SIMULATION OF NONLINEAR RF AND MICROWAVE CIRCUITS

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ABSTRACT

Nonlinear analysis methods for RF and microwave circuits can be classified as time domain, frequency domain, or as hybrid (time and frequency domain) depending on how the linear and nonlinear elements are analyzed. Some promising techniques include harmonic balance methods and series methods. These approaches are reviewed and BJT and MESFET amplifiers are analyzed using harmonic balance (a hybrid method) and generalized power series analysis (a frequency domain method). Results of the analyses are compared.

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I. INTRODUCTION

With the progress of manufacturing techniques in monolithic integration of microwave circuits, computer-aided-design has become an essential tool for the microwave circuit design engineer. A predominant interest and the most troublesome problem faced in the design of analog microwave circuits is the analysis of nonlinear circuits having sinusoidal excitation. At present, the methods of nonlinear circuit analysis can be classified into three categories, depending on how the linear and nonlinear circuit elements are handled: time-domain methods, where all elements are analyzed in the time domain; frequency-domain methods, where all elements are analyzed in the frequency domain; and hybrid methods, including the harmonic balance methods, which are combinations of time and frequency-domain techniques.

Most time domain nonlinear circuit simulations are not oriented toward RF and microwave applications, since the only available means of accurately computing and measuring linear microwave components is in the frequency domain under sinusoidal excitation. Furthermore, distributed linear microwave components, such as transmission lines, are represented in the frequency domain via simple algebraic relationships. Analysis based on the direct integration of the time domain network equations would devote much of its computational effort to transient evaluation, while most of the engineer's interest is concentrated on steady-state information. Consequently, harmonic balance techniques which interface the efficient frequency domain analysis of the linear part of a circuit with conventional time domain analysis of the nonlinear part of a circuit are increasingly being used. Recently, a new numerical nonlinear analysis technique based on modified power series descriptions of the nonlinear elements working entirely in the frequency domain has been presented [1]. In this paper, we compare several different aspects such as efficiency, accuracy and memory requirements of these two nonlinear circuit analysis techniques. These two techniques have been implemented in the program FREDA2, which is a CAD tool used to simulate both autonomous and nonautonomous microwave analog circuits. The theoretical basis of FREDA2 is used to present a unified discussion and comparison of harmonic balance and a frequency domain series technique (generalized power series analysis — GPSA) of nonlinear RF and microwave circuits.

II. BASIC THEORY

The basic idea of nonlinear circuit analysis is to separate the entire circuit into a linear subcircuit and a nonlinear subcircuit, and then to analyze each subcircuit separately. The linear subcircuit contains all the linear elements, and voltage and current sources, and the nonlinear portion includes all the nonlinear elements. These two subcircuits are connected with L different nodes; the number L will be determined by the characteristics of the whole network, such as the number of admittance type nonlinear elements (e.g., resistors), impedance type nonlinear elements (e.g., inductors), sources and the requirements for output information. Following separation of the network, the linear and nonlinear subcircuits can be analyzed separately. The required values of node voltage and branch current at the interface of the subcircuits are obtained for each subcircuit. Using these values and Kirchhoff's laws, the steady state "balance" point of the whole system can be determined.

The modified nodal admittance matrix is used to analyze the linear subcircuit [2]. Using this matrix and estimated node voltages and branch currents at the linear/nonlinear interface, the induced driving voltage and current sources, v and i, of the linear subcircuit can be obtained. Comparison of the induced voltages v and currents i of the linear subcircuit with the induced voltages v' and currents i' of the nonlinear subcircuit results in the objective function $E = \sum | v - v' |^2 + \sum | i + i' |^2$. The balance point of the whole system can be obtained by finding the zero of this objective function.

To demonstrate how we obtain the system equations in FREDA2 from Kirchhoff's laws, let us simplify our system to only one nonlinear element as shown in Figure 1. In Figure 1, i and v are induced current and voltage of the linear subcircuit and i' and v' are induced current and voltage of the nonlinear element. The nonlinear element can be either an admittance type or impedance type.

Assume $I_{\omega k}$, $V_{\omega k}$, $I'_{\omega k}$ and $V'_{\omega k}$ represent the phasor forms of i, v, i' and v' at a particular radian frequency ω_k, and consider N different ac frequency components where the different ac frequencies are not necessarily harmonically related. For a nonlinear admittance type element, the induced current component of frequency ω_k is a function of all input voltage components $V'_{\omega k}$ $(k = -N, \ldots, -1, 0, 1, \ldots, N)$ across this element :

$$I'_{\omega k} = f(V'_{-\omega N}, \ldots, V'_{-\omega 1}, V'_{\omega 0}, V'_{\omega 1}, \ldots, V'_{\omega N}) \tag{1}$$

and

$$V_{\omega k} = V'_{\omega k} \tag{2}$$

For a nonlinear impedance type element, the induced voltage component of frequency ω_k is a function of all input current components $I'_{\omega k}$ $(k = -N, \ldots, -1, 0, 1, \ldots, N)$ which flow through this element :

$$V'_{\omega k} = g(I'_{-\omega N}, \ldots, I'_{-\omega 1}, I'_{\omega 0}, I'_{\omega 1}, \ldots, I'_{\omega N}) \tag{3}$$

and

$$I_{\omega k} = -I'_{\omega k} \tag{4}$$

where functions $f(x)$ and $g(x)$ can be determined either in the time domain (hybrid harmonic balance method) or in the frequency domain (GPSA).

To formulate the system solutions, Kirchhoff's current law and Kirchhoff's voltage law must be satisfied. In other words, we need to find the zero of the objective function

$$E = \sum_{k=0}^{N} | I_{\omega k} + I'_{\omega k} |^2 + \sum_{k=0}^{N} | V_{\omega k} - V'_{\omega k} |^2 \tag{5}$$

Equation (5) is applied to each node which connects the linear and nonlinear subcircuits with one nonlinear element in the nonlinear subcircuit. In general, if we have L different nodes between the two subcircuits and M different nonlinear impedance type elements in the nonlinear subcircuit, the system objective function will be

$$E = \sum_{k=0}^{N} \left(\sum_{p=1}^{L} | I_{p,\omega k} + I'_{p,\omega k} |^2 + \sum_{q=1}^{M} | V_{q,\omega k} - V'_{q,\omega k} |^2 \right) \tag{6}$$

which is usually minimized using Newton's method. For Newton's method, we minimize the objective function E with respect to $\mathbf{x}$ using the iterative procedure

$${}^{i+1}\mathbf{x} = {}^{i}\mathbf{x} - \mathbf{J}^{-1}({}^{i}\mathbf{x})\mathbf{f}({}^{i}\mathbf{x}) \tag{7}$$

where the leading superscripts are iteration numbers and the matrix $\mathbf{J}$ is the Jacobian matrix.

The simulation algorithm just described is implemented in FREDA2 and summarized in Figure 2.

Generalized Power Series Analysis

The generalized power series is a time-domain power series with the addition of order-dependent time delays and frequency-domain complex coefficients so that the frequency domain output of the system is represented by

$$Y_{\omega q} = A \sum_{l=0}^{\infty} \left[a_l \left\{ \left[\sum_{k=0}^{N} b_k x_k (t - \tau_{k,l}) \right]^l \right\}_{\omega q} \right], \tag{8}$$

where $Y_{\omega q}$ is the output component of the system at frequency ω_q, $\{f(x)\}_{\omega q}$ represents the phasor form of the ω_q component of the time domain function $f(x)$, l is the order of the power series terms, a_l is a complex coefficient in frequency domain, $\tau_{k,l}$ is a time delay that depends on both power series order and the index of the input frequency component, and b_k is a real coefficient in time domain. The power series representation of nonlinearities allows efficient calculation of the frequency domain output signal at frequencies of interest.

Two different GPSA methods are used by FREDA2 to convert the time domain power series $\left[\sum_{k=0}^{N} x_k(t)\right]^l$ into the frequency domain component $\left\{\left[\sum_{k=0}^{N} x_k(t)\right]^l\right\}_{\omega q}$. One is based on direct calculation and is called the Arithmetic Operator Method [3] (GPSA-AOM), the other method is the Table Method [4] (GPSA-TM). GPSA-AOM is the more efficient method to use in calculating strongly nonlinear circuits with single frequency excitation. GPSA-TM, however, is more efficient at handling nonlinear circuits with multifrequency excitation.

GPSA has the advantage of retaining the time-domain description of the nonlinearities but requires no explicit time-domain calculations to calculate the frequency-domain representation for the output. In addition, a formula for the derivatives of all the output components with respect to the input quantities can be calculated directly. Generalized power series differs from ordinary power series in that it uses complex frequency domain coefficients and time delays. This enables a broad class of nonlinear circuits and systems to be described. However, some restrictions which apply to the ordinary power series also apply to the generalized power series. Some nonlinear expressions, such as exponential curves which apply to p-n junctions, are difficult to represent accurately with power series, since the significant terms of that power series are not just the first several terms. Increasing the order l of the power series can always improve the accuracy, but will degrade the simulation efficiency too.

Hybrid Harmonic Balance Method

In harmonic balance analysis the linear subcircuit is analyzed in the frequency domain while the nonlinear subcircuit is analyzed in the time domain. The objective function is formed after the result of the time domain nonlinear analysis is converted to the frequency domain via a Fourier transform [5]. Linear elements are handled identically by most frequency domain analysis techniques and time domain calculation for the nonlinear element is a simple application of the algebraic model. Much work has been performed on the transform which links the calculated time domain waveform to the frequency domain coefficient vector [7,9] — this topic will now be discussed.

Without loss of generality we can consider the circuit of Figure 1. For illustrative purposes assume that the time domain current through the nonlinear element is expressed as a function of the time domain potential across it, i.e., the element is an admittance (the analysis of general nonlinear elements with algebraic constitutive relations follows in a straight forward manner). We may write this relationship as

$$i'(t) = f(v'(t)) \tag{9}$$

If the steady state solution is assumed to be almost periodic, the time domain current can be written in terms of a countable number of harmonic series:

$$i'(t) = \prod_{l=1}^{L} \Lambda_l$$

$$\Lambda_l = \sum_{m=0}^{\infty} \left(A_{l,m}\cos(m\omega_l t) + B_{l,m}\sin(m\omega_l t)\right) \tag{10}$$

Equation (10) can be rewritten in the form

$$i'(t) = \sum_{k=0}^{R} \left(I_k^c \cos(\omega_k t) + I_k^s \sin(\omega_k t)\right) \qquad R \to \infty \tag{11}$$

where I_k^c and I_k^s are real numbers. To perform analysis we must truncate (11) to a finite number of terms even though, physically, an infinite number of harmonics will be generated. It is common to select R to include frequencies generated by mixing below a maximum intermodulation order [6,7]. A unique relationship between the coefficients I_k^c, I_k^s and $i'(t)$ is desired. The required *independent* equations can be generated by evaluating (11) at B discreet instants in time, τ_b. Here it is assumed that $\omega_0 = 0$. The resulting system of equations can be expressed as

$$\mathbf{i}' = \mathbf{\Gamma}^{-1}\mathbf{I}' \tag{12}$$

$$\mathbf{I}' = \mathbf{\Gamma}\mathbf{i}' \tag{13}$$

where

$$\begin{aligned} \mathbf{i}' &= [i'(\tau_b)] = [i'(\tau_1)\, i'(\tau_2) \ldots i'(\tau_B)]^T \\ \mathbf{I}' &= [I_k^c\, I_k^s] = [I_0^c\, I_1^c\, I_1^s \ldots I_R^c\, I_R]^T \end{aligned} \tag{14}$$

$$\mathbf{\Gamma}^{-1} = \begin{bmatrix} 1 & \cos(\omega_1\tau_1) & \sin(\omega_1\tau_1) & \ldots & \cos(\omega_R\tau_1) & \sin(\omega_R\tau_1) \\ 1 & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & \cos(\omega_1\tau_B) & \sin(\omega_1\tau_B) & \ldots & \cos(\omega_R\tau_B) & \sin(\omega_R\tau_B) \end{bmatrix} \tag{15}$$

The frequency domain nonlinear current is now available from (9), (12) and (13).

$$\mathbf{I}' = \mathbf{\Gamma} f(\mathbf{\Gamma}^{-1}\mathbf{V}') \tag{16}$$

Equations (12) and (13) define the inverse and forward transform. If the network has a single tone input, ω_1, then we can replace ω_k with $k\omega_1$ and use a constant sampling interval $\tau_b = b\,\Delta t$. We can select the number of time samples $B = 2R + 1$. Under these conditions $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^{-1}$ are square matrices and the operations (12) (13) become the standard discreet Fourier transform. For this strictly periodic circuit response, fast transform methods may be used to efficiently perform these operations [10].

If multiple input tones are present $\mathbf{\Gamma}^{-1}$ may be ill conditioned. Inversion of (15) will then introduce significant numerical error. Three methods have been successfully used to deal with poor conditioning of $\mathbf{\Gamma}^{-1}$.

Ushida et. al. [6] use a constant sampling interval but select $2R + 1 \leq B \leq 3(2R + 1)$. Thus, $\mathbf{\Gamma}^{-1}$ becomes a rectangular matrix. The frequency domain coefficients are determined by premultiplying (12) by $(\mathbf{\Gamma}^{-1})^T$.

$$(\mathbf{\Gamma}^{-1})^T \mathbf{i}' = (\mathbf{\Gamma}^{-1})^T \mathbf{\Gamma}^{-1} \mathbf{I}'$$

This is solved using the least squares method.

Rizzoli [8] uses a multidimensional Fourier transform. For two input tones ω_1 and ω_2, each tone is considered to be an independent variable and a two dimensional sample matrix is constructed. Application of a 2-D FFT results in a 2-D frequency coefficient matrix, which contain coefficients for the frequencies $p\omega_1 + n\omega_2$ where p, n are the matrix indices. Sample times are unequally spaced and selected to force the correct periodicity for each analysis frequency. This method is efficient because an FFT algorithm can be used and has been extended to the three tone input case [11]. The use of a multidimensional transform matrix forces the use of a 'rectangular' truncation scheme.

FREDA2 uses a ***near orthogonal*** time point selection algorithm, which was proposed by Kundert et. al. [7]. Considering the rows of $\mathbf{\Gamma}^{-1}$ as vectors, a good selection of time points will result in the rows of (15) being nearly mutually orthogonal which results in a well conditioned matrix. This $\mathbf{\Gamma}^{-1}$ matrix is constructed as follows: a collection of $B > 2R + 1$ rows is constructed using random sample times and a pivot row is selected and normalized. A dot product is taken between the normalized row and each other row to determine the degree of each row's orthogonality to the pivot. The collection of rows is then sorted using orthogonality as a criterion. The row which is most nearly normal becomes the next pivot row. This process is continued until $B = 2R + 1$ rows, and the associated sample times, have been selected.

The use of Newton's method to solve the determining equations requires that frequency domain derivatives be calculated for the Jacobian. These can be obtained from time domain calculations by use of the chain rule for differentiation and by exploiting the linearity of the transform operation. The derivatives may be found using

$$\frac{\partial \mathbf{I}'}{\partial \mathbf{V}'} = \mathbf{\Gamma} \frac{\partial \mathbf{i}'}{\partial \mathbf{v}'} \mathbf{\Gamma}^{-1} \tag{17}$$

The matrix derivative is of the form

$$\frac{\partial \mathbf{i}'}{\partial \mathbf{v}'} = \left[\frac{\partial i'(\tau_b)}{\partial v'(\tau_d)} \right] \; ; b, d = 1, 2, \ldots B \tag{18}$$

and its calculation places constraints on the type of time domain model which can be used in the analysis. The model must be algebraic, i.e. have no memory, which results in

$$\frac{\partial i'(\tau_b)}{\partial v'(\tau_d)} = 0 \; ; b \neq d$$

The matrix (18) cannot be found for elements with memory. Nonlinear elements having ordinary differential or integral constitutive relations can be handled using the equivalence of multiplication by $j\omega$ in the frequency domain to time domain differentiation. This property permits use of nonlinear inductors and capacitors. The multiplication by $j\omega$ supresses transients and permits steady state solution.

III. SIMULATION RESULTS and DISCUSSION

The class C BJT amplifier and the class A MESFET amplifier of Figure 3 and Figure 5 were simulated. Each circuit represents a different type of nonlinearity — strongly and moderately nonlinear respectively. The BJT amplifier was simulated with a single input frequency and the MESFET amplifier was also simulated with three incommensurate input tones.

The number of ac frequencies considered, N, in each circuit simulation is the major factor affecting accuracy and computer run time. After each iterative process, all induced ac output components which are not included in the N frequencies are assumed to be negligible. Therefore, simulation with a lower N will generally produce higher errors.

The BJT class C amplifier is considered to be more highly nonlinear than the MESFET amplifier, since the maximum ac input voltage used is larger than the dc bias base-emitter voltage. Power series which are used to describe nonlinear elements of the BJT were up to 18th order in this example. Simulation results, as shown in Figure 4 and table 1, show that with increasing N, simulation errors of the GPSA while using the Arithmetic Operator Method drop faster than the error of the harmonic balance method. However, the computer run time of the harmonic balance method is much less than the time required by the GPSA method. The reason for the difference in accuracy is that fourier transforms in the harmonic balance method introduces aliasing when N is too low. For lower N, GPSA always obtains higher accuracy than does harmonic balance. However, in each iterative convergence process, the GPSA Arithmetic Operator Method has to calculate the induced current or voltage of each nonlinear element using complex multiplications up to the 17th or 18th order, while the harmonic balance method can get the same results from just one analytical relation, the constituitive equation. Circuits with highly nonlinear components and periodic outputs, such as the BJT class B or class C amplifiers, require longer simulation time when using the GPSA method. In this case, the hybrid harmonic balance method is much more efficient than GPSA method.

The second example, an RF MESFET amplifier, represents a moderately nonlinear circuit with a periodic output signal. Table 2 shows that both GPSA and Harmonic Balance method have similar performance when comparing memory requirements, computer run time, and accuracy. The highest order of the power series required in this circuit is 7.

For the third circuit simulation, three incommensurable frequencies, 3GHz, 3.243GHz and 3.449GHz, are chosen as the inputs of the RF MESFET amplifier. Simulation results are shown in Figure 6 and table 3. In this example, the GPSA Table method is more efficient than harmonic balance with respect to accuracy, computer run time and memory requirements. Accuracy was determined by comparison of the values of the IF output power (the 243MHz component), simulation errors can reach acceptable levels with $N = 4$ using GPSA method, while harmonic balance method needs $N = 26$ to decrease the error to the same level. This discrepancy arises because GPSA intrinsicly retains higher intermodulation order products. For example the GPSA Table method can account for the effect of third order intermodulation on the IF for a circuit with three input frequencies using $n = 3$ and $N = 4$, while harmonic balance requires $N = 31$. Thus, for the same simulation accuracy, the GPSA method is faster than the harmonic balance method for this case.

V. CONCLUSION

In this paper, some general concepts of microwave nonlinear analog circuit analysis and both GPSA and harmonic balance techniques have been reviewed. A BJT class C amplifier and MESFET amplifiers having one and three incommensurable input frequencies are used to compare the performances of these two different techniques. In general, hybrid harmonic balance method performs better in strongly nonlinear circuits, and the GPSA method will dominate in any circuit which has several incommensurable input frequencies. We propose that a hybrid simulator, mixing harmonic balance and generalized power series analysis is required for RF and microwave circuit simulation.

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	Memory (Kbytes)		Computer Time(seconds)		Fundamental Power(dbm)	
N	GPSA	HB	GPSA	HB	GPSA	HB
02	216	—	013	—	4.48147	—
03	258	217	022	04	3.42565	5.77175
04	284	231	039	06	2.84273	4.29847
05	310	249	059	10	2.57679	4.42579
06	372	297	089	12	2.50879	2.08840
07	478	325	130	22	2.52956	2.50594
08	520	361	173	19	2.53803	2.54132
09	560	387	215	22	2.53729	2.61638
10	598	415	272	25	2.53263	2.71859
12	—	474	—	31	—	2.52075
14	—	690	—	36	—	2.28547
16	—	817	—	54	—	2.60666
18	—	920	—	69	—	2.56813

Table 1: Comparison of memory usage, simulation time, and simulated fundamental output for a BJT amplifier analyzed using generalized power series Arithmetic Operator Method and harmonic balance. Computer run time are for VAXstation 2000.

	Memory (Kbytes)		Computer Time(seconds)		Fundamental Power(dbm)	
N	GPSA	HB	GPSA	HB	GPSA	HB
02	220	218	05	04	0.145898	0.143930
03	240	232	06	05	0.145966	0.145627
04	256	256	08	07	0.145987	0.145951
05	316	314	10	08	0.145987	0.145954
06	342	340	12	11	0.145987	0.145954
07	382	368	13	13	0.145987	0.145951
08	412	414	16	15	0.145987	0.145952
09	440	444	17	18	0.145987	0.145954
10	608	610	19	20	0.145987	0.145953
12	702	707	22	28	0.145987	0.145956

Table 2: Comparison of memory usage, simulation time, and simulated fundamental output for a MESFET amplifier analyzed using generalized power series Arithmetic Operator Method and harmonic balance. Computer run time are for VAXstation 2000.

N	Memory (Kbytes)		Computer Time(seconds)		IF Power(dbm)	
	GPSA	HB	GPSA	HB	GPSA	HB
04	0450	—	011	—	−39.6632	—
06	0654	—	016	—	−39.6632	—
09	1814	0799	050	026	−39.6074	−48.1242
12	2923	1062	106	036	−39.6074	−46.8312
16	—	1790	—	062	—	−40.6979
21	—	2487	—	101	—	−40.9009
26	—	2791	—	159	—	−39.3873
31	—	3497	—	239	—	−39.5181

Table 3: Comparison of memory usage, simulation time, and simulated IF output power for a MESFET amplifier with three incommensurable input frequencies. GPSA is in Table Method. Computer run time are for VAXstation 2000.

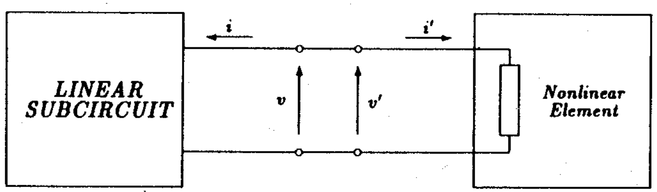

Figure 1: A network partitioned into a linear and nonlinear subcircuit.

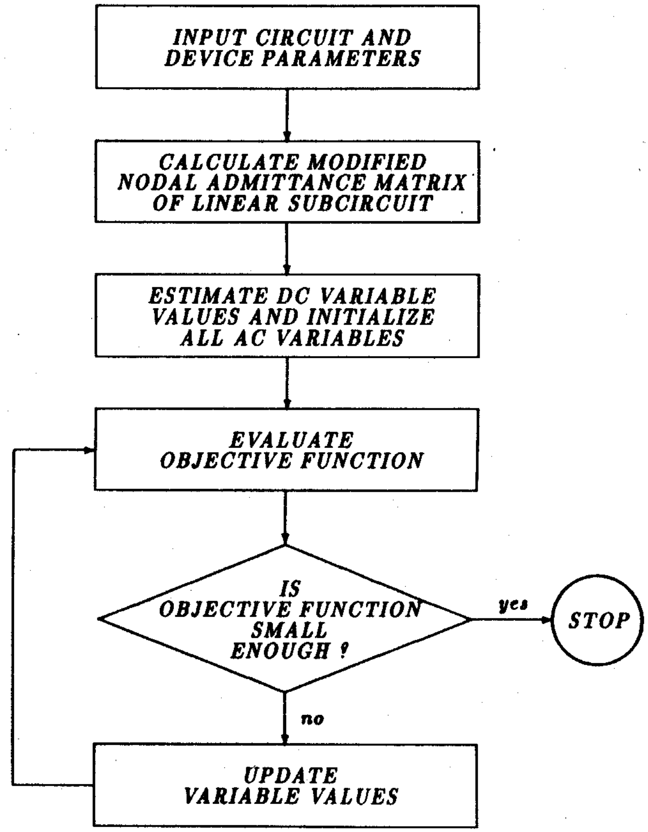

Figure 2: General algorithm for nonlinear circuit simulation.

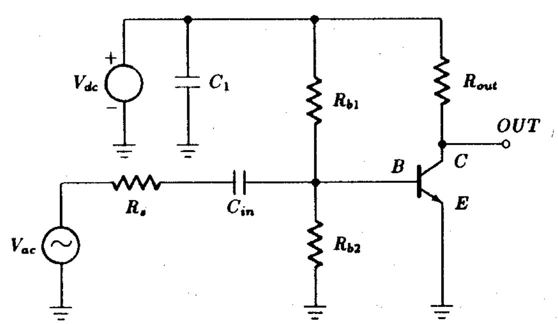

Figure 3: Schematic of BJT class C amplifier.

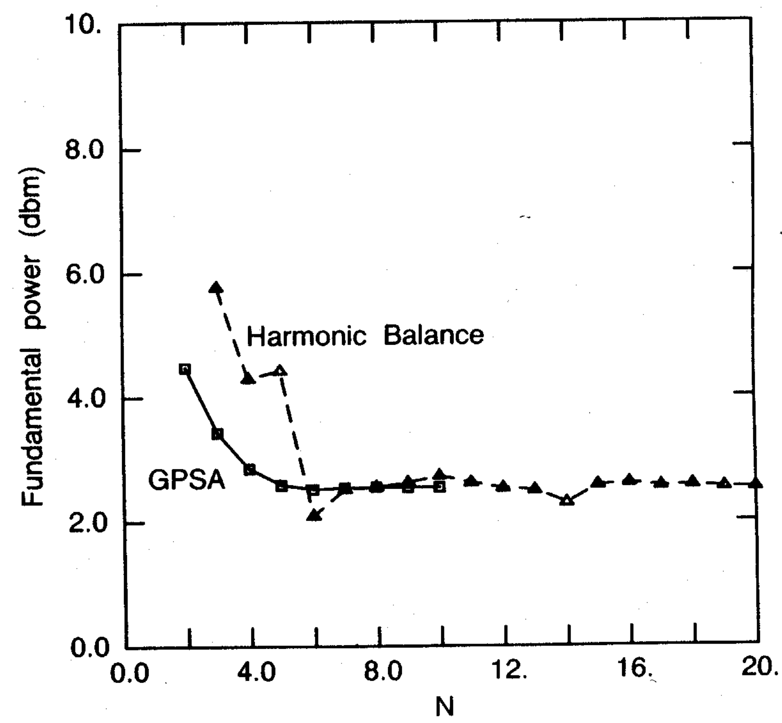

Figure 4: Output power at the fundamental frequency vs. number of frequencies used in the analysis of a BJT amplifier.

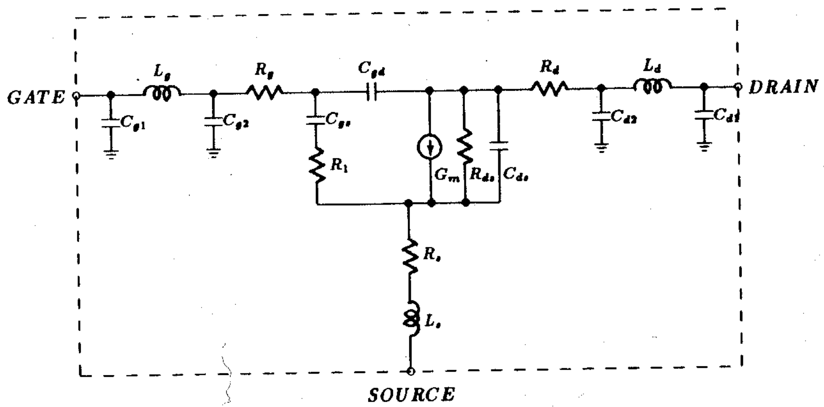

Figure 5: Circuit used to model the MESFET. Includes linear as well as nonlinear elements. Nonlinear elements include C_{gs}, C_{ds}, G_m, and R_{ds}.

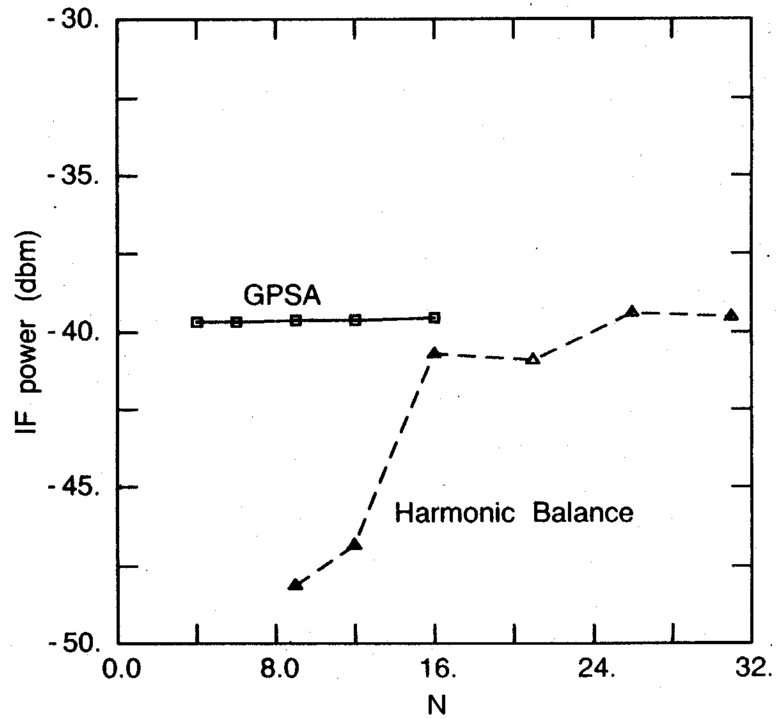

Figure 6: IF power vs. number of analysis frequencies for a MESFET amplifier with three incommensurable input frequencies.