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CONTROL of ALIASING in the HARMONIC BALANCE SIMULATION of NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

The simulation of nonlinear microwave circuits using harmonic balance requires Fourier transformation to interface the frequency domain analysis of the linear subcircuits with the time domain analysis of the nonlinear subcircuits. Subsequent aliasing can unacceptably reduce the accuracy of harmonic balance simulation. Simulation error can be reduced by selecting a large set of frequencies for use in the circuit analysis. Unfortunately, such analyses require extended simulation time. A *dual frequency set* analysis scheme is proposed which reduces aliasing without requiring excessive simulation time.

I. *INTRODUCTION*

The reported transform algorithms used in the implementation of a general purpose harmonic balance simulator for nonlinear circuits are the Discrete Fourier Transform (DFT), the Almost Periodic Discrete Fourier Transform (APDFT)[1,2] and the Multidimensional Fast Fourier Transform (NFFT) [3]. Spectrum selection is of critical importance in each of these methods as it effects simulation accuracy as well as simulation time and memory requirements. If the set of frequencies selected for the analysis is small, simulation time will be reduced. However, the accuracy of the solution may be degraded. On the other hand, analysis using a large set of harmonics or intermodulation frequencies can require an inordinate amount of computer time and memory. That is, a major portion of the simulation time and memory is associated with minimization of the determining equations rather than evaluation of the frequency domain response of the nonlinear subcircuit.

If, for example, Newton's method is used to solve the system of determining equations for an analysis with M circuit variables and K frequencies, the Jacobian will be a $(2MK)^2$ real matrix; where $M = N + B$, N is the number of nonlinear nodes, and B is the number of branches with impedance type elements or sources. Then a major portion of the simulation time and memory is required to form and invert the Jacobian matrix (This is particularly true if a circuit is excited by multiple incommensurate tones because a large number of intermodulation frequencies may be generated).

Efficient simulation requires use of a minimal set of frequencies which will *rapidly* produce acceptably accurate results. The simulation error introduced by truncation to an insufficient set of frequencies is due to two mechanisms (which will be discussed for admittance type nonlinear elements):

1. Node voltage phasors are transformed into a time domain voltage sequence via a discrete inverse transform. This sequence is then applied as an input to a nonlinear element. If an insufficient number of frequencies are used, the instantaneous voltage at the nonlinearity is incorrect and the current response is also incorrect. This alters the error surface so that the simulation may be inaccurate.

2. If a sufficient number of frequencies are used so that the time domain voltage sequence at the nonlinearity is accurate during a particular iteration, the resulting current sequence may not contain necessary spectral information. The discrete time current sequence may be undersampled and thus be incapable of representing the increased signal spectral content caused by modulation. For example, assume that the input voltage phasors include frequencies up to the K^{th} harmonic, and a K^{th} order inverse transform is used to produce the voltage sequence. As a result of the fixed sampling rate, a K^{th} order transform must also be used to convert the resulting current sequence back to the frequency domain. The current waveform bandwidth now exceeds the bandwidth of the transform due to modulation effects of the nonlinear element. Thus significant error can be introduced into the transform due to aliasing. The dilemma is that analysis using a sufficiently large frequency set will result in inefficient simulation.

Reduction of aliasing requires that a sufficiently large frequency set be used in the transformation. However a much smaller frequency set often suffices for the evaluation of the linear subcircuit and in error minimization of the determining equations. Thus the use of a *dual frequency set* harmonic balance can reduce simulation time while controlling aliasing. In section II we investigate the origin of aliasing and present a condition for determining the smallest acceptable transform frequency set which will reduce aliasing. This derivation is for periodic signals, however, the results can be extended for multiple incommensurate input tones if the transform is accomplished by a NFFT. Section III presents the results of some dual set analyses.

II. *ALIASING*

The spectral error introduced by aliasing during the discrete Fourier transform can be determined by examining the continuous Fourier transform of a sampled data signal. Consider, for example, the continuous periodic signal

$$v(t) = \sum_{k=-K}^{K} a_k \exp(jk\omega_0 t)$$

where $a_{-k} = \overline{a_k}$, the complex conjugate. We will examine the problems encountered in recovering the coefficients a_k from the discreet spectrum.

The spectrum is obtained by observing the signal over a finite period of time. This effect is modeled by multiplying the signal by a rectangular window distribution.

$$x(t,T) = H(t+T) - H(t-T)$$

where $H(t)$ is the unit Heaviside distribution.

Next the windowed waveform, $v_x(t)$, is sampled at discrete time instants. This may be accomplished by multiplying by a sampling impulse train $p(t,\tau_s)$. For periodic signals we may use equally spaced sampling intervals.

$$p(t,\tau_s) = \sum_{n=-\infty}^{\infty} \delta(t - n\tau_s)$$

The sampled data waveform,

$$\hat{v}_x(t) = p(t,\tau_s)v_x(t) = p(t,\tau_s)x(t,T)v(t)$$

is transformed to the frequency domain using the Fourier transform operator which is defined as

$$\mathcal{F}(\cdots) \triangleq \int_{-\infty}^{\infty} (\cdots)\exp(-j2\pi ft)dt$$

Time domain multiplication is equivalent to frequency domain convolution, so the spectrum of the signal is

$$\mathcal{F}\hat{v}_x(t) = \mathcal{F}p(t,\tau_s) * (\mathcal{F}x(t,T) * \mathcal{F}v(t)) \qquad (1)$$

Expanding (1) and sampling the frequency domain spectrum by evaluating at discrete frequencies $\omega = p\omega_0$, we get

$$\hat{V}_x(p\omega_0) = \mathcal{F}\hat{v}_x(t) = \frac{T}{2\pi\tau_s}\sum_{n=-\infty}^{\infty}\sum_{k=-K}^{K} a_k \mathrm{Sa}\left(\frac{T}{2}\left((p-k)\omega_0 - \frac{2\pi n}{\tau_s}\right)\right) \qquad (2)$$

where $Sa(\cdots)$ is the sampling function. A consequence of sampling is that the entire spectrum is replicated and is offset in frequency by multiples of $2\pi/\tau_s$. The interference of other spectra with the primary ($n = 0$) spectrum is called *aliasing*. To insure that the coefficient a_p is not corrupted by adjacent spectra, we can see from (2) that we must satisfy the condition

$$\tau_s \leq \left(\frac{\tau_0}{K+p+1}\right) \qquad (3)$$

If we wish to recover all coefficients including the $p = K-1$ term, then (3) reduces to the standard sampling theorem. This result permits selection of an appropriate Fourier transform size based upon estimated bandwidth and simulation accuracy requirements.

Aliasing during simulation of circuits which are driven by multiple incommensurate tones can be understood by applying the previous ideas to a multidimensional frequency space. If R incommensurate input frequencies are to be considered, then a R-dimensional complex frequency space can be constructed. The result of sampling and windowing are analogous to those discussed for the single input tone case with the operations of equation (2) performed in a hypervolume.

	1	3	7	15
1	8.880	8.920	8.921	8.921
3	9.063	9.120	9.124	9.124
7	9.062	9.121	9.125	9.125
15	9.061	9.121	9.125	9.125

Table 1:
Output power (dBm) for large signal analysis of a MESFET amplifier driven by two incommensurate tones vs. highest order of intermodulation used in the FFT for f_1 (row) and f_2 (column).

Table 1 illustrates the effect of aliasing in two dimensions during the large signal simulation of a MESFET amplifier. The amplifier was driven by two incommensurate input frequencies, f_1 with an input power of 1.0dBm, and f_2 with an input power of 3.0dBm. The amplifier was analyzed using a rectangular frequency truncation scheme with only first order intermodulation included in the Newton method and linear calculations, i.e. all positive analysis frequencies such that $f_{analysis} = (\pm n_1 f_1 \pm n_2 f_2)$ with $|n_1|, |n_2| \leq 1$, were used. The row and column heading indicate the order of intermodulation used by the 2-D FFT in the f_1 and f_2 directions respectively. The table entries represent the amplifier output power (dBm) at the frequency f_1. Aliasing is seen to effect the solution for transforms using less than seventh order intermodulation in each dimension (up to $7f_2 \pm 7f_1$).

An important, and expected, feature of multidimensional transforms is suggested in table 1. If a transform is of sufficiently high order in all dimensions but one, *all* frequency domain coefficients may be corrupted due to aliasing. Therefore the elimination of aliasing in multifrequency simulation necessitates the use of a large set of transform frequencies.

III. *RESULTS*

Dual frequency set harmonic balance has been incorporated into *FREDA* (FREquency Domain Analysis), a general nonlinear microwave circuit simulation program. FREDA uses a modification of the Sămanskii method to solve the system determining equations. Analysis can be performed using the Generalized Power Series method of Rhyne and Steer [4,5] or by harmonic balance using either the NFFT or the APDFT. The dual frequency set analysis oversamples the output waveforms of nonlinear elements to accommodate the expanded signal bandwidth. This technique may be applied to any type of Harmonic Balance simulator regardless of the optimization method used.

The relative sensitivity of a simulation to the two factors discussed in section I will depend upon the nature of the nonlinearities in the circuit, the input power level, and the impedance presented to each nonlinear element by the remainder of the circuit. There are two advantages to *dual frequency set analysis*:

1. In many cases it has been found that use of a *dual frequency set* simulation scheme will significantly reduce simulation time without substantial degradation of simulation accuracy. This is because a smaller analysis frequency set will require less time and memory for the linear circuit and numerical solution computations.

2. A general N-dimensional FFT algorithm can be used in the simulator. The most efficient transform algorithms require that the number of samples in each dimension be an integer power of two ($2^{(n-1)}-1$ a.c. frequencies in each dimension for a transform with 2^n samples in each dimension). Thus the use of a multidimensional FFT leads naturally to dual set analysis with a large transform frequency set and a reduced analysis frequency set.

   The FFT requires $O(n\log_2(n))$ operations for an n-element transform. Matrix based discreet transform methods require $O(n^2)$ operations [6]. Thus simulation time and memory will be reduced significantly.

A MESFET amplifier was simulated under single tone large signal conditions using the APDFT and both traditional and dual frequency set harmonic balance. The results are presented in figure 1. Similar results were observed for the first, second, and fourth harmonics. During traditional harmonic balance simulation the solution was observed to be stable for sixteen or more a.c. analysis and transform frequencies (solid lines). A dual set analysis was performed using the APDFT and sixteen constant a.c. transform frequencies with a variable number of analysis frequencies (broken line). Virtually the same result was obtained using only six analysis frequencies as for traditional simulation with sixteen frequencies. Dual set analysis using Sămanskii's method with six analysis frequencies required 907kb and 20.8sec on a *Micro Vax* 3000. Similar traditional analysis with sixteen a.c. frequencies required 1989kb and 135sec. If the FFT is used, a traditional simulation with fifteen a.c. frequencies required 1601kb and 8.5sec using a modified Sămanskii method. A similar dual frequency set FFT analysis with fifteen transform frequencies and six a.c. analysis frequencies required 623kb and 4.0sec.

A class-C BJT amplifier was simulated using *dual set analysis* and a modified Sămanskii method. The solution was observed to be stable fifteen or more transform frequencies and any fixed number of transform frequencies. Next, simulations were performed with 15 transform frequencies while the number of analysis frequencies was varied from 4 through 15. The relative solution error for the output power at the fourth harmonic versus relative simulation time are plotted in figure 2. Error is a function of the number of analysis frequencies used.

The two dimensional MESFET simulation of table 1 gives some indication of the usefulness of *dual set analysis* for simulation of circuits with multitone excitation. The simulation result with a third order transform in the f_1 direction and seventh order for f_2 (with only first order intermodulation in the analysis) is 8.174mw (9.124dBm). This result was obtained via the modified Sămanskii method using 687kb and 24sec. Traditional harmonic balance simulation using third and fourth order intermodulation (use of higher order intermodulation for f_2 exceeded memory limits of the computer) required 32 analysis frequencies, 8987kb, and 170sec. The output power for f_1 was 8.491mw. The two results are within 4% of each other. Use of a triangular analysis frequency truncation scheme (rather then rectangular) may help to reduce the number of unnecessary frequencies used in multitone simulations. This feature has not been implemented yet.

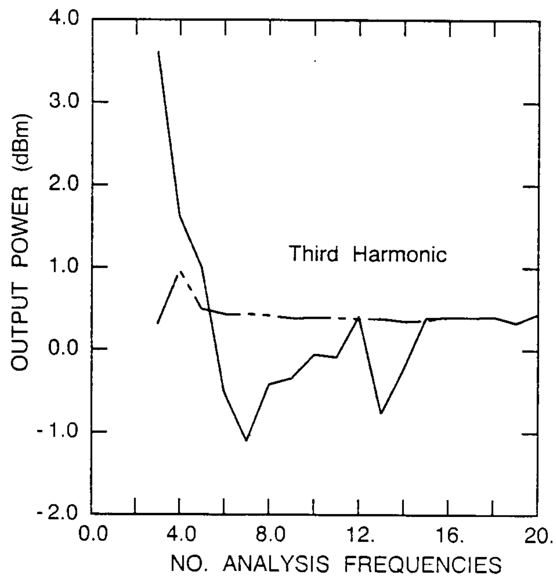

Figure 1:
MESFET simulation results using the APDFT with traditional (solid line) and dual frequency set (broken line) analysis.

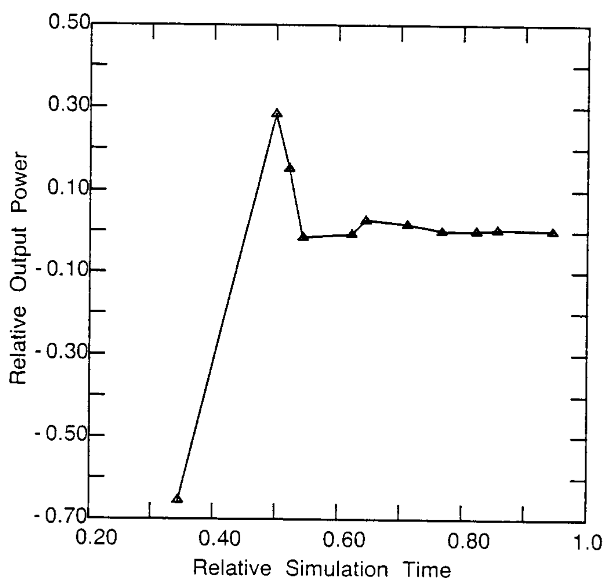

Figure 2:
BJT amplifier simulation using the FFT with dual frequency set analysis and a variable number of analysis frequencies.

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