Intermodulation Analysis of Mixers

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This paper presents a method for the large-signal, frequency domain analysis of mixers. The numerical results obtained for a single diode mixer are compared to those obtained using Sonine's expansion of the diode characteristics and those obtained experimentally.

1. INTRODUCTION

The purpose of this paper is to present a method for the large signal frequency domain analysis of microwave resistive mixers. The method is based on the calculation of individual intermodulation products using an algebraic formula previously derived by the authors [1].

Mixers involve the frequency components $f_1, \ldots, f_N$, and all sum and difference frequencies, of the original components $n_1 f_1 + \ldots n_N f_N$; where the n_k's are any positive or negative integers. Note that many intermodulation products, IP's, contribute to a frequency component. One of the frequencies (usually the largest) is designated as the local oscillator frequency LO, and the frequency component to be mixed RF.

Mixer analyses can be categorized into the frequency domain, time domain and two-state-switch approaches depending on how the nonlinear diode is handled.

Frequency domain analyses have involved obtaining expressions for the IP's, which are in turn summed to obtain the overall response of the system. These IP's have been determined from the expansion of the diode-circuit equation

$$I = I_s \{\exp [cU - I(R + r_s)] - 1\} \qquad (1)$$

where I_s, r_s and c are the diode's reverse saturation current, spreading resistance and nonlinearity index respectively. The other symbols are defined in figure 1.

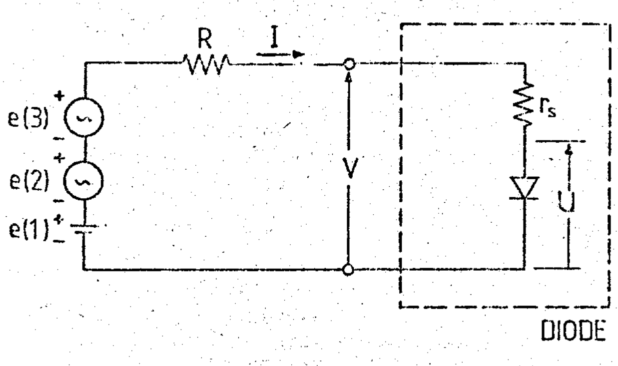

Figure 1 Frequency-independent mixer circuit.

Orloff [2] and Gretsch [3] have shown that, if R and r_s are ignored, the IP's can be obtained as a product of Bessel functions.

Some workers, again ignoring R and r_s, have expanded the diode equation into either a power series or a Taylor series. This is followed either by derivation of a multi-dimensional Fourier transform [4, 5] or by substitution of a sum of sinusoids into a power series [6] to determine the IP's.

Rutz-Phillip [7] and Beane [8] used the special recursive polynomials of Mills [9] to expand (1) in a power series. Subsequently a Fourier series was obtained.

The frequency domain analyses above, in attempting to obtain a closed form solution to the mixer problem, have ignored any frequency dependence in the mixer circuit. Thus the analyses are limited to circuits containing resistors and one or more diodes.

Most time-domain analyses initially solve for the LO waveform at the diode although two-state-switch analyses assume a square LO waveform. The LO waveform involves the LO and its harmonics only. Using the waveform a small signal conversion matrix, for the RF and the intermodulation frequencies is obtained [10, 11]. These methods are restricted to small-signals (apart from the LO), as these signals are applied to a linear time-varying network, rather than to a nonlinear element. An apparent exception to this is the recent work of Akaike and Onishi [12]. However, here the RF is a subharmonic of the LO and thus the total device waveform is periodic.

Hines [13] has pointed out that, for a proper solution, the classical time-domain treatment above leads to incorrect results through neglect of conversion of RF power to dc. He further states that a frequency domain method is required to incorporate such conversion.

Here the diode mixer circuit of figure 2 is analyzed using an harmonic balance technique. For the mixer the results obtained using our algebraic formula are compared to those obtained using the Sonine expansion of the diode characteristic (similar to that of Gretsch [3]). Good agreement is obtained with the three sets of results. It is seen that the power series approach is preferable to that using Sonine's expansion.

2. DEVELOPMENT OF MIXER ANALYSIS

In a previous paper [1] the authors obtained an algebraic formula for the complex amplitude of any frequency component of the output, y, of a nonlinear system described by the power series

$$y = A \sum_{\ell=0}^{\infty} a_\ell x^\ell$$

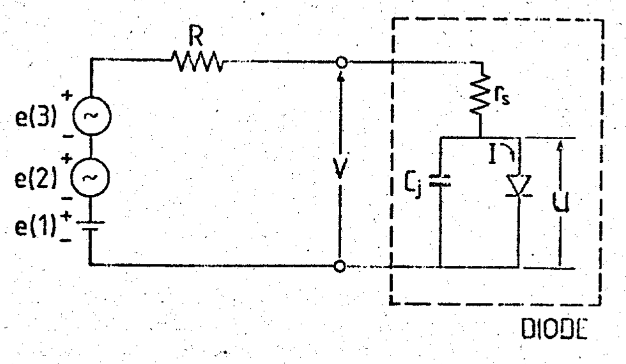

Figure 2 Mixer circuit, e(1) = 1V, dc; e(2) = 1.4Vp, 1100 Hz; e(3) = .1Vp, 900 Hz.

where the input is given by

$$x = \sum_{k=1}^{N} |x_k| \cos(\omega_k t + \theta_k) = \sum_{k=1}^{N} x_k \cos \omega_k t$$

If y_ω is the component of y at frequency ω then

$$y_\omega = \sum_{IP} [Re]_\omega \, \varepsilon_n A \left[\prod_{k=1}^{N} X_k^{|n_k|} \right] . T$$

Here the summation is over all intermodulation product descriptions, IPD's that is, sets of n_k's satisfying

$$\omega = \sum_{k=1}^{N} n_k \omega_k .$$

Now $n = \sum_{k=1}^{N} |n_i|$; $X_k = \begin{cases} x_k & \text{if } n_k \geq 0 \\ x_k & \text{if } n_k < 0 \end{cases}$

$$T = \sum_{\sigma=0}^{\infty} \frac{a_{n+2\sigma}(n+2\sigma)!}{2^{(n+2\sigma)}} . z \qquad (2)$$

and $z = \sum_{\substack{s_1,\ldots,s_N \\ s_1+\ldots+s_N=\sigma}} \prod_{k=1}^{N} \frac{|x_k|^{2s_k}}{s_k!(|n_k|+s_n)!}$

$[Re]_\omega$ is defined such that the real part of the right hand side is taken for ω=0 and ignored otherwise.

For a system with noninteracting inputs and outputs it is only necessary to apply the formula once. For a system with interacting inputs and outputs, such as a two terminal nonlinear element with current as input and voltage as output, it is necessary to iterate between the current/voltage solution of the nonlinear element and that of the external linear circuit. (A suitable iteration procedure is described by Hicks and Khan [14]).

When iterating, the parameter T (see equation (2)) tends to change slowly, particularly as convergence is approached. Thus it need not be calculated during every iteration. As convergence is approached the interval between the calculations of T can be increased, giving up to an order of magnitude reduction in computation time.

The ideal diode characteristic

$$I = I_s \{\exp [cU] - 1\} \qquad (3)$$

was expanded into a power series of current in terms of ac voltage about the dc value of U. The use of this power series eliminates dc as one of the input frequencies and thus considerably reduces the number of intermodulation product descriptions, set's of n_k's, required.

3. RESULTS

The mixer circuit of figure 2, incorporating a low frequency model of a 100 GHz Schottky barrier diode was constructed. The element values, r_s and C_j, of the diode were obtained experimentally by Held and Kerr [11]. (Note that the impedance of 1 μF at 1 kHz is equal to that of 10 nF at 100 GHz). The ideal diode characteristic (3) was used where I_s and c were determined from the diode's dc characteristic.

The experimental waveform and spectrum of the voltage, V, at the diode terminals are given in figures 3 and 4 respectively.

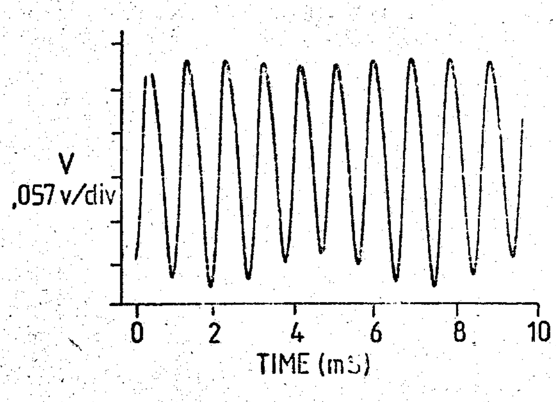

Figure 3 Experimental waveform of V of figure 2.

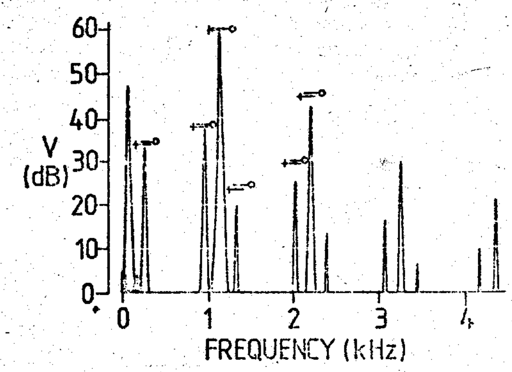

Figure 4 Spectrum of V of figure 2; solid line - experimental, Vdc = .523V; +--- algebraic formula, Vdc = .514V; 0--- Sonine expansion, Vdc = .509V.

The results obtained using the algebraic formula, indicated by crosses, are overlayed on the experimental spectrum, figure 4. In obtaining these results seven frequencies were considered: dc; 200 Hz; 900 Hz; 1100 Hz; 1300 Hz; 2000 Hz; and 2200 Hz. The frequency components of V were calculated to a fractional accuracy of .01. The initial values of current and voltage were taken as zero.

When the parameter T was calculated every iteration the solution required 61 iterations and 14.1s on a PDP-KL10 computer. Calculating T every iteration for the first three iterations and thereafter at every 20th iteration the corresponding figures are 83 iterations and 4.7s. In both cases 220 IP's were considered. The equivalent of 15 terms in the power series expansion was considered. (Note that .26 milliseconds were required per IP. This can be compared to the 1 minute per IP of Surana and Gardiner [4]).

Numerical results were also obtained by expanding the diode characteristic using Sonine's expansion. The results are given by the circles in figure 4. This method took 57 iterations and 14.1s to converge.

4. DISCUSSION

From figure 4 it can be seen that the two sets of numerical results are in very good agreement with each other and in good agreement with the experimental results. Any disagreement with the experimental results is probably due to the limited number of frequencies considered numerically. Other contributory causes are the experimental inaccuracies, particularly in determining c of equation (3).

The power series method converges faster than that using Sonine's expansion. However the major advantage of the power series method is that, for strong non-linearities or large signals, the Sonine expansion method behaves erratically and no information is available as to when this situation begins to occur. This behaviour has been traced to the inaccuracies involved in determining the Bessel functions of high order and large argument.

5. CONCLUSION

A power series method previously reported by the authors was applied to the analysis of a resistive mixer. The set of results obtained using this method was compared tothat obtained using the Sonine expansion of the diode characteristic. Good agreement was obtained for the three sets of results. It was seen that the power series approach is preferable to that using the Sonine expansion in terms of both numerical speed and accuracy of results.

The research reported here was supported by the Australian Research Grants Committee, Telecom Australia and the United States Army Research Office.

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