line simulator being developed at NCSU, the conversion from S-parameters to Z0 and gamma is necessary. This is done by means of Equations 1 and 2.

The mean and standard deviation of Z0 and gamma is generated as a function of frequency, and a routine is written to collect the data and calculate these numbers. The standard deviation describes the repeatability of a measurement.

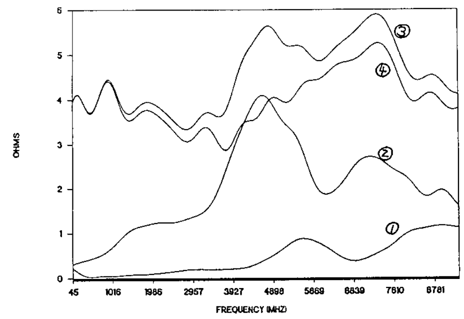
The deviation of just the manufacturing variations must be extracted from the additional random effects added by the measurement technique. This is accomplished by an analysis of variance technique called "the total sum of squares of deviations" described in reference [3] and given by Equation 5. This result is used to create statistical models which are implemented in a transmission line simulator to do a worst case analysis of the random effects of the PCB on signal propagation.

Results

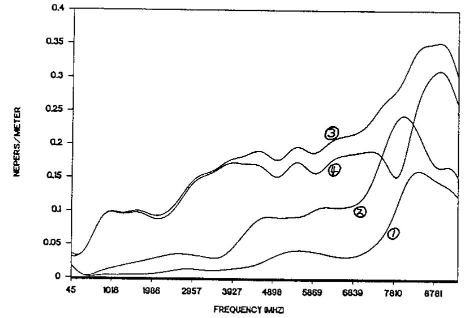
The results describe, as a function of frequency, the repeatability (standard deviation) of the different errors encountered in the measurement and manufacturing process :

1) Connecting ANA cables to SMA adapters on a PCB using a torque wrench (connecting deviation only)
2) Soldering SMA adapters to a PCB (includes: soldering deviation plus connecting deviation)
3) Total measurement (includes: boards deviation, soldering deviations, plus connecting deviation)
4) Manufacturing a PCB line (board deviation only).

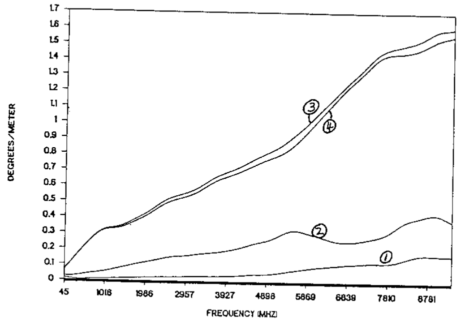
STANDARD DEVIATION (MAG ZO)



STANDARD DEVIATION (ALPHA)



STANDARD DEVIATION (BETA)



An important result is the repeatability of the aspects of this measuring technique. Even though a torque wrench is used, connecting the ANA to the SMA causes measurable random deviation. Soldering connectors will cause significant errors even at relatively low frequencies. Therefore, it is important to take these effects into consideration when using this measuring technique in the analysis of the measured data.

Conclusion

The lack of sufficient repeatability in PCB manufacturing can cause inaccurate simulation of high-speed digital circuits. One solution to this problem is to characterize the deviation so that a worst case analysis can be performed. This is accomplished by measuring a number of structures and generating the characteristic impedance and propagation constant for each structure. These two parameters are used in the statistical modeling of PCB lines and discontinuities. If the circuit performs properly under the worst possible conditions, the designers can be confident that they will have fewer post-prototype problems.

Reference

[1] Riedell, Heyward, Kay, M., Steer, M., Basil, M., and Pomerleau, R. 'Dielectric Characterization of Printed Circuit Substrates', Proceedings IEEE Southeast Con 89.

[2] Kasten, J., Steer,, M., and Pomerleau, R. 'Through Symmetric Fixture: A Two-Port S-Parameter Calibration Technique', Proceedings RF Expo East 88.

[3] Menden, Schaeffer, and Wackerly. Mathematical Statistics with Applications. 3rd ed Duxbury, 1986.